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Review

FINANCIAL ENGINEERING COMPUTING: BINOMIAL MODELING USING MAPLE

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Summary: One of the fastest growing areas of scientific computing is in the financial industry. Even, many of the most basic problems in financial analysis, in common and in financial engineering in special, are still unsolved, and are surprisingly resilient to the onslaught of legions of talented researchers from many diverse disciplines. In this article, I hope to give readers a sense of these challenges by describing a relatively simple problem that all investors face--managing a portfolio of financial securities over time to optimize a particular objective function employing binomial tree; showing how complex such a problem can become as real-world considerations such as preferences, and portfolio constraints are incorporated into its formulation. I am using Maple to show how software meant for mathematics can be made to work for financial engineering that deals primarily with stochastic calculations. The simple scenario used here is sufficient enough to show readers how this software may be used for more complex scenarios.

Keywords: Two notal model, European option, two period portfolios, State prices

Introduction

Two decades ago, only a handful of Wall Street specialists needed the mathematical knowledge to create pricing algorithms and risk curves. Today's global financial transactions and rapid advances in technology have led to a critical demand for professionals who can quantify, assess, price and forecast increasingly complex financial outcomes. This has led to the development of an important new field in the domain of finance--financial engineering.

Financial engineering, in the simplest possible vocabulary, is the application of mathematical and statistical techniques for the solution of risk management problems in finance. Financial engineering uses tools from finance, applied mathematics, computer science, statistics and economic theory. Investment banks, hedge funds, insurance companies, corporate risk managers and regulatory agencies apply the methods of financial engineering to such problems as derivative securities valuation, risk management, strategic planning and dynamic investment strategies. As the pace of financial innovation increases, the need for highly qualified and efficient tools and techniques intensifies.

The use of statistical techniques to measure uncertainties in a market of traded assets dates back at least to Bachelier [1] (translated by Cootner [2]) at the twist of previous century but a rigorous justification based on an axiomatic treatment of the theory of choice and utility representation of preferences may be found in Kreps [3], Debreu [4] among others. This opens

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the gates of statistical solutions to diehard problems in finance and we have a series of articles and papers in different magazines and research journals.

Arrow's development of a general equilibrium model of security markets based on agents trading in an idealized market and the search for *market clearing* [5] provides the foundation for the key idea of *state prices* (positive discount factors, one for each date and *state of the world*) such that any security price is the state-price weighted sum of its future payoffs. The existence of state prices is equivalent to the existence of *risk neutral* probabilities for the states (see Arrow [6]). All of this work was done in a discrete time setting. Merton [7,8] initiated the study of continuous time financial modeling, introducing his general equilibrium model in 1973 (see also Merton [9]), in the same year as the publication of the Black-Scholes option pricing formula (see Black and Scholes [10]). Since 1970, there has been an explosion of interest in the theoretical aspects of finance. Harrison and Kreps [11], Harrison and Pliska [12], and Jarrow [13] are just a few names in a very long list of authors who shaped the theoretical base of today's financial engineering.

This theoretical development should have been accompanied at least by a similar, if not superior, development in the relevant computing instruments. Unfortunately, this was not the case. Many important and potentially efficient discoveries are still not solicited by computing strategies and thus are not applied in the actual financial markets to gauge their practical efficiency. Some attempts, however, are there in this regard, such as special *option* routine in *R* and a free-for-all open source command driven statistical package. Even this routine is capable only in certain given circumstances and is absolutely not user-friendly. Similar attempts have also been made by others with some *macros* for MS Excel but all those attempts are situation-dependent and the results are not reliable. There are some exclusive packages in the market but are very expensive and their usage is limited only to high profile firms and organizations. The current article is an attempt in the same direction aimed at using renowned and reliable mathematical package, like *Maple*, for financial engineering modeling. The prime objective is to show how complex stochastic financial engineering calculations are performed by these packages.

In this article is considered the case where the stock price follows a simple, stationary *binomial* process. After each moment in time, the stock price can go either up or down by a given known percentage (or rate) which is governed by the binomial process. When the stock price follows such a process and there exist a risk free asset, options written on the stock are easy to price. Furthermore, given appropriate limiting conditions, the binomial process converges to a lognormal price process and the binomial formula converges to the Black and Scholes [10] formula.

Option pricing is the key issue in this whole scenario, which is the topic of the current paper. The price of the option is typically a non-linear function of the underlying asset (and some other variables, e,g. interest rates, strike) The basis of any option pricing model is a description of the stochastic process followed by the underlying asset on which the option is written. The following sections introduce, in the first hand, the financial engineering modeling with simple but very practical two nodal binomial models. Section 0 provides a simple description, while the subsequent sections explore the model for state prices and parameters. Maple codes are available at each step to facilitate computing.

Two Nodal Binomial Models

A two nodal binomial option pricing model, also known as Cox-Ross-Robinstein model after their seminal paper (see Cox and Ross [14]) in this field, is a simple but powerful technique that can effectively be used to solve many complex option-pricing problems. In contrast to the Black and Scholes [10] and other complex option pricing models that require solutions to stochastic differential equations, the binomial option pricing model (two-state option pricing model) is mathematically simple. It is based on the assumption of no arbitrage.

Model Description

Running time is denoted by t and by definition we have two points in time, $t = 0$, i.e., today and $t = 1$, i.e., tomorrow or any next moment in time. In a typical model, we have two assets; a *bond* and a *stock*. Table 1 shows more comprehensively the bond and stock mechanisms for a binomial pricing model.

Table 1. A typical 2 nodal binomial model.

	Bond	Stock
Initial Position, $t = 0$	$B_0=1$	$S_0 = s$
Final Position, $t = 1$	$B_1=1+R$	$S_1 = sZ = \begin{cases} su \\ sd \end{cases}$

where Z is a stochastic variable defined at two levels; one each for up and down stock positions may go in the next moment of time. In other words,

$$Z = \begin{cases} u & \text{with probability } p_u \\ d & \text{with probability } p_d \end{cases}$$

We assume here that S_0, today's stock price is known, as are the positive constants u, d, p_u, and p_d. R is the spot interest rate. In the contextualized vocabulary, R is the mark-up rate. These probabilities p_u and p_d are also called as objective probabilities and are based on market experiences. The model is also described in Fig. 1 which depicts bond and stock 1 period mechanism, respectively, in left and right panels.

For Bond Mechanism, next moment in time results in a change in value (1+R) times the original value. While, the stock dynamics, includes an uncertainty; it can go either up or down with certain calculable probabilities. So the next moment in time, for stock, results in a change in value either u or d times the original value.

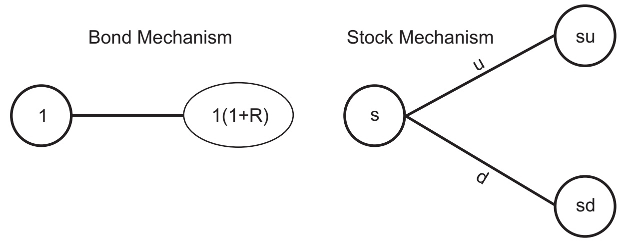

Figure 1. The binomial model.

Benninga and Wiener [15] have shown a 4 period model with 6% interest (mark-up) rate. The tree is shown here in Fig. 2. The *up* jump is expected to be to be 10% (u=1.1), while the *down* jumps to be 5% (d=0.95) each period. If the initial stock price is Rs.50, then the development of the stock price is described in the Figure 2.

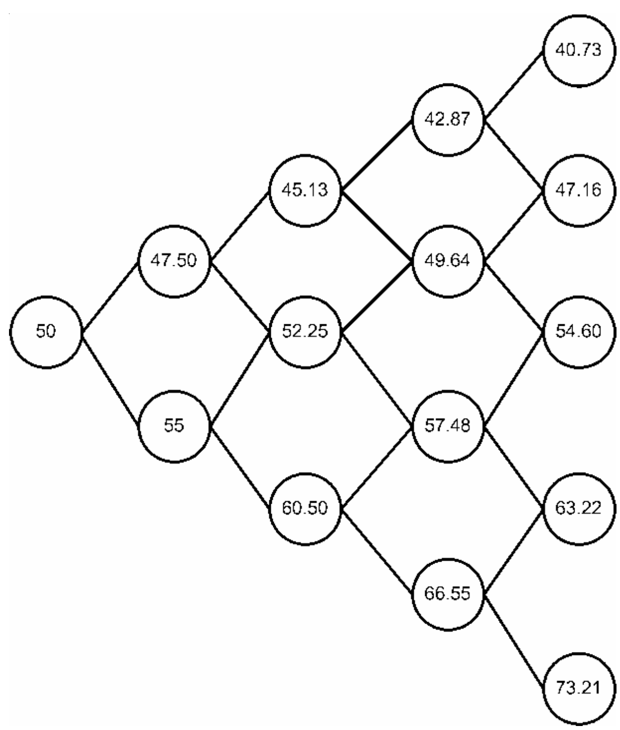

Figure 2. A 4-period binomial model.

So the stock portfolio, as suggested by binomial model described in Fig. 2, after 4 moments in time, may have any of the five values shown in the very last nodes of the Figure. There is only one set of price changes which will take the prices to Rs.73.21 at the expiry and this is if the price goes up in each step. Likewise, there is only one path to the lowest expiry day price, Rs.40.73 and this is if price falls in each step. There are 4 possible paths which would take the stock price to the 2nd highest final price, Rs.63.22. These numbers of paths may be readily found from binomial theorem. King [16], (page 288), has detailed the construction of these paths. Such trees are quite difficult to construct for a higher number of periods which is very much possible in real world. Fortunately, Maple is here to help us out. Vivaldi [17] and Heck [18] have presented beautiful explanations for the Maple functions. Here is a Maple function (not macro) that can be used to generate such binomial trees,

```
>v:=(u,d,s,n)->seq(
              {seq(s*u^(j-1)*d^(i-j),
                  j=1..i)},
                  i=1..(n+1));
```

The function asks for up jump rate, u, down jump rate, d, initial stock price, S_0, and number of periods, n, required in the tree. Applying the function to the data described in Figure 2 for 4-period binomial model, it results in

```
> v(1.1,0.95,50,4);
   {50.0}
   {47.50, 55.0}
   {45.1250, 52.250, 60.50}
   {42.868750, 49.63750, 57.4750, 66.550}
   {40.72531250, 47.1556250, 54.601250,
   63.22250, 73.2050}
```

The assumption of no arbitrage, that runs through this binomial model, implies that all risk-free investments earn the risk-free rate of return and no investment opportunities exist that require zero rupees of investment but yield positive returns. It is the activity of many individuals operating within the context of financial markets that, in fact, upholds these conditions. The activities of arbitrageurs, or speculators, are often maligned in the media, but their activities insure that our financial markets work. They insure that financial assets such as options are priced within a narrow tolerance of their theoretical values.

One of the biggest problems in the world of financial engineering is to price the options and derivatives. Of course, this price should be fair enough for parties at both ends of the contract. Options are available usually at a selection of exercise prices. An option holder faces great upside potential and little downside risk, while the opposite applies to the writer. An American style option can be exercised at any time until expiry. European style options can only be exercised at maturity. Besides this, there are other tailor-made *exotic* options. Hull [19,20] and Binnewies [21] have presented excellent discussions on different types of options and derivatives. Presently, here the simplest model is being considered, yet most frequently used are the European options under binomial processes. This would give one a fairly good idea of how computing-instrument-convertible mathematics can be developed for such pricing. The other types of option may be discussed separately.

European Option Prices under Binomial Pricing Model

In this section, the attempt is to develop binomial pricing model for the options, starting with a simple European call option. Suppose a single period is divided into T intervals, so that the price of the stock at the end of the last interval can be written s_T. Then the Call and Put option price at maturity with a strike price K, coincides with its payoff function, using symbol system employed in Bjork [22], is given by

$$\Phi^{\text{European Call}} = \max(s_T - K, 0)$$
$$\Phi^{\text{European Put}} = \max(K - s_T, 0)$$

A day prior to maturity at each state, one can calculate the call price as a weighted average (with risk-neutral probabilities as weights) of its price at maturity. Obviously this procedure can be continued to the root of the stock tree, giving a *fair* price of the option today. The procedure will work for both the European and the American options. However, in the case of European options a simpler procedure based on the observation that only the distribution of payoffs at maturity matters would be used. State prices of each state at maturity can also be calculated and take a weighted average of payoffs with these weights.

One Period Portfolios

Let us start with an illustrative example used by Bjork [22] (page 13). Consider the simplest possible portfolio with a single bond and a single stock, the price of which today is Rs.100. Suppose that over the next year, the stock price can go either up or down by 20 percent, so that the stock price at the end of the year is either Rs.120 or Rs.80. In such a case we then we have a binomial tree as given in Fig. 3.

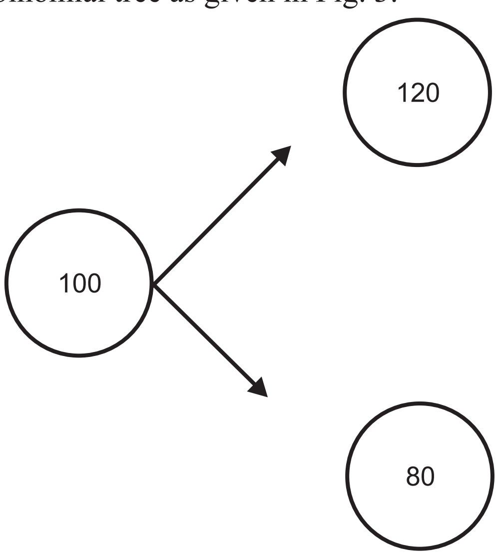

Figure 3. One-period two-nodal binomial tree for European Call.

If there exists a call, (supposing a European) with strike price $K = 110$, this constitutes a complete one period portfolio. This one period portfolio, consisting of bonds and stocks only, added with the European Call, constitutes an arbitrage-free, risk-neutral hedging portfolio. Using Cox and Ross [14] and Rendlemen and Bartter [23], the portfolio may be expressed mathematically as

$$(1+R)x + 120y = \Phi_{K=110} = \max(120-K,0) = 10$$
$$(1+R)x + 80y = \Phi_{K=110} = \max(80-K,0) = 0$$

where x and y are respectively the number of bond and stock in the portfolio. These equations show both the branches of binomial tree and how the portfolio changes over one period for a European Call option. This turns out to be a system of simultaneous equations whose solution can be approached by using Maple. Vivaldi [17] enlisted a function that can be used here to solve such a system of simultaneous equations. So, a hedging portfolio is approached by solving

```
> solve({(1+R)*x+120*y=10,
        (1+R)*x+80*y=0},
        {x,y});
```

$$x = -20\left(\frac{1}{1+R}\right), y = \frac{1}{4}$$

where R is the spot rate for the period. Hedging portfolio depends directly on its value. Fig. 4 shows logarithmic dynamics for such a hedging portfolio with differing values of R. If it is taken to be zero for computational ease, then x turns out to be -20 while y would remain to be 0.25. In everyday terms, this means that the replicating portfolio is formed by borrowing Rs. 20 from the bank, and investing the money in a quarter of a share in the stock. Such a portfolio would give us a payoff of Rs. 10 if the stock price goes up and Rs. Nil if it goes down.

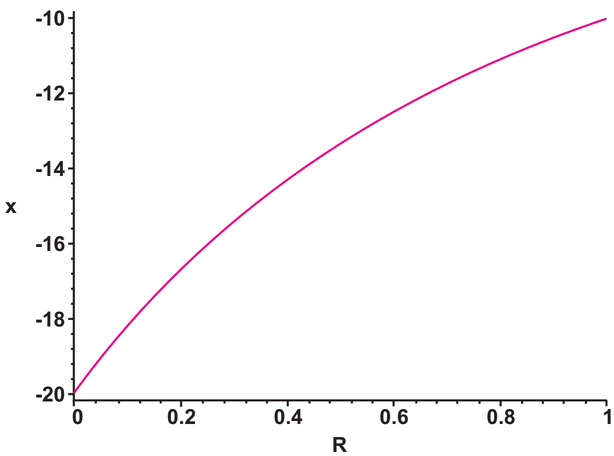

Figure 4. Dynamics of hedging portfolio with R.

The next step, after establishing a hedging portfolio, is the calculation of a fair price for this option. It follows that the price of the given call option, $\Pi^{\text{European Call}}$ must be equal to the cost of replicating its payoff, i.e.,

$$\Pi^{\text{European Call}} = (-20) + (0.25 \times 100) = 5$$

where the first bracket shows bonds' price and the second stock's price, the only two assets for the portfolio. Such logic for evaluating option price is often referred to as *pricing by arbitrage*; if two assets, or sets of assets, (in our case - the call option and the portfolio of Rs.0.25 of the stock and -Rs.20 of the bonds) have the same payoffs, they must have the same market price.
Two Periods Portfolios

This arbitrage pricing logic does work for multi-period scenarios. Let us extend the same example to 2 periods. Here, for instance, consider a stock--a bond portfolio with a European Call (Table 2).

Table 2. A Bond-Stock Portfolio with a European Call.

Bond	Stock	Option
$R = 0.06$	$s = \text{Rs.}50$ $Z = \begin{cases} u = 1.10 \\ d = 0.95 \end{cases}$	European Call $K = 50$

The Maple function v(1.10,0.95,50,2) described in Sec.0 results in a 2-period binomial tree shown here in the left panel of Fig. **5**. The price for such an option would again be calculated using arbitrage pricing, discussed in the previous section, with the only difference that the logic has to be applied twice; one for each period starting from the right. The right panel of Fig. **5** shows these calculations step by step. This example will be discussed again in the coming sections to develop a single step Maple function for such a pricing logic.

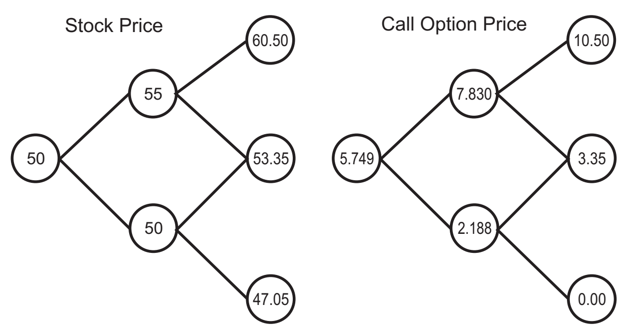

Figure 5. Two-period binomial process for a European Call.

State Prices

State prices are basically a measure of relative scarcity of price at a certain state of nature. Moffat [24], however, defines state prices, or discount factors, not as a reduction in the scarcity at some future time but as a subjective probability that the agent will die before the next period, and so discounts the future experiences not because they aren't valued, but because they may not occur. Suppose that the states of the world called **up** and **down** representing all the uncertainty in the next time period, one can derive market prices for these states by making use of the proposition discussed and derived in Neftci [25], and Bjork [22] among others, and given by

$$d \leq (1+R) \leq u$$

where u and d represents the stochastic variable for up and down states, as defined and used earlier. In other words, using Royden [26], the proposition says that (1+R) is a convex combination of u and d, or

$$(1+R) = u.q_u + d.q_d$$

where q_u, q_d satisfy

$$q_u \geq 0$$
$$q_d \geq 0$$
$$q_u + q_d = 0$$

q_u and q_d are the constants known as the martingale, or real time, probabilities. These probabilities can easily be calculated by using the above system of simultaneous equations in q_u and q_d. Using the symbolic computational feature of Maple to solve this system of simultaneous equations results in

```
> solve({1+R=qu*u+qd*d,
        qu+qd=1},
        {qu,qd});
```

$$qu = \frac{1+R-d}{u-d}$$
$$qd = \frac{u-(1+R)}{u-d}$$

The literature also calls these martingales as state prices and the above expressions are used to calculate these prices. A comparison of these martingale probabilities with objective probabilities enables us to calculate arbitrage profit.

The Portfolio discussed in Sec.0 gives identical martingales ($q_u = q_d = 0.5$). The net value of the portfolio, employing the expression available in Bjork [22] (page 9) using these identical martingales and non-identical objective probabilities ($p_u = 0.6, p_d = 1 - p_u$), turns out to be

$$\frac{E(S)}{1+R} = \begin{cases} 10 \times 0.5 + 0 \times 0.5 = 5 \text{ using martingale probabilities} \\ 10 \times 0.6 + 0 \times 0.4 = 6 \text{ using objective probabilities} \end{cases}$$

So we have indeed replicated the option. If anyone is foolish enough to buy the option from us for Rs.6, then we can make a riskless profit. We sell the option, thereby obtaining Rs.6. Out of these Rs 6 we invest Rs 5 in replicating the portfolio and invest the remaining Rs 1 in the bank. At time $t = 1$, the claims of the buyer of the option are completely balanced by the value of the replicating portfolio, and we still have Rs.1 invested in the bank. If, on the other hand, someone is willing to sell the option to us at a price lower than Rs.5, we can also make an arbitrage profit, like in the previous case, by selling the portfolio short.

To make these formulas work for multi-periods portfolios, let us divide the interval between 0 and 1 into T subintervals. Suppose that in each of these subintervals the price of the stock can go up by u or down by d and suppose that the risk less interest rate over a subinterval is R. Using the state prices for each of these subintervals, one can have a multi-period tree for state prices at each node. Here is a simple Maple function to generate these state prices at each node of a multi-period binomial pricing model. The following function generates q_u for each node present in the binomial model, while the other q_d may be calculated by using the convex relationship between the two.

```
> sp:=(u,d,R,n)->seq(
{seq(((1+R-d)/(u-d))^(j-1)*((1+R-u)/(d-u))^(i-j),
j=1..i)},
i=1..(n+1));
```

The function asks for up jump rate, u, down jump rate, d, the risk free interest rate, R, and number of periods, n, required in the tree. Application of the function to the portfolio described in Table 2 results in:

```
> sp(1.1,0.95,0.06,4);
    {1.}
    {0.7333333333, 0.2666666667}
    {0.07111111113, 0.1955555556, 0.5377777777}
    {0.01896296297, 0.05214814816, 0.1434074,
    0.394370}
    {0.0050567, 0.01390617, 0.03824198,
    0.105165, 0.289205}
```

For a typical binomial process, the price of the stock at time t can be written as

$S_t = s(u)^k (d)^{t-k}$, k = Number of up moves in the process = 0,1,…,t

This follows the price of an option, under binomial process. The following are the culminated expressions for the option prices for both European Calls and Put, respectively,

$$\Pi_{t=0} = \sum_{j=0}^{T} \frac{T!}{(T-j)!\,j!}(q_u)^j (q_d)^{T-j}\,\Phi(S_j) = \begin{cases} \sum_{j=0}^{T} \frac{T!}{(T-j)!\,j!}(q_u)^j (q_d)^{T-j}\left[\max\left(s(u)^k (d)^{t-k} - K, 0\right)\right] \\ \sum_{j=0}^{T} \frac{T!}{(T-j)!\,j!}(q_u)^j (q_d)^{T-j}\left[\max\left(K - s(u)^k (d)^{t-k}, 0\right)\right] \end{cases}$$

These expressions are iterative in nature and one has to go through step by step to reach at the final node. Here are Maple functions to calculate these prices for a European Call and Put, respectively,

```
> call:=(u,d,R,s,k,n)->sum('binomial(n,j)*
      ((d-1-R)/(-u-u*R+d+R*d))^j*
      ((1+R-u)/(-u-u*R+d+R*d))^(n-j)*
      max(s*u^j*d^(n-j)-k,0)','j'=0..n);

> put:=(u,d,R,s,k,n)->sum('binomial(n,j)*
      ((d-1-R)/(-u-u*R+d+R*d))^j*
      ((1+R-u)/(-u-u*R+d+R*d))^(n-j)*
      max(k-s*u^j*d^(n-j),0)','j'=0..n);
```

```
>plot(call(1.5,0.5,0,80,80,j),j=1..7,
    labels=["Number of
          Nodes","Price"],
    Label directions=[HORIZONTAL,
          VERTICAL]);
```

```
>plot(call(1.5,0.5,0,80,80,j),j=1..100,
    labels=["Number of
          Nodes","Price"],
    Label directions=[HORIZONTAL,
          VERTICAL]);
```

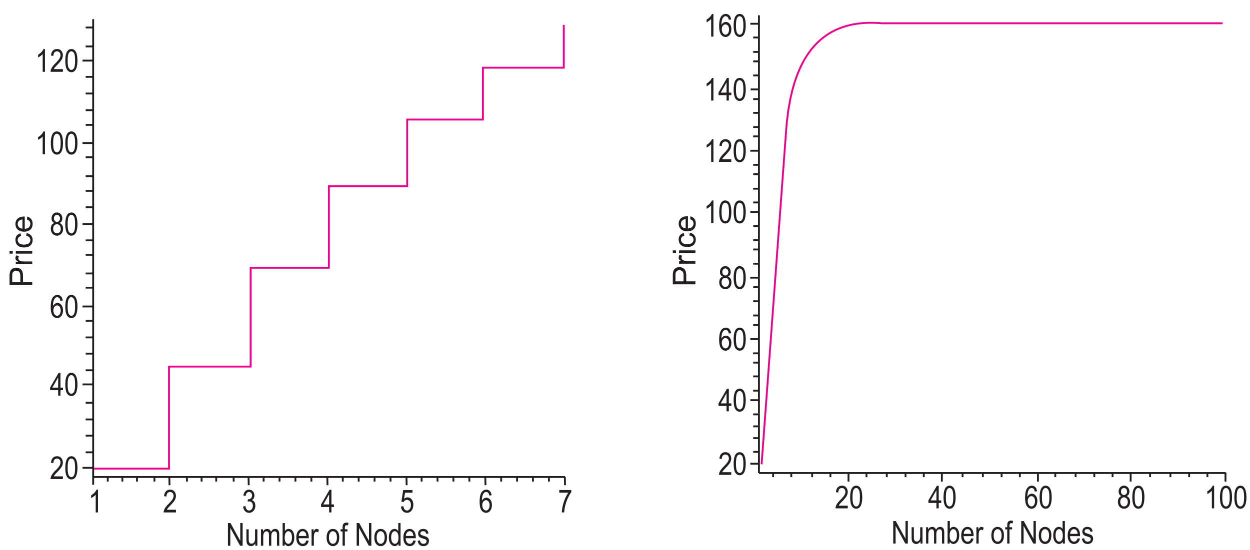

Figure 6. The left panel shows change for smaller number of nodes and the right panel shows change for larger number of nodes; the price becomes constant at some point.

These two functions are name-specific and ask for up jump rate, *u*, down jump rate, *d*, the risk free interest rate, *R*, the initial stock price, S_0, strike price, *K*, and the number of periods, *n*, required in the tree. Applying these functions to the portfolio described in Table 2 with a European Call/Put option with a strike price, *K*=50, the Maple function (defined above) yields

> call(1.1,0.95,0.06,50,50,2);

it turns out to be 5.74917. In a similar way, the function for the European put can be applied to have its price, also indicated in Fig. **5**. These expressions are number of node dependents and it is interesting to observe how the option price at *t*=0, i.e., $\text{Ï}_{t=0}$ changes with the number of nodes. Let us use Maple to see this change graphically. The increase in price seen in Fig. 6 is omnipresent but the dynamics of this increase is quite interesting in the case of larger number of nodes. It increases with the increasing number of modes, but this increase is not permanent; at some number of nodes, the option price becomes constant. The right panel of Fig. **6** shows this feature. A similar behavior can be observed by plotting Put option.

This completes the process of pricing an option. There are different factors that affect this price. King [16] (page 218) has given an exhaustive list of such factors. Among the most important factors are the strike price, *K*, current price of the stock, S_0, the time remaining until the option expires. The spot rate does affect the price but its dynamics is not that significant.

In conclusion**,** Financial Engineering discusses arbitrage theory and its applications to pricing problems for financial derivatives. In recent years, there have been many developments in the variety and the use of financial instruments to these financial engineering problems. Unfortunately, these developments are not matched by soliciting software. Pathetically, the complex nature of financial engineering problems needs such solicitation. The market has a few *e*-solutions to these complications but most of these are exclusive, tailor-made, and above all very expensive. Benninga and Wiener [27] have described a somewhat different approach for a wide variety of contingent claims including path-dependent, American and exotic options wherein normality is assumed in dealing with pricing. However, normality assumption is not true for all possible cases. In fact, it is not true for most of the cases as the financial engineers are moving towards other distributions such as binomial, trinomial and log-Normal. The present article has used binomial distribution for the type of portfolio that can offshoot in only two directions (hence binomial). Its focus is on how the presently available software, especially Maple, can be used for these complex financial engineering problems. A very simple yet equally effective and practical scenario regarding two nodal binomial models for European options is discussed here for their solution through Maple. Special functions have been developed, in the Maple language, to show how this language is used to achieve more complex scenarios. Financial engineering is very new here in Pakistan but it is creeping in very fast. The State Bank of Pakistan has allowed trade options. The University of Management & Technology, Lahore, is among the very first institutes in Pakistan to introduce this the subject.

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