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MAGNETO-THERMO ELASTIC PLANE WAVES IN A ROTATING TRANSVERSELY ISOTROPIC MEDIUM

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Abstract: In this article we have extended the work from isotropic to transversely isotropic material. A dispersion relation in general form is derived, by using it with appropriate replacements the results for rotating and non-rotating isotropic materials are deduced.

Keywords: Magneto-thermo elasticity, dispersion relation, wave speed

1. Introduction

The study of wave propagation in elastic solids has a long and distinguished history. Schoenberg and Censor [1] have considered the effect of rotation on plane wave propagation in an isotropic medium. They showed that in a rotating isotropic medium three waves can propagate. They also showed that longitudinal or transverse wave can exist only if the direction of propagation and axis of rotation are either parallel or perpendicular. Wave propagation in a transversely isotropic solid has been discussed by a number of researchers [2-8] but Chadwick [2] has discussed it in detail. The effect of rotation does not increase the number of waves in transversely isotropic medium but plays a significant role on their speeds. Longitudinal waves can propagate along the axis of rotation and the plane normal to it, whereas transverse waves can also propagate along the coordinate axes.

Chandrasekharaiah and Srikantiah [9] have discussed thermo-elastic plane waves in a rotating isotropic solid. Chandrasekharaiah and Srinath [10] have discussed thermo-elastic plane waves without energy dissipation in a rotating isotropic body. Chadwick [12] discussed thermoelasticity, the dynamic theory, in progress in solid mechanics. Ahmad and Khan [13] have discussed the theory of thermo-elastic plane waves in a rotating isotropic material and find that in any given direction there are in general four waves. Three of them generalize the elastic waves discussed by Schoenberg and Censor [1], while the fourth is a thermal wave.

None of these waves is dilatational or transverse in character unless special propagation directions are considered. Their results are at variance with those of Chandrasekharaiah and Srikantiah [9], and Chandrasekharaiah and Srinath [10] who assume that the displacement vector of a dilatational or a transverse wave in a rotating medium can make an arbitrary angle with the axis of rotation.

Green and Naghdi [11] have formulated a theory of thermo-elasticity without energy dissipation. Chandrasekharaiah and Srinath [10] have considered this theory for a uniformly rotating medium. They have considered only dilatational and shear modes and have claimed that the phase velocity of both of these modes is reduced by a factor $\left(1+q^2\sin^2\varphi\right)^{-\frac{1}{2}}$ where q is the ratio of the frequency of rotation and the frequency of the wave and φ is the angle between the polarization vector of the displacement and the axis of rotation. Effects of rotation enter the theory through this factor and disappear when φ = 0, i.e. when the displacement is in the direction of the axis of rotation. Ahmad and Khan [13] showed that for the pure dilatational or shear modes considered in [10] the angle cp must vanish, and the results of Chandrasekharaiah and Srikantiah [9] reduce to earlier published results in [1].

Chaudhuri and Debnath [16] made an investigation of the propagation of plane harmonic waves in an infinite thermo-elastic isotropic solid permeated by a primary uniform magnetic field when the entire elastic medium is rotating with a uniform angular velocity.

2. Magneto-Thermo elastic plane Waves in a Rotating Transversely Isotropic Medium

Boit [18] equations were derived to investigate the plane thermo-elastic waves, but being dependent upon Fourier law of heat conduction, these equations predict an infinite speed of propagation which is physically unacceptable. To remedy this paradox, Lord and Shulman [19] have derived equations of dynamics thermo-elasticity based on modified Fourier equation. These equations are usually regarded as the basis of generalized thermoelasticity and have been used by several authors including Puri [20], Nayfeh and Nemat-Nasser [21] to study the plane thermo-elastic waves in unbounded isotropic homogeneous elastic solids. Relevant works of Paria [14], Willson [15], and Purushothama [22] on the propagation of magneto-thermo-elastic plane waves are also remarkable. Furthermore, Roy and Debnath [16] and Agarwal [17] have worked on magneto-thermo-elastic and electromagneto-thermo-elastic plane waves in infinite rotating and non-rotating isotropic materials, respectively. In this article we extend the work of Roy and Debnath [16] from isotropic to transversely isotropic material. It is assumed that the body is rotating about z-axis, i.e. $\underline{\Omega} = \Omega\,(0, 0, 1)$, hence equations of motion in component form can be written as

$$\sigma_{11,1} + \sigma_{12,2} + \sigma_{13,3} + (J \times B)_1 = \rho\,\{\ddot{u}_1 - \Omega^2 u_1 - 2\,\Omega\,\dot{u}_2\},$$

$$\sigma_{21,1} + \sigma_{22,2} + \sigma_{23,3} + (\quad \times B)_2 = \rho\,\{\ddot{u}_2 - \Omega^2 u_2 + 2\,\Omega\,\dot{u}_1\},$$

$$\sigma_{31,1} + \sigma_{32,2} + \sigma_{33,3} + (J \times B)_3 = \rho\,\ddot{u}_3\,. \qquad (2.1)$$

Equations of motion (2.1) on simplification can further be written as

$$c_{11}u_{1,11} + c_{12}u_{2,21} + c_{13}u_{3,31} + \frac{c_{11}-c_{12}}{2}(u_{1,22}+u_{2,12}) + c_{44}(u_{1,33}+u_{3,13}) - \beta_1 \frac{\partial(T - T_r + \alpha\frac{\partial T}{\partial t})}{\partial x} + (J\times B)_1 = \rho\,\{\ddot{u}_1 - \Omega^2 u_1 - 2\,\Omega\,\dot{u}_2\},$$

$$\frac{c_{11}-c_{12}}{2}(u_{1,21}+u_{2,11}) + c_{12}u_{1,12} + c_{11}u_{2,22} + c_{13}u_{3,32} + c_{44}(u_{2,33}+u_{3,23}) - \beta_1 \frac{\partial(T - T_r + \alpha\frac{\partial T}{\partial t})}{\partial y} + (J\times B\,)_2 = \rho\,\{\ddot{u}_2 - \Omega^2 u_2 + 2\,\Omega\,\dot{u}_1\},$$

$$c_{44}(u_{1,31}+u_{3,11}) + c_{44}(u_{2,32}+u_{3,22}) + c_{13}u_{1,13} + c_{13}u_{2,23} + c_{33}u_{3,33} - \beta_2 \frac{\partial(T - T_r + \alpha\frac{\partial T}{\partial t})}{\partial z} + (J\times B)_3 = \rho\,\ddot{u}_3. \qquad (2.2)$$

In order to solve (2.2) we choose the field quantities of the form [16] as follows

$$u_i = (p,q,r) = (p_0,q_0,r_0)\,exp\left[i(k\,x-\omega t)\right],$$
$$T = T_0\,exp\left[i(k\,x-\omega t)\right],$$
$$J = (J_x,J_y,J_z) = (J_1,J_2,J_3)\,exp\left[i(k\,x-\omega t)\right],$$
$$b = (b_x,b_y,b_z) = (b_1,b_2,b_3)\,exp\left[i(k\,x-\omega t)\right],$$
$$E = (E_x,E_y,E_z) = (E_1,E_2,E_3)\,exp\left[i(k\,x-\omega t)\right] \qquad (2.3)$$

where p_0, q_0, r_0; J_1, J_2, J_3; b_1, b_2, b_3; E_1, E_2, E_3 and T_0 are all constants.

The dispersion relation by using (2.2), (2.3) becomes

$$\begin{vmatrix} \left[-\rho(\Omega^2+\omega^2)+c_{11}k^2+\beta_1 k(i+\alpha\omega)\gamma\right] & \left[2\,i\,\rho\,\Omega\,\omega\right] & \left[\frac{ik}{\mu_c}B_2\right] \\ \left[-2\,i\,\rho\,\Omega\,\omega\right] & \left[-\rho(\Omega^2+\omega^2)+\frac{(c_{11}-c_{12})}{2}k^2\right] & \left[-\frac{ik}{\mu_c}B_1\right] \\ \left[-\sigma\,(i\,\omega B_2+\Omega B_1)\right] & \left[\sigma\,(i\,\omega B_1-\Omega B_2)\right] & \left[-\left(\frac{ik}{\mu_c}+\frac{\sigma\omega}{k}\right)\right] \end{vmatrix} = 0\,, \qquad (2.4)$$

$$\text{where } T_0 = \frac{T_r\, \beta_1\, k\, \omega\, p_0}{\left[\rho\, c_v \left(i\, \omega + \alpha^* \omega^2\right) - \kappa_1\, k^2\right]} = \gamma\, p_0 \ (say), \qquad (2.5)$$

$$\text{where } \gamma = \frac{T_r\, \beta_1\, k\, \omega}{\left[\rho\, c_v \left(i\, \omega + \alpha^* \omega^2\right) - \kappa_1\, k^2\right]} .$$

Remarks

By $\beta_1 = \beta$, putting in (2.4), one can obtain the result for Magneto-Thermo elastic plane wave speed in a rotating isotropic medium [16]. By putting $\Omega = 0$ and $\beta_1 = \beta$, in (2.4) one can obtain the result for Magneto-Thermo elastic plane wave speed in a non-rotating isotropic medium.

By using equations of motion (2.2), if one puts $B = 0$ one can deduce the result of thermoelastic plane wave speed in a rotating transversely isotropic medium. If one putS both $B = 0$ and $\beta_1 = 0$, then one can obtain the result of elastic wave speed in rotating transversely isotropic medium. In the same manner, the result of thermo-elastic and elastic plane wave speed in a rotating isotropic medium can also be deduced. For this purpose in general one has to put $\beta_1 = \beta_2 = \beta$, $\kappa_1 = \kappa_2 = \kappa$, $c_{13} = c_{12}$, $c_{33} = c_{11}$, and $c_{44} = \left(c_{11} - c_{12}\right)/ 2$.

Thus, this article is more generalized through which one can deduce the results of wave speed in rotating and non-rotating, conducting and non-conducting and magnetic and nonmagnetic mediums in isotropic as well as transversely isotropic materials.

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