Proc. Pakistan Acad. Sci. 45(1):31-34.2008

KEPLERíS LAWS UNDER MODIFIED NEWTONIAN DYNAMICS

S.B. Faruque[1] and M.M. Huq Chowdhury

Department of Physics, Shahjalal University of Science and Technology, Sylhet 3114, Bangladesh

Received July 2007, accepted December 2007

Communicated by Prof. Dr. M. Iqbal Chaudhary

Abstract: We show that, despite its successes, the proposal of Romero and Zamora of modified Newtonian dynamics (MOND), alternative to that of Milgrom, leads to violation of Keplerís laws in the MOND limit. This also indicates that the modification of MOND proposed by Romero and Zamora suffers the same failing it was supposed to fix: non-conservation of momentum and energy.

Keywords: Modified Newtonian dynamics, Keplerís laws.

Introduction

Nowadays there are various observational results in astrophysics whose explanation represents a challenge for physics. One of the problems is to explain the rotation curves of galaxies. Observation indicates a relationship of the form $v^4 \propto M$, for the speed v of the distant stars in a galaxy of mass M. However, as the only acting force on these stars is gravity and their trajectories are circles, Newtonian dynamics manifests that the relationship to hold is $v^2 = GM/r$, where r is the distance from the star to the center of galaxy. This leads one to search for a gravity theory that can explain galaxy dynamics without the need for exotic dark matter. In 1983, Milgrom [1] made a remarkable observation called as Milgromís law which is the foundation for his modified Newtonian dynamics (MOND) that is the alternative to particle dark matter. By considering the behavior of the speed of the distant stars, Milgrom proposed a modification to Newtonís second law, now known as ìmodified Newtonian dynamicsî (MOND) which can solve the galaxy rotation curve problem. MOND is a purely phenomenological theory but it explains most of the galaxy rotation curves without introducing dark matter. Its simplicity is what it makes attractive. Extension to MOND at the level of gravitational field can be found in [2]. MONDs relation to dark matter can be found in [3] and its relation to cosmology can be found in [4].

Milgromís proposal of MOND comprises the following form of the Second Law of Motion [1]:

$$m\mu(z)\frac{d^2x^i}{dt^2} = F^i,\ i = 1,2,3; \qquad (1)$$

where $z = |\ddot{x}|/a_0$, $a_0 \approx 10^{-8}$ cm/s^2 and $\mu(z)$ is a function satisfying

$$\mu(z) = \begin{cases} 1, z >> 1 \\ z, z << 1. \end{cases} \qquad (2)$$

One can show that this gives, for circular orbits in Newtonian gravitational field, a velocity satisfying $v^4 = a_0GM$. In spite of all of its successes, MOND is not free from limitations. A crucial problem is the lack of conserved quantities, like angular momentum and energy, in the MOND limit [2], i.e. when $\mu(z) = z$.

*Corresponding author
E-mail: awsbf62@yahoo.com

To overcome this particular conservation problem, Romero and Zamora [5] have recently proposed a MOND equation alternative to Eq.(1) which is

$$m\mu(z)\frac{d^2x^i}{dt^2}+m\frac{\mu'(z)}{2}\dot{z}\frac{dx^i}{dt}=F^i,\ i=1,2,3; \quad (3)$$

where the terms are of the same meaning as before. In the Newtonian limit, i.e when μ (z) = 1, Eqs. (1) and (3) are identical. In the MOND limit, Eqs. (1) and (3) become identical only for circular trajectories, and differ in any other case. We show in this short paper that Eq. (3) violates the second and third laws of planetary motion, i.e. Keplerís laws are not valid once Eq. (3) is regarded to be true. One thing to be noted here is that Milgromís MOND, i.e. Eq. (1) does not lead to such violation of Keplerís laws. This amounts to the fact that the modification proposed by Romero and Zamora suffers the same failing it was supposed to fix: non-conservation of angular momentum and energy. We emphasize on obeying the Keplerís laws by any gravitational dynamics for these laws are the foundations of universal gravitation. Once these laws are violated, then one can consider that there are serious problems underneath.

Keplerís laws under MOND of Romero and Zamora

The first conserved quantity which resembles angular momentum in the Newtonian limit that issues from Eq. (3) is the following:

$$L=\sqrt{z}mr^2\dot{\varphi}=\text{constant}, \quad (4)$$

where we have considered orbits only in the equatorial plane. Clearly, in the MOND limit, the areal velocity is not constant along trajectory since one can show that Eq. (4) leads to

$$\frac{d}{dt}\left(\frac{dA}{dt}\right)=-\frac{L\dot{z}}{4z^{3/2}}, \quad (5)$$

where A is the area subtended by the trajectory of the particle in the orbit. Clearly, Eq. (5) shows that areal velocity is conserved only for circular trajectories, for which $\dot{z}=0$. For elliptic orbits, areal velocity is not a conserved quantity, violating Keplerís second law. Note that if Milgromís MOND is true then the particle does not violate Keplerís second law. This feature is further indicated by Keplerís third law following from Eq. (3) and the second conserved quantity which resembles the total energy in the Newtonian limit. This conserved quantity is given by

$$E=\frac{m}{2}\mu(z)\frac{dx^i}{dt}\frac{dx^i}{dt}+U(x). \quad (6)$$

where Ù is the period and a is the semi-major axis and k=GMm. Note that we are assuming an equivalent one body problem of the two-body problem defined by the masses M and m, where m<<M, wherefore the mass m is the reduced mass essentially. Now, Eq. (7) is exactly the Keplerís third law in the Newtonian limit, i.e. when we replace z by μ(z) and equate it to unity. However, in the MOND limit z is a variable and we no longer get $\tau^2 \propto a^3$, required by Keplerís third law statement. That is, once again Keplerís one law is being violated. But for circular orbits we recover the third law of Kepler from Eq. (7).

Conclusion

We have shown that the alternative proposal of Romero and Zamora [5] of modified Newtonian dynamics, i.e. Eq. (3) leads to violation of Keplerís law of areas and the law of period. Hence, we can say that the proposal of Romero and Zamora does not cure the problem of non-conservation of angular momentum and energy associated with the original modified Newtonian dynamics of Milgrom Eq. (1). Only for circular trajectories the proposal of Romero and Zamora conforms with Keplerís laws. Hence, we can say that it carries partial conservation of energy and angular momentum. In conclusion, the proposal of Romero and Zamora suffers the same lacking there was in the original proposal of Milgrom.

References

1. **Milgrom, M.** 1983. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophys. J.* 270:365-370.
2. **Bekenstein, J.D.** 2004. Relativistic gravitation theory for the modified Newtonian dynamics paradigm. *Phys. Rev. D* 70:083509.
3. **Kaplinghat, M. and Turner, M.** 2002. How cold dark matter theory explains Milgromís law. *Astrophys. J.* 569:L19-L22.
4. **Sanders, R.H.** 1998. Cosmology with modified Newtonian dynamics (MOND). *Mon. Not. R. Astron. Soc.* 296:1009-1018.
5. **Romero, J.M. and Zamora, A.** 2006. Alternative proposal to modified Newtonian dynamics. *Phys. Rev. D* 73:027301.
6. **Thornton, M.** 1995. *Classical Dynamics of Particles and Systems*. Harcourt, Inc., New York.