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WEAK AND STRONG FORMS OF FUZZY IRRESOLUTE MAPS

R.K.Saraf*, Seema Mishra, M.Caldas*****

Department of Mathematics Govt. K.N.G. College Damoh (M.P.) India,Department of Mathematics, Govt. D. B. GirlsíP.G. College, Raipur (C.G.), India, ***Departmento De Matematica Aplicada Universidade Federal Fluminense Ruamario, Santos Braga s/n CEP 24020-140 Niteroi, RJ Brasil*

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Abstract: We consider new weaker and stronger forms of irresolute and semi closed maps via the concepts of Fsg-closed sets, which we call Fap irresolute maps, Fap-semi closed maps, and fuzzy contra -irresolute maps, and use them to obtain a characterization of Fuzzy semi $T_{1/2}$ spaces.

Keywords**:** Fuzzy Topological spaces, Fsg- closed sets, Fuzzy semi Open sets, Fuzzy semi closed Maps, Fuzzy irresolute maps.2000 Mathematics Subject Classification: 54A40.

Introduction

Ever since the introduction of fuzzy sets by Zadeh [1] and fuzzy topological spaces by Chang [2] various notions in classical topology have been extended to fuzzy topological spaces .The concept of generalized fuzzy semi closed sets was introduced by Balasubramanian, Chandrasekar [3], later the same concept was studied by Tapi, Thakur, and Rathore [4] under the name Fuzzy semi generalized closed sets. In 2000 M.Caldas [5] defined and studied weak and strong forms of irresolute maps in General Topology.

In this paper we introduce the concept of irresoluteness called Fap-irresolute maps and Fap-semi closed maps by using Fsg-closed sets and study some of their basic properties this definition enables us to obtain conditions under which maps and inverse maps preserve sg-closed sets. Also, in this paper we present a new generalization of irresoluteness called fuzzy contra irresolute map, we define this last class of maps by the requirement that the inverse image of each fuzzy semi open set in the co-domain is fuzzy semi closed in the domain. This notion is a stronger form of Fap-irresoluteness. Finally, we also characterize the class of fuzzy semi $T_{1/2}$ ñ spaces in terms of Fap- irresolute and Fap-semi closed maps.

Email: ** mkseema@yahoo.com :*** gmamccs@vm.uff.br

Throughout this work (X,τ),(Y,σ) and (Z,γ) stand for fuzzy topological spaces (in short fts),with no separation axioms assumed unless otherwise stated. Let A be a fuzzy subset of X, the closure of ìAî and the interior of ìAî will be denoted by Cl (A) and Int (A), respectively.

Preliminaries

For the concept of fuzzy topological spaces we refer to Chang [2], since we require the following known definitions, notations and some properties, we recall them in this section.

***Definition* 2.1**

A fuzzy set A of (X, τ) is said to be fuzzy semi generalized closed (Fsg-closed) in (X, τ) [3] if $sCl(A) \leq H$ whenever $A \leq H$ and H is fuzzy semi open in (X,τ).A fuzzy set is said to be semi generalized open (Fsg-open) in (X,τ) [3] if its complement B^c =X ìB is Fsg-closed in (X,τ).

***Definition* 2.2**

A fuzzy set ìAî is fuzzy semi open [6] if there exists $H \in \tau$ if $H \leq A \leq (Cl(H))$. The semi interior [7] of A denoted by sInt (A) is defined by the union of all semi open sets of (X,) contained in A.

***Remark* 2.3**

A fuzzy set A is fuzzy semi open iff A = sInt (A) [6] By FSO (X,τ) we mean the collection of all fuzzy semi open sets in (X, τ).

***Definition* 2.4**

A fuzzy set B of (X,τ) is said to be fuzzy semi closed [6] if its complement B^c is fuzzy semi open in (X,τ) The semi closure [7] of a set B of (X,τ) denoted by sCl(B), is defined to be the intersection of all fuzzy semi closed sets of (X,τ), containing B.

***Remark* 2.5**

A fuzzy set B is fuzzy semi closed iff sCl(B) =B [6]

***Definition* 2.6**

A map f: (X,τ) (Y, σ) is called fuzzy irresolute [7] if $f^{ì1}$(A) is fuzzy semi open in (X,τ) for every A ∈ FSO (Y,σ).

***Definition* 2.7**

A map f: (X,τ) (Y,σ) is called fuzzy pre semi-closed [8](respectively fuzzy pre semi-open)[9] if for every fuzzy semi -closed (res. Fuzzy semi open) set B in (X, τ), f (B) is fuzzy semi-closed (resp. fuzzy semi-open) in (Y,σ).

Fap-Irresolute, Fap-Semi-closed and Fuzzy contra ñirresolute maps

Let f: (X,τ) (Y,σ) be a map from fuzzy topological spaced (X,τ) in to a fuzzy topological space (Y,σ)

***Definition* 3.1**

A map f: (X,τ) (Y,σ) is said to be fuzzy approximately irresolute (or Fap-irresolute) if sCl (A)≤$f^{ì1}$ (B) whenever B is a fuzzy semi open set of (Y,σ). A is Fsg-closed set of (X,τ) and A ≤ $f^{ì1}$ (B).

***Definition* 3.2**

A map f: (X,τ) (Y,σ) is said to be fuzzy approximately irresolute (Fap-semi-closed) if f (B)≤ sInt (A) whenever A is an Fsg-open set of (Y,σ). B is fuzzy semi-closed set of (X, τ) and f (B) ≤A. Clearly fuzzy irresolute maps are Fap-irresolute and fuzzy pre semi-closed maps are Fap-semi-closed, but not conversely. The proof follows from definition 3.1 and definition 13 [3], {resp. Definition 3.2 and proposition 17 [3]}. The following examples show that the converse implications do not hold.

***Example* 3.3**

Let X = {a, b}, define τ = {0, A, B, C, D, E, 1} and σ={0, A, B, C, D,1}, where A,B,C,D,E : X → (0,1) are such that:

A (a) = D (a) = D (b) = B (b) = 0.3,
A (b) = B (a) = C (a) = C (b) = 0.4, E (a) = E (b) = 0.5,

The identity mapping f: (X,τ) → (Y,σ) satisfies the condition of Fap -irresolute mapping but not that of fuzzy irresolute.

***Example* 3.4**

Let X = {x, y, z} Define τ = {0, 1, A} and σ = {0,1,A, B}, where A, B: X → (0,1) is such that

A (x) = 0, A (y) = 0.3, A (z) = 0.2;
B (x) = 0.9, B (y) = 0.6, B (z) = 0.7.

The identity mappings f: (X,τ) → (Y,σ) satisfies the condition of Fap-semi-closed, but not that of fuzzy-semi-pre-closed.

***Theorem* 3.5**

(1) f: $(X,\tau) \rightarrow (Y,\sigma)$ is Fap-irresolute if f^{-1} (A) is fuzzy semi-closed in (X, τ) for every A$\in$ FSO (Y, σ).

(2) f : $(X,\tau) \rightarrow (Y,\sigma)$is Fap-semi-closed if f(B) $\in$ Fso(Y,σ) for every fuzzy semi-closed set B of (X,τ).

Proof

(1) Let E $\leq f^{-1}$(A), where A $\in$ FSO(Y,σ) and E is Fsg closed set of X therefore, sCl(E) $\leq$ sCl(f^{-1} (A)) = f^{-1} (A). Thus f is Fap-irresolute.

(2) Let f (B) $\leq$ A where B is a fuzzy semi-closed set of X, and A is Fsg-open set of Y.

Therefore, sInt f (B) $\leq$ sInt (A). Then f (B) $\leq$ sInt(A).Thus f is Fap-semi-closed.

***Observation* 3.6**

Converse of the Theorem 3.5 does not hold. Example 3.3 and 3.4 serve the purpose. In the following Theorem, we, under certain conditions, get that the converse of the Theorem 3.5 is true.

***Theorem* 3.7**

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a map from a fuzzy topological space (X,τ) in a fuzzy topological space (Y,σ).

(1) If the fuzzy semi-open and fuzzy semi-closed sets of (X,τ) coincide, then f is Fap-irresolute if and only if f^{-1} (A) is fuzzy semi closed in (X,τ) for every A $\in$ FSO (Y,σ).

(2) If the fuzzy semi-open and fuzzy semi-closed sets in (Y,σ) coincide, then f is Fap-semi-closed if and only if f (B) $\in$FSO (Y,σ) for every fuzzy semi-closed set B of (X,τ)

Proof

(1) Assume f is Fap irresolute. Let A be an arbitrary set of (X,τ) such that A $\leq$ Q where Q $\in$ FSO (X,τ), then by hypothesis sCl (A) $\leq$ sCl (Q) = Q. Therefore all sets of X are Fsg-closed (and hence all are Fsg-open). So, for any A $\in$ FSO (Y,), f^{-1} (A) is Fsg -closed in (X, τ). Since f is Fap-irresolute, sCl (f^{-1} (A)) $\leq f^{-1}$ (A). Therefore sCl (f^{-1} (A)) = f^{-1} (A), i.e. f^{-1} (A) is fuzzy semi-closed in (X,τ) .The converse is clear by Theorem 3.5

(2) Assume f is fap-semi-closed. Reasoning as in (1), we obtain that all sets of (Y,σ) are Fsg-open, therefore for any fuzzy semi-closed set B of (X, τ) f(B) is Fsg-open in Y. Since f is Fap-semi-closed, f (B) $\leq$ sInt (f (B)). Therefore f (B) = sInt (f (B)), i.e. f(B) is fuzzy-semi-open. The converse is clear by Theorem 3.5 .

As immediate consequence of Theorem 3.7, we have the following,

***Corollary* 3.8**

f: $(X,\tau) \rightarrow (Y,\sigma)$ be a map such that:

(1) If the fuzzy semi open and fuzzy semi-closed sets of (X,τ) coincide, then f is Fap-irresolute if and only if f is fuzzy irresolute.

(2) If the fuzzy semi open and fuzzy semi-closed sets of (Y,σ) coincide, then f is Fap-semi-closed if and only if f is fuzzy pre-semi-closed.

***Definition* 3.9**

A map f: (X,τ) (Y,σ) is called fuzzy contra - irresolute if f^{-1} (A) is fuzzy semi-closed in (X, τ) for each A $\in$ FSO(Y,σ).

***Definition* 3.10**

A map f: (X,τ) (Y,σ) is called fuzzy contra

-pre-semi-closed if f(B) $\in$ FSO (Y, σ), for each fuzzy semi closed set B of (X, τ).

***Observation* 3.11**

Fuzzy contra irresoluteness and fuzzy irresoluteness are independent notions. Example 3.3 shows that fuzzy contra-irresoluteness does not imply fuzzy irresoluteness. The following example shows that fuzzy irresoluteness does not imply fuzzy contra irresoluteness.

***Example* 3.12**

Let X = {a, b} and Y = {x, y}. Fuzzy sets A and H are defined as:

A (a) = 0.6, A (b) = 0.7; H (x) = 0.7, H (y) = 0. 8.

Let τ = {0,A, 1} and σ = {0, H, 1} then the mapping f: (X,τ) $\rightarrow$ (Y,σ) defined by f(a) = x and f (b) = y is fuzzy irresolute but not fuzzy contra irresolute. In the same manner, we can prove that fuzzy contra-pre-semi-closed maps and fuzzy pre-semi-closed maps are independent notions. Following result can easily be verified; the proof is straightforward.

***Theorem* 3.13**

Let f: (X,τ) (Y,σ) be a map. Then the following conditions are equivalent.

(1) f is fuzzy contra-irresolute.

(2) The inverse image of each fuzzy-semi-closed set in Y is fuzzy semi-open in X.

***Observation* 3.14**

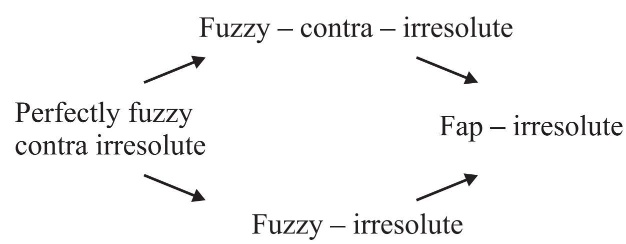
By Theorem 3.5 we have that every fuzzy contra-irresolute map is fap-irresolute and every fuzzy contra-pre-semi-closed map is Fap-semi closed, but the converse implication does not hold.

***Definition* 3.15**

A map f: (X,τ) (Y,σ) is called perfectly fuzzy-contra-irresolute if the inverse image of every fuzzy semi-open set in Y is fuzzy semi-clopen in X.

***Observation* 3.16**

Every perfectly fuzzy contra -irresolute map is fuzzy contra irresolute. Clearly, the following diagram holds and none of its implications are reversible.



The next two theorems establish conditions under which maps and inverse maps preserve fuzzy semi-generalized-closed sets.

***Theorem* 3.17**

If a map f: (X,τ) (Y,σ) is Fap-semi irresolute and fuzzy pre-semi closed then for every Fsg-closed set E of (X,τ), f(E) is Fsg-closed set of (Y,σ).

Proof

Let E be Fsg-closed set of (X,τ). Let f(E) $\leq$ H where H $\in$ FSO(Y,σ). Then E $\leq$ f^{-1}(H) holds. Since f is Fap-irresolute sCl(E) $\leq$ f^{-1}(H), and hence f(sCl(E)) $\leq$ H. Therefore, we have sCl(f(E)) $\leq$ sCl(f(sCl(E))) = f(sCl(E)) $\leq$ H. Hence f(E)is Fsg-closed in Y.

***Theorem* 3.18**

Let f: (X,τ) $\rightarrow$ (Y,σ), g: (Y, σ) $\rightarrow$(Z,γ) be two maps such that gof: (X, τ) $\rightarrow$(Z,γ), then

(1) gof is fuzzy contra-irresolute, if g is irresolute and f is fuzzy contra irresolute.

(2) gof is fuzzy contra irresolute if g is fuzzy contra irresolute and f is fuzzy irresolute.

Proof

The proof of this theorem is easy and hence it is omitted. In an analogous way, we have the following.

***Theorem 3*.19**

Let f: $(X,\tau) \to (Y,\sigma)$ g: $(Y,\sigma) \to (Z,\gamma)$ be two maps such that gof: $(X,\tau) \to (Z,\gamma)$, then

(1) gof is Fap-semi-closed, if f is fuzzy pre semi-closed and g is Fap-semi-closed.

(2) gof is Fap-semi-closed, if f is Fap semi closed and g is fuzzy pre-semi-open and g^{-1} preserves Fsg-open sets.

(3) gof is Fap-irresolute, if f is Fap-irresolute and g is fuzzy irresolute.

Proof

(1) Suppose B is fuzzy semi-closed set in (X,τ) and A is Fsg-open subset o (Z,γ) and, (gof): (B) $\le$ A. Then f (B) is fuzzy semi-closed in Y because f is fuzzy pre-semi-closed. Since g is Fap-semi-closed, g (f(B)) $\le$ sInt(A). This implies gof is Fap-semi-closed.

(2) Suppose B is fuzzy semi-closed subset of (X,τ) and A is Fsg-open set of (Z,γ) for which, (gof) (B) $\le$ A. Hence f(B) $\le g^{-1}$ (A). Then f(B) $\le$ sInt(g^{-1}(A)), because g^{-1} (A) is Fsg-open and f is Fap-semi-closed. Thus (gof)(B) = g (f (B)) $\le$ g (sInt (g^{-1}(A))) $\le$ sInt (g g^{-1}(A)) $\le$ sInt (A). This implies that gof is Fap-semi-closed map.

(3) Suppose E is Fsg-closed subset of (X,τ) and H $\in$ FSO (Z,γ), for which E $\le$ $(gof)^{-1}$ (H). Then g^{-1}(H) $\in$ FSO (Y) because g is fuzzy irresolute. Since f is Fap-irresolute, sCl (E) $\le f^{-1}(g^{-1}(H)) = (gof)^{-1}(H)$. This proves that gof is Fap-irresolute.

We recall that a fuzzy topological space (X,τ) is said to be fuzzy semi-$T_{1/2}$-space [3] if every Fsg-closed set is fuzzy semi closed in it.

***Theorem* 3.20**

Let (X,τ) be a fuzzy topological space, then the following statements are equivalent:

(1) (X,τ) is fuzzy semi -$T_{1/2}$ space.

(2) For every fuzzy topological space (Y,σ) and every map f: $(X,\tau) \to (Y,\sigma)$ f is Fap-irresolute.

Proof:

(1) $\Rightarrow$ (2)

Let E be a Fsg-closed subset of (X,τ) and suppose that E $\le f^{-1}$ (H), where H $\in$ FSO (Y). Since, (X,τ) is fuzzy semi $T_{1/2}$ space, E is fuzzy semi-closed (i.e. E = sCl (E)). Therefore, sCl (E) $\le f^{-1}$ (H), Then f is Fap-irresolute.

(2) $\Rightarrow$ (1)

Let B be a Fsg-closed subset of (X,τ) and let Y be the set X with topology $\sigma = \{1,B,0\}$. Finally let f: $(X,\tau) \to (Y,\sigma)$ be the identity map. By assumption, f is Fap-irresolute, since B is Fsg-closed in (X,τ) and fuzzy semi-open in (Y,σ) and B $\le f^{-1}$(B). It follows that sCl(B) $\le$ B. Hence B is fuzzy semi-closed in (X,τ) and therefore it is fuzzy semi-$T_{1/2.}$

***Theorem* 3.21**

Let (Y,σ) be a fuzzy topological space. Then the following statements are equivalent:

(1) (Y,σ) is fuzzy semi-$T_{1/2}$ space.

(2) For every space (X, τ) and every map f: (X,τ) → (Y,σ) f is Fap-semi-closed.

Proof

Analogous to theorem 3.20 making the obvious changes.

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