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ON THE CONVERGENCE OF THREE-STEP ITERATION PROCESS WITH ERRORS IN THE CLASS OF QUASI-CONTRACTIVE OPERATORS

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Abstract: We establish a general theorem to approximate the unique common fixed point of three quasi-contractive operations on an arbitrary Banach space through the three-step iteration process with errors.

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Introduction and preliminaries

We recall the following definitions in a metric space (X,d). A mapping $T:X\to X$ is called an $a-\lim\limits_{x\to\infty}$ contraction if

$$d(Tx,Ty)\le ad(x,y) \quad \text{for all } x,y\in X, \text{ where } a\in(0,1). \tag{1.1}$$

The mapping T is called Kannan mapping [1] if there exists $b\in(0,\frac{1}{2})$ such that

$$d(Tx,Ty)\le b[d(x,Tx) + d(y,Ty)] \text{ for all } x,y\in X \tag{1.2}$$

A similar definition is due to Chatterjee [2], there exists $c\in(0,\frac{1}{2})$ such that

$$d(Tx,Ty)\le c[d(x,Ty) + d(y,Tx)] \text{ for all } x,y\in X \tag{1.3}$$

Combining these three definitions, Zamfrescu [3] proved the following important result.

Theorem 1

Let (X,d) be a complete metric space and $T:X\to X$a mapping for which there exists the real numbers a, b and c satisfying $a\in(0,1),b,c\in(0,\frac{1}{2})$ such that for each pair $x,y\in X$, at least one of the following conditions holds:

(z_1) $d(Tx,Ty)\le ad(x,y)$,
(z_2) $d(Tx,Ty)\le b[d(x,Tx)+d(y,Ty)]$,
(z_3) $d(Tx,Ty)\le c[d(x,Ty)+d(y,Tx)]$.

Then T has a unique fixed point p and the Picard iteration $\{x_n\}$ defined by

$$x_{n+1}=Tx_n,\ n\in\mathbb{N}$$

converges to p for any arbitrary but fixed $x_0\in X$.

An operator T satisfying the contractive conditions $(z_1),(z_2)$ and (z_3) in Theorem 1 is called Zamfirescu operator (Z-operator).

Let K be a nonempty convex subset of normed space E and T: $K\to K$ be a mapping. In recent years, Mann [4] and Ishikawa [5] iterative process have been studied extensively by many authors.

(a) The Mann iteration process is defined by the sequence $\{x_n\}$

$$\text{(M)}\quad \begin{cases} x_0\in K, \\ x_{n+1}=(1-b_n)x_n+b_nTx_n, n\ge 0, \end{cases}$$

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where $\{b_n\}$ is sequence in [0,1].

(b) The sequence $\{x_n\}$ defined by

$$\text{(I)}\begin{cases} x_0 \in K, \\ x_{n+1} = (1-b_n)x_n + b_n T\, y_n \\ y_n = (1-b'_n)x_n + b'_n T x_n,\ n \geq 0, \end{cases}$$

where $\{b_n\}, \{b'_n\}$ are sequence in [0,1] is known as the Ishikawa iteration process.

In 1995, Liu [6] introduced iterative processes with errors as follows:

(c) The sequence $\{x_n\}$ iteratively defined by

$$\text{(LM)}\begin{cases} x_0 \in K, \\ x_{n+1} = (1-b_n)x_n + b_n T\, x_n + u_n,\ n \geq 0 \end{cases}$$

where $\{b_n\}$ is a sequence in [0,1] and $\{u_n\}$ a sequence in K satisfying $\sum \|u_n\| < \infty$, is known as Mann iterative process with errors.

(d) The sequence $\{x_n\}$ in K iteratively defined by:

$$(LI)\begin{cases} x_0 \in K, \\ x_{n+1} = (1-b_n)x_n + b_n T\, y_n + u_n, \\ y_n = (1-b'_n)x_n + b'_n T x_n + v_n,\ n \geq 0, \end{cases}$$

where $\{b_n\}, \{b'_n\}$ are sequences in [0,1] and $\{u_n\}, \{v_n\}$ are sequences in K satisfying $\sum \|u_n\| < \infty$, $\sum \|v_n\| < \infty$, is known as Ishikawa iterative process with errors.

While it is clear that consideration of error terms in iterative process is an important part of the theory, it is also that the iterative process with errors introduced by Liu [6], as in (c) and (d) above, is not satisfactory. The errors can occur in a random way. The conditions imposed on the error terms in (c) and (d) which say that they tend to zero as *n* tends to infinity are, therefore, unreasonable.

In 1998, Xu [7] introduced a more satisfactory error term in the following iterative process:

(e) The sequence $\{x_n\}$ iteratively defined by:

$$\text{(XM)}\begin{cases} x_0 \in K, \\ x_{n+1} = a_n x_n + b_n T\, x_n + c_n u_n,\ n \geq 0, \end{cases}$$

with $\{u_n\}$ a bounded sequence in K and $a_n + b_n + c_n = 1$, is known as Mann iterative process with errors.

(f) The sequence $\{x_n\}$ iteratively defined by:

$$\text{(XI)}\begin{cases} x_0 \in K, \\ x_{n+1} = a_n x_n + b_n T\, y_n + c_n u_n, \\ y_n = a'_n x_n + b'_n T x_n + c'_n v_n,\ n \geq 0, \end{cases}$$

with $\{u_n\}, \{v_n\}$ bounded sequences in K and $a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n$, is known as Ishikawa iterative with errors.

Note that the error terms are now improved; Mann and Ishikawa iterative follow as special cases of the above processes, respectively.

(g) The sequence $\{x_n\}$ defined by

$$\text{(N)}\begin{cases} x_0 \in K, \\ x_{n+1} = b_n T\, y_n + (1-b_n)x_n, \\ y_n = b'_n T z_n + (1-b'_n)x_n, \\ z_n = b''_n T x_n + (1-b''_n)x_n,\ n \in \mathbb{N}, \end{cases}$$

where $\{b_n\}, \{b'_n\}, \{b''_n\}$ are appropriate sequence in [0,1] is known as Noor iteration process [8].

Recently, Agarwal *et al.* [9] studied the following iteration process for a couple of quasi-contractive mappings.

(h) The sequence $\{x_n\}$ iteratively defined by

$$(RCHI)\begin{cases} x_0 \in K, \\ x_{n+1} = a_n x_n + b_n S\, y_n + c_n u_n, \\ y_n = a'_n x_n + b'_n T x_n + c'_n v_n,\ n \geq 0, \end{cases}$$

where $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}, \{c'_n\}$ are sequence in [0,1] such that $a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n$ and $\{u_n\}, \{v_n\}$ are bounded sequences in K.

(i) It can be easily seen that for $c_n = 0 = c'_n$, (RCHI) reduces to

$$(DDI)\begin{cases} x_0\in K,\\ x_{n+1}=(1-b_n)x_n+b_nS\,y_n,\\ y_n=(1-b'_n)x_n+b'_nTx_n,\ n\geq 0,\end{cases}$$

where $\{b_n\},\{b'_n\}$ are sequence in [0,1] and known as Das and Debata iteration process [10].

In 2004, Berinde [11] introduced a new class of operators on an arbitrary Banach space E satisfying

$$\begin{aligned}&\|Tx-Ty\|\leq\delta\|x-y\|\\&+2\delta\|x-Tx\|,\end{aligned}\qquad 1.4$$

for all $x,y\in E$ and $0\leq\delta<1$.

He proved that this class is wider than the class of *Z*-operators and used the Ishikawa iteration process (I) to approximate fixed points of this class of operators in an arbitrary Banach space given in the form of the following theorem:

Theorem 2

Let K be a nonempty closed convex subset of an arbitrary Banach space E. Let $T:K\rightarrow K$ be an operator satisfying (1.4). For $x_0\in K$, let $\{x_n\}$ be defined through the iterative process (I). If $F(T)\neq\varphi$ and $\sum b_n=\infty$, then $\{x_n\}$ converges strongly to the unique fixed point of *T*.

Inspired and motivated by the above said facts, we suggest the following three-step iterative process with errors as follows

$$(RAHN)\begin{cases} x_0\in K,\\ x_{n+1}=a_n\,x_n+b_nT^n{}_1\,y_n+c_nu_n,\\ y_n=a'_nx_n+b'_nT^n{}_2z_n+c'_nv_n,\\ z_n=a''_nx_n+b''_nT^n{}_3x_n+c''_nw_n,\ n\geq 0,\end{cases}$$

where $\{a_n\},\{a'_n\},\{a''_n\},\{b_n\},\{b'_n\},\{b''_n\},\{c_n\},\{c'_n\}$ and $\{c''_n\}$ are appropriate real sequences in [0,1] and $\{u_n\},\{v_n\},\{w_n\}$ are bounded sequences in *K*, to approximate the common fixed point of the three *Z*-operators. Our results generalize and improve upon, among others [9,2,1,12,13,14-17], the corresponding results of Berinde [11].

(j) For $T_1=T_2=T_3$, (RAHN) reduces to

$$(ARXN)\begin{cases} x_0\in K,\\ x_{n+1}=a_n\,x_n+b_nT\,y_n+c_nu_n,\\ y_n=a'_nx_n+b'_nTz_n+c'_nv_n,\\ z_n=a''_nx_n+b''_nTx_n+c''_nw_n,\ n\geq 0,\end{cases}$$

where $\{a_n\},\{a'_n\},\{a''_n\},\{b_n\},\{b'_n\},\{b''_n\},\{c_n\},\{c'_n\}$ and $\{c''_n\}$ are appropriate real sequences in [0,1] and $\{u_n\},\{v_n\},\{w_n\}$ are bounded sequence in *K*.

(k) For $c_n=0=c'_n$, (RAHN) reduces to

$$(ARN)\begin{cases} x_0\in K,\\ x_{n+1}=(1-b_n)x_n+b_nT_1^n\,y_n,\\ y_n=(1-b'_n)x_n+b'_nT_2^nz_n,\\ z_n=(1-b''_n)x_n+b''_nT_3^nx_n,\ n\geq 0,\end{cases}$$

where $\{b_n\},\{b'_n\},\{b''_n\}$ are sequences in [0,1].

Results

Lemma 1 [18]

Let $\{r_n\},\{s_n\},\{t_n\}$ and $\{k_n\}$ be sequences of nonnegative numbers satisfying

$r_{n+1}\leq(1-s_n)r_n+s_nt_n+k_n$ for all $n\geq 0$. If $\sum s_n=\infty,\lim_{n\rightarrow\infty}t_n=0$ and $\sum k_n<\infty$ hold, then $\lim_{n\rightarrow\infty}r_n=0$.

Theorem 3

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T_i:K\rightarrow K;i=1,2,3,$ be three *Z*-operators. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (RAHN). If $F(T_1)\cap F(T_2)\cap F(T_3)\neq\varphi,\sum b_n=\infty,\sum c_n<\infty$ and $\lim_{n\rightarrow\infty}c'_n=0=\lim_{n\rightarrow\infty}c''_n$, then $\{x_n\}$ converges strongly to a unique common fixed point of $F(T_1)\cap F(T_2)\cap F(T_3)$.

Proof

Since each $T_i;i=1,2,3$ is an *Z*-operator, so according to (1.4),

(B) $\|T_i x - T_i y\| \leq \delta \|x - y\| + 2\delta \|x - T_i x\|$,

For all $x, y \in E$ and $0 \leq \delta < 1$. First we prove the uniqueness, for this purpose let $w, w_1 \in F(T_1) \cap F(T_2) \cap F(T_3)$, then from (B) we have

$$\|w - w_1\| = \|T_i w - T_i w_1\|$$
$$\leq \delta \|w - w_1\| + 2\delta \|w - T_i w\|$$
$$= \delta \|w - w_1\|,$$

giving us $(1-\delta)\|w - w_1\| \leq 0$, which implies with the help of $0 \leq \delta < 1, w = w_1$ a contradiction. As $F(T_1) \cap F(T_2) \cap F(T_3) \neq \varphi$, so let $w \in F(T_1) \cap F(T_2) \cap F(T_3)$

Set

$$M = \max\{\sup_{n \geq 0} \|u_n - w\|, \sup_{n \geq 0} \|v_n - w\|, \sup_{n \geq 0} \|w_n - w\|\}$$

Using (RAHN), we have

$$\|x_{n+1} - w\| =$$
$$\|a_n x_n + b_n T_1 y_n + c_n u_n - (a_n + b_n + c_n) w\|$$
$$= \|a_n (x_n - w) + b_n (T_1 y_n - w) + c_n (u_n - w)\|$$
$$\leq a_n \|x_n - w\| + b_n \|T_1 y_n - w\| + c_n \|u_n - w\| \quad (2.1)$$

$$\leq (1 - b_n)\|x_n - w\| + b_n \|T_1 y_n - w\| + M c_n.$$

Now for $x = w$ and $y = y_n$, (B) gives

$$\|T_1 y_n - w\| \leq \delta \|y_n - w\| \quad (2.2)$$

In a similar fashion, we can get

$$\|y_n - w\| = \|a'_n x_n + b'_n T_2 z_n + c'_n v_n - (a'_n + b'_n + c'_n) w\|$$
$$= \|a'_n (x_n - w) + b'_n (T_2 z_n - w) + c'_n (v_n - w)\|$$
$$\leq a'_n \|x_n - w\| + b'_n \|T_2 z_n - w\| + c'_n \|v_n - w\|$$
$$\leq (1 - b'_n)\|x_n - w\| + b'_n \|T_2 z_n - w\| + M c'_n \quad 3.3$$

Using (B), for $x = w$ and $y = z_n$, we get

$$\|T_2 z_n - w\| \leq \delta \|z_n - w\|. \quad (2.4)$$

Also,

$$\|z_n - w\| = \|a''_n x_n + b''_n T_3 x_n + c''_n v_n - (a''_n + b''_n + c''_n) w\|$$
$$= \|a''_n (x_n - w) + b''_n (T_3 x_n - w) + c''_n (v_n - w)\|$$
$$\leq a''_n \|x_n - w\| + b''_n \|T_3 x_n - w\| + c''_n \|v_n - w\| \quad (2.5)$$
$$\leq (1 - b''_n)\|x_n - w\| + b''_n \|T_3 x_n - w\| + M c''_n.$$

Again by (B), if $x = w$, and $y = x_n$, we get

$$\|T x_n - w\| \leq \delta \|x_n - w\|. \quad (2.6)$$

From (2.1-2.6), we obtain

$$\|x_{n+1} - w\| \leq (1 - b_n)\|x_n - w\| +$$
$$\delta b_n [(1 - b'_n)\|x_n - w\| + \delta b'_n \|z_n - w\|$$
$$+ M c'_n] + M c_n$$
$$= [1 - b_n [1 - \delta(1 - b'_n)]] \|x_n - w\|$$
$$+ \delta^2 b_n b'_n \|z_n - w\| + \delta M b_n c'_n + M c_n$$
$$\leq [1 - b_n [1 - \delta(1 - b'_n)]] \|x_n - w\|$$
$$+ \delta^2 b_n b'_n [[1 - (1-\delta) b''_n \|x_n - w\|$$
$$+ M c''_n] + \delta M b_n c'_n + M c_n$$
$$= [1 - (1-\delta) b_n [1 + \delta b'_n (1 + \delta b''_n)]] \|x_n - w\|$$
$$+ \delta^2 M b_n b'_n c''_n + \delta M b_n c'_n + M c_n. \quad (2.7)$$

It may be noted that for $\delta \in [0,1)$ and $\{\eta_n\} \in [0,1]$, the following inequality is always true

$$1 \leq 1 + \delta \eta_n \leq 1 + \delta. \quad (2.8)$$

From (2.7) and (2.8), we get

$$\|x_{n+1} - w\| \leq [1 - (1-\delta) b_n] \|x_n - w\| +$$
$$\delta M b_n (\delta b'_n c''_n + c'_n) + M c_n.$$

By Lemma 1, with $\sum b_n = \infty$, $\sum c_n < \infty$ and $\lim_{n\to\infty} c'_n = 0 = \lim_{n\to\infty} c''_n$, we get that $\lim_{n\to\infty} \|x_n - w\| = 0$. Consequently $x_n \to w \in F(T_1) \cap F(T_2) \cap F(T_3)$ and this completes the proof. □

Corollary 1

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T_i : K \to K; i = 1, 2$, be two Z-operators. For

$x_0 \in K$, let $\{x_n\}$ be defined by the iterative process (RCHI). If $F(T_1)\bigcap F(T_2)\neq\varphi, \sum b_n=\infty$, $\sum c_n<\infty$ and $\lim_{n\to\infty} c'_n=0$, then $\{x_n\}$converges strongly to a unique common fixed point of $F(T_1)\bigcap F(T_2)$.

Corollary 2

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T_i:K\to K; i=1,2,3,$ be three Z-operators. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (ARN). If

$F(T_1)\bigcap F(T_2)\bigcap F(T_3)\neq\varphi$ and $\sum b_n=\infty$, then $\{x_n\}$ converges strongly to a unique common fixed point of $F(T_1)\bigcap F(T_2)\bigcap F(T_3)$.

Corollary 3

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T_i:K\to K$; $i=1,2,$ be two Z-operators. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (DDI). If $F(T_1)\bigcap F(T_2)\neq\varphi$ and $\sum b_n=\infty$, then $\{x_n\}$ converges strongly to a unique common fixed point of $F(T_1)\bigcap F(T_2)$.

Corollary 4

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T:K\to K$ be an Z-operator. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (ARXN). If

$F(T)\neq\varphi, \sum b_n=\infty, \sum c_n<\infty$ and $\lim_{n\to\infty} c'_n=0=\lim_{n\to\infty} c''_n$, then $\{x_n\}$ converges strongly to a unique fixed point of $F(T)$.

Corollary 5

Let K be nonempty closed convex subset of an arbitrary Banach space E and $T:K\to K$ be an Z-operator. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (XI). If $F(T)\neq\varphi, \sum b_n=\infty, \sum c_n<\infty$ and $\lim_{n\to\infty} c'_n=0$, then $\{x_n\}$ converges strongly to a unique fixed point of $F(T)$.

Corollary 6

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T:K\to K$ be an Z-operator. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (XM). If

$F(T)\neq\varphi, \sum b_n=\infty$ and $\sum c_n<\infty$, then $\{x_n\}$ converges strongly to a unique fixed point of $F(T)$.

Corollary 7

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T:K\to K$ be an Z-operator. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (N). If $F(T)\neq\varphi$ and $\sum b_n=\infty$, then $\{x_n\}$ converges strongly to a unique fixed point of $F(T)$.

Corollary 8

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T:K\to K$ be a Z-operator. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (I). If $F(T)\neq\varphi$ and $\sum b_n=\infty$, then $\{x_n\}$ converges strongly to a unique fixed point of $F(T)$.

Corollary 9

Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T:K\to K$ be an Z-operator. For $x_0\in K$, let $\{x_n\}$ be defined by the iterative process (M). If $F(T)\neq\varphi$ and $\sum b_n=\infty$,then $\{x_n\}$ converges strongly to a unique fixed point of $F(T)$.

Remarks

1. The contractive condition (1.1) makes *T* a continuous function on *X* while this is not the case with the contractive conditions like (1.2-1.3) and (1.4).

2. The Chatterjee's and the Kannan's contractive conditions (1.3) and (1.2) are both included in the class of *Z*-operators and so their convergence theorems for the Ishikawa iteration process are obtained in Corollary 8.

3. Theorem 4 of Rhoades [17] in the context of Mann iteration on a uniformly convex Banach space has been extended in Corollary 9.

4. In Corollary 9, Theorem 8 of Rhoades [19] is generalized to the setting of Banach spaces.

5. Our results also generalize Theorem 5 of Osilike [20] and Theorem 2 of Osilike [18].

References

1. **Kannan, R.** 1968. Some results on fixed points. *Bull. Calcutta Math. Soc.* 10:71-76.
2. **Chatterjee, S.K**. 1972. Fixed point theorems. *C. R. Acad Bulgara Sc*i. 25:727-730.
3. **Zamfirescu, T**. 1972. Fix point theorems in metric spaces, *Arch. Math.* (Basel) 23:292-298.
4. **Mann, W.R.** 1953. Mean value methods in iterations. *Proc. Amer. Math. Soc.* 4:506-510.

5 **Ishikawa, S.**1974. Fixed points by a new iteration method. *Proc. Amer. Math. Soc*. 44:147-150.

6. **Liu, L.S.** 1995. Ishikawa and Mann iterative process with errors for nonlinear strongly accretive mappings in Banach spaces. *J. Math. Anal. Appl.* 194:114-125.
7. **Xu, Y.** 1998. Ishikawa and Mann iteration process with errors for nonlinear strongly accretive operator equations, *J. Math. Anal. Appl*. 224:91-101.
8. **Noor, M.A.** 2001. Three-step iterative algorithms for mutlivalued variati-onal inclusions. *J. Math. Anal. Appl.* 255:1589-604.
9. **Agarwal, R.P., Cho, Y.J., Li, J. and Huang, N.J.** 2002. Stability of iterative procedures with errors approximating fixed points for a couple of quasi-contractive mappings in q-uniformly smooth Banach spaces. *J. Math. Anal. Appl.* 272:435-447.
10. **Das, G. and Debata, J.P**. 1986. Fixed points of Quasi-nonexpansive mappings. Indian *J. Pure Appl. Math.* 17:1263-1269.
11. **Berinde, V**. 2004. On the convergence of the Ishikawa iteration in the class of quasi contractive operators. *Acta Math. Univ. Comenianae* LXXIII 1**:**119-126.
12. **Kannan, R.** 1971. Some results on fixed points III., *Fund. Math.* 70:169-177.
13. **Kannan, R.** 1973. Construction of fixed points of class of nonlinear mappings. *J. Math. Anal. Appl.* 41:430-438.
14. **Rafiq, A**. 2006. A convergence theorem for Mann fixed point iteration procedure, *Appl. Math. E-Notes* 6:289-293.
15. **Rafiq, A.** 2006. On the convergence of the three step iteration process in the class of quasi-contractive operators, *Acta Mathemat. Academiae Paedagogicae Nyireghaziensis* 22:305-309.
16. **Rafiq, A**. 2006. Strong convergence of a modified implicit iteration process for a finite family of Z-operators. *Internat. J. Math. Math. Sci.* 2006,1-6, Article ID 10328.
17. **Rhoades, B.E**. 1974. Fixed point iteration using infinite matrices *Trans. Amer. Math. Soc.* 196:161-176.
18. **Osilike, M.O.** 1999. Short proofs of stability results for fixed point iteration procedure for a class of contractive-type mappings, *Indian J. Pure Appl. Math.* 30:1229-1234.
19. **Rhoades, B.E.** 1976. Comments on two fixed point iteration method. *J. Math. Anal Appl.* 56:741-750.
20. **Osilike, M.O**. 1995/1996. Stability results for fixed point iteration procedures. *J. Nigerian Math. Soc.* 14/15:17-29.