DIFFRACTION OF ELECTROMAGNETIC WAVE ON THE SYSTEM OF METALLIC STRIPS IN THE STRATIFIED MEDIUM

***Ahmad Maher**

Department of Mathematics, Faculty of Science, Assiut University, 71516, Egypt

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Abstract: In this paper we study diffraction of electromagnetic wave on the system of metallic strips in the stratified medium. The integral equation which represents this problem is solved by Galerkin's method. To this equation the plane diffraction problem for TE-polarizated electromagnetic wave on the system of metallic strips in the stratified medium was reduced.

***Keywords**:* Partial differential equations; The Jump problem; Diffraction problem, AMS Subject Classification 35J, 35Q.

Introduction and formulation of the problem

Let planes $z = h_j$; $j = 1,...,n$ separate the space (x, y, z) into domains $D_0 : z < h_1$, $D_j : h_j < z <$ h_{j+1}, j = 1,...,n – 1 and D_n : $z > h_n$ filled with dielectric with dielectric indexesε_j , j = 0,..., n. Let the ideal conductive and infinitely thin metallic strips be placed on the media interfaces parallel to the axis y and segments $[\alpha_{jk}, \beta_{jk}]$, k = 1,..., m_j corresponding to the strips on the line $z = h_j$; j = 1,..., n in the plane y = 0.

Consider plane electromagnetic fields, the components of which do not depend on the coordinate y. Denote the complement of M_j with respect to the whole real axis by

$M_j = \mathrm{Y}_{k=1}^{m_j} (\alpha_{jk}, \beta_{jk})$ and by N_j

We need to seek a field arising under diffraction of the plane TE-wave with the potential function $\tilde{u}(x, z)$ which falls down from above on the stratified structure. The potential function $u(x, z)$ of the unknown filed should be a solution of the Helmholtz equation in every layer

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + k_j^2 u(x,z) = 0 , \quad (x,z) \in D_j \qquad (1)$$

and should satisfy the conjugation conditions

$$u(x, h_n + 0) = -\tilde{u}(x, h_n + 0) , \quad u(x, h_n - 0) = 0 , \quad x \in M_n ;$$

$$u(x, h_n + 0) - u(x, h_n - 0) = -\tilde{u}(x, h_n + 0) , \quad x \in N_n ;$$

$$\frac{\partial u}{\partial z}(x, h_n + 0) - \frac{\partial u}{\partial z}(x, h_n - 0) = -\frac{\partial \tilde{u}}{\partial z}(x, h_n + 0) , \quad x \in N_n ; \qquad (2)$$

$$u(x, h_j \pm 0) = 0 , \quad x \in M_j , \quad j = 1,..., n-1 ;$$

$$u(x, h_j + 0) - u(x, h_j - 0) = 0 , \quad x \in N_j , \quad j = 1,..., n-1 ;$$

$$\frac{\partial u}{\partial z}(x, h_j + 0) - \frac{\partial u}{\partial z}(x, h_j - 0) = 0 , \quad x \in N_j , \quad j = 1,..., n-1.$$

It is convenient to consider the solution of the problems (1) and (2) as a sum of two functions $u_j(x,z) = u\ (x,z)$ in D_j completed by zero with respect to the whole plane. To justify the Fourier integral transformation method we will seek $u_j(x,z)$

*Present Address:
The Teacher's College in Makkah, PO Box 2064, Makkah, Kingdom of Saudi Arabia
E-mail: a_maher69@yahoo.com

in the Sobolev spaces of distributions of slow growth at infinity $H_1^{loc}(D_j)$. We can show that the generalized solutions coincide with the classical solutions after the unknown distributions are found.

Consider the supplementary conditions providing for uniqueness of solution of the conjugation problem. Let $U_j(\xi,\varsigma)$ be the Fourier transform of distribution $u_j(x,z)$. We will seek a solution of the problems (1) and (2) in the domains D_0 and D_n in the class of the outgoing at infinity solutions, i.e., we will assume that the representations

$$u_j(x,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}U_j(\xi,\varsigma)e^{-i\xi x}e^{-i\varsigma z}\,d\xi\,d\varsigma \qquad (3)$$

should contain no elementary harmonics corresponding to those coming from the infinity plane waves under $j=0$ and $j=n$. Besides, we assume that the unknown solution $u(x,z)$ has no addends corresponding to eigne waves of the stratified structure going along the axis x (if such waves exist).

The jump problem

Consider the auxiliary jump problem for the Helmholtz equation in the stratified medium [1]. We need to seek a solution of the equation (1) in the domain D_j in the class of the outgoing at infinity solutions satisfying the conditions

$$\left.\begin{aligned} &u(x,h_j+0)-u(x,h_j-0)=a_j(x),\\ &\frac{\partial u}{\partial z}(x,h_j+0)-\frac{\partial u}{\partial z}(x,h_j-0)=b_j(x)\end{aligned}\right\} \qquad (4)$$

under $j=1,...,n$. Assume that conditions (4) are fulfilled on the axis x everywhere except on finite number of points, probably. We will seek functions $u_j(x,z)$ as solutions of the auxiliary Cauchy problems [2] for domains D_j with boundary conditions

$$u_n(x,h_n+0)=u_n^+(x)\,,\quad \frac{\partial u_n}{\partial z}(x,h_n+0)=v_n^+(x);$$

$$u_j(x,h_j+0)=u_j^+(x)\,,\quad \frac{\partial u_j}{\partial z}(x,h_j+0)=v_j^+(x);\quad j=1,...,n-1;$$

$$u_j(x,h_{j+1}-0)=u_{j+1}^-(x)\,,\quad \frac{\partial u_j}{\partial z}(x,h_{j+1}-0)=v_{j+1}^-(x); \qquad (5)$$

$$u_0(x,h_1-0)=u_1^-(x)\,,\quad \frac{\partial u_0}{\partial z}(x,h_1-0)=v_1^-(x).$$

where $u_j^\pm(x)$, $v_j^\pm(x)$ are the auxiliary boundary functions. Note that the Cauchy problems for the Helmholtz equation are over-determined. The boundary functions cannot be given arbitrarily.

We denote $\Delta h_j=h_{j+1}-h_j$ and

$$\gamma_j^0(\xi)=\left\{|\xi|>k_j:\ i\sqrt{\xi^2-k_j^2}\ ;\quad |\xi|<k_j:\ -\sqrt{k_j^2-\xi^2}\ \right\}.$$

From (1), (2) and (4), one can obtain the following.

Theorem 1. *The solution of the jump problem for the Helmholtz equation in the stratified medium exists if and only if the Fourier transforms of the auxiliary boundary functions* $V_j^\pm(\xi)$, $U_j^\pm(\xi)$ *satisfy the system of equations*

$$V_n^+(\xi)-i\gamma_n^0(\xi)U_n^+(\xi)=0,$$

$$[V_j^+(\xi)-i\gamma_j^0(\xi)U_j^+(\xi)]-e^{i\Delta h_j\gamma_j^0(\xi)}[V_{j+1}^-(\xi)-i\gamma_j^0(\xi)U_{j+1}^-(\xi)]=0, \qquad (6)$$

$$e^{i\Delta h_j\gamma_j^0(\xi)}[V_j^+(\xi)+i\gamma_j^0(\xi)U_j^+(\xi)]-[V_{j+1}^-(\xi)+i\gamma_j^0(\xi)U_{j+1}^-(\xi)]=0,$$

$$V_1^-(\xi)+i\gamma_0^0(\xi)U_1^-(\xi)=0,$$

$$U_j^+(\xi)-U_j^-(\xi)=A_j(\xi)\,,\qquad V_j^+(\xi)-V_j^-(\xi)=B_j(\xi)\,;\quad j=1,...,n.$$

Here

$$\sqrt{2\pi}\,(k_n^2-\xi^2-\varsigma^2)U_n(\xi,\varsigma)=e^{ih_n\varsigma}[V_n^+(\xi)-i\varsigma U_n^+(\xi)],$$

$$\sqrt{2\pi}\,(k_j^2-\xi^2-\varsigma^2)U_j(\xi,\varsigma)= \qquad (7)$$

$$=e^{ih_j\varsigma}[V_j^+(\xi)-i\varsigma U_j^+(\xi)]-e^{ih_{j+1}\varsigma}[V_{j+1}^-(\xi)-i\varsigma U_{j+1}^-(\xi)];\quad j=1,...,n-1,$$

$$\sqrt{2\pi}\,(k_0^2-\xi^2-\varsigma^2)U_0(\xi,\varsigma)=-e^{ih_1\varsigma}[V_1^-(\xi)-i\varsigma U_1^-(\xi)].$$

The integral equation

We consider a solution of the diffraction problem in the form

$$u(x,z) = u_d(x,z) + u_m(x,z)\,,$$

where the first addend in the right hand side is a solution of the problem on the fall of the wave at the media interfaces without metallic strips and the second addend is a new unknown function.

The function $u_d(x,z)$ can be found as a solution of the jump problem under the following conditions.

$$a_n(x) = -\tilde{u}(x,h_n+0)\,, \qquad b_n(x) = -\frac{\partial \tilde{u}}{\partial z}(x,h_n+0)\,,$$

$$a_j(x) = 0\,, \qquad b_j(x) = 0\,; \qquad j = 1,\ldots,n.$$

The function $u_m(x,z)$ is also a solution of the jump problem under the following conditions

$$a_j(x) = 0\,, \qquad x \in (-\infty,+\infty)\,; \qquad j = 0,\ldots,n\,,$$

$$b_j(x) = 0\,; \quad x \in N_j\,, \qquad b_j(x) = \vartheta_j(x)\,; \quad x \in M_j\,, \qquad j = 0,\ldots,n,$$

where $\vartheta_j(x)$ are the auxiliary unknown functions which can be found from the boundary conditions on the metallic strips

$$\begin{aligned} &u(x, h_n + 0) = -u(x,h_j + 0), \\ &u(x,h_j \pm 0) = 0,\ j = 1,\ldots, n-1. \end{aligned} \tag{8}$$

Let us transform equations (8) into integral equation with respect to the function $\vartheta(x) = \vartheta_j(x)$ on $M = Y_j\, M_j$ in the following way. Having solved SLAE (6), we express the Fourier transforms of the auxiliary boundary functions $V_j^{\pm}(\xi), U_j^{\pm}(\xi)$ in terms of the functions $\vartheta(x)$. Having substituted them into equations (7), we seek the Fourier transforms $U_j(\xi,\varsigma)$ of the unknown functions $u_j(x,z)$ and by formula (3) we obtain the expressions of these functions. Thus, we have the proof of the following theorem.

Theorem 2. *The diffraction problem for TE-wave on the system of the metallic strips in the stratified medium is equivalent to the integral equation of the form*

$$\left.\begin{aligned} &\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} U_n(\xi,\varsigma)e^{-i\xi x}e^{-i\varsigma h_n}\,d\xi\,d\varsigma = -\tilde{u}(x,h_n+0)\,, \quad x \in M_n\,, \\ &\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} U_j(\xi,\varsigma)e^{-i\xi x}e^{-i\varsigma h_j}\,d\xi\,d\varsigma = 0\,, \quad x \in M_j\,, \quad j = 0,\ldots n-1. \end{aligned}\right\} \tag{9}$$

with respect to the function $\vartheta(x)$.

In the particular case under $n = 1$ and $m_1 = 1$ (there is only strip $\alpha < x < \beta$ on the boundary $z=h$ of two mediums) the equation (9) has the form

$$\int_{\alpha}^{\beta}\vartheta(t)\Big[\frac{-i}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{\gamma_0^0(\xi)+\gamma_1^0(\xi)}e^{i(t-x)\xi}\,d\xi\Big]dt =$$

$$= -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{U}(\xi)\frac{2\gamma_1^0(\xi)}{\gamma_0^0(\xi)+\gamma_1^0(\xi)}e^{-ix\xi}\,d\xi\,, \qquad x \in (\alpha,\beta). \tag{10}$$

If dielectrics in the upper and lower half-planes are the same, i.e. $k_0 = k_1 = k$, then also $\gamma_0^0(\xi) = \gamma_1^0(\xi)$. Having calculated the interior integral by formulas (130) and (134) from [4], we obtain the well known integral equation

$$\frac{1}{4\sqrt{\pi}}\Gamma(1/2)\int_{\alpha}^{\beta}\vartheta^M(t)H_0^{(1)}(k|t-x|)dt = -\tilde{u}(x)\,, \qquad x \in (\alpha,\beta)\,,$$

where $H_0^{(1)}(x)$ is the Hankel function.

4. The Galerkin method

Different particular cases of the integral equation of the 1st kind with logarithmic singularity in the kernel (9) are solved numerically by Galerkin method with decomposition of the unknown function on every segment $[\alpha_{jk}, \beta_{jk}]$ by Chebyshev polynomials with weight. Note that SLAE with respect to coefficients of decomposition approximating the initial integral equation (9) can be obtained by another method. The Galerkin method can be applied not to the equation (9) with respect to the function $\vartheta(x)$ but to the integral equation with respect to its Fourier transform $F(\xi)$ which has been obtained at the preceding stage, e.g., for the following equation

$$\int_{-\infty}^{+\infty} F(\xi) \frac{1}{\gamma_0^0(\xi)+\gamma_1^0(\xi)} e^{-ix\xi}\, d\xi =$$

$$= -i \int_{-\infty}^{+\infty} \tilde{U}(\xi) \frac{2\gamma_1^0(\xi)}{\gamma_0^0(\xi)+\gamma_1^0(\xi)} e^{-ix\xi}\, d\xi\,, \qquad x \in (\alpha, \beta).$$

Since the Bessel functions are the Fourier transforms of the Chebyshev polynomials together with the weight function, the solution of the last should be decomposed into the sum by these functions. From the Parseval formula, it follows that such approach has as a result just the same SLAE.

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