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Research Article

Study of Fractal Structure of Plasma Turbulence at Pakistan Atmospheric Region

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Abstract: This study focused on the concept of fractal structure in plasma turbulence. This phenomenon is a multifaceted action of charged particles and electromagnetic field and is an unstable state of fluid. Therefore, it occurs as a consequence of the collective character of mutual interactions of charged particles in the plasma. Using the rescaled range analysis approach, we estimated the fractal dimension of plasma turbulence data at the Pakistan atmospheric region to the observed dynamical behavior. The estimated value of fractal dimension of plasma turbulence was approximately 1.8; the plasma turbulence being greater than 1.5, the behavior was anti-persistent, i.e., indicating a deceasing trend in plasma turbulence. This study revealed that the nature of plasma turbulence was randomness and intermittence. In summary, it was observed that fractal analysis is helpful to examine the persistent, anti-persistent and Brownian nature for long-range behavior of chaotic systems.

Keywords: Plasma turbulence, nonlinear dynamics, fractal analysis, Hurst exponent

1. INTRODUCTION

The solar-terrestrial environment consists of the upper part of terrestrial atmosphere (ionosphere) which is affected by outer part of geomagnetic filed and solar emissions [1]. Low frequency radio signals are degraded during their transmission through plasma in the interstellar and interplanetary medium, and the Earth's ionosphere [2]. The interaction between electromagnetic signals and plasma is termed as plasma turbulence [3]. The dynamic turbulence is not only active on a large range of length sizes, but also is filled with restricted coherent morphologies which makes some quantities extremely irregualr such as energy, anisotropy, pressure.The plasma turbulence is generally described by hypothesizing that the variations are randomly distributed in space [4]. The study of plasma turbulence is important because it makes possible a time, frequency, and location dependent transmission phase on electromagnetic signals [2]. The edge region of plasma turbulence is directed by variation in density and potential that exhibits such turbulent nature which is continuously influenced on the transfer of energy and particles at the edge region [5-6]. Such plasmas are said to be turbulent in nature and produce a nonlinear effect on radio signal transmission. There are two main characteristics of plasma turbulence, i.e., nonlinearity and intermittency [7].

The purpose of fractal dimension analysis presents suitable approach to depict naturally occurring irregular structures, therefore, there are several articles whose dynamics reveal rough or chaotic behavior, dedicated to fractal analysis of the so complex dynamic systems as magnetosphere, ionosphere [8]. A lot of work has been done on plasma turbulence and other phenomena of plasma using fractal dimension analysis. Narayanan et al. [4] applied continuous

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wavelet algorithum on plasma tubulence to estimate fractal dimension.Another study about multifractality on solar wind turbulence and summarized the results that as the solar activity increased, the solar wind becomes somewhat more multifractal while the fast wind is slighty less multifractal. Macek et al. [9] reported that the rate of irregularity of the dimension scale for the slow wind is rather antipersistence with the phase of the solar activity [9]. Another study discussed the complexity and smoothing natureof solar flare that are associated with changes in the magnetic stucrure of the active region using fractal dimension[10]. Therefore, it is necessary to use appropriate statistical approach (fractal analysis) to study the rough nature of plasma turbulence at Pakistan atmospheric region. This study comprises two sections; the first one described the statistical analysis of plasma turbulence and second illustrated its complex nature on perspective of fractal dimension.

2. DATA AND METHODOLOGY

The daily time series data of plasma turbulence for Pakistan atmospheric region for the year 1989 has been taken from Pakistan Space and Upper Atmospheric Research Commission (SPARCO).

Abroad-spectrum technique of estimating the fractal dimensions of sporadic signals of plasma turbulence are by the box counting method [11]. But, to obtain more precise results, it is better to use Rescaled Range Analysis (R/S) technique to find fractal dimension of plasma turbulence.

2.1 Rescaled Range Analysis

G. Hurst first time proposed this useful method in 1951 [19] which is still an acceptable tool for fractal time series research of different complex domain. Several studies applied Rescaled Range analysis to compute Hurst exponent of time series data [12, 13, and 14]. This method estimates the fractal dimension of a sample (N) by iteratively dividing it into subsamples (n) with decreasing subsample so that n= N, N/2, N/4, N/8,….. , so on and performing the following procedure for each of iteration.

Suppose the time series X (t) of length N, the rate R/S is defined, where R (N) indicated range of cumulative series X_{cum} (t, N), S (N) stands for standard deviation of original time series:

$$\text{R/S (N)} = \frac{\max(X_{cum}(t,N)) - \min(X_{cum}(t,N)}{\sqrt{\frac{1}{N-1}\sum_{t=1}^{N}[X(t)-\bar{X}]^2}}$$

Where,

$$\bar{X} = \frac{1}{N}\sum_{t=1}^{N} X(t)$$

$$X_{cum}(t,N) = \sum_{i=1}^{t}[X(i) - \bar{X}]$$

For fractal process and large values of N, the range R depends on series of random variables has fixed standard deviation (S) and is independent[15], and then R/S has follow power law as

$$\text{R/S} \sim (C.\text{N})^{\text{H}}$$

Where, ‘C’ is a constant. H (Hurst exponent) can be determined by rescaled range analysis

$$\text{H} = \frac{log_2(R/S)}{log_2(N)}$$

The Hurst exponent (H) is the slop of the fitted curve relating to $\log_2$ (R/S) against$\log_2$ (N). This procedure can be completed least square method that gives the estimation of Hurst exponent [12]. Finally, the relation between fractal dimension and Hurst exponent then becomes D= 2 – H as defined by [11, 16].

Practically, the Hurst exponent (H) evaluates the persistency, anti-persistency and Brownian character of the time series data. The value of H varies between $0 \leq H \leq 1$. For the case when $H > 0.5$ then we have persistence in time series shows the occurrence of strong positive correlation. In the second case when the value of H is between 0 and 0.5($0 < H < 0.5$) considered as anti-persistence tends to move toward mean i.e. negative correlation and negative trend in time series data. Finally, H=0.5 confirm Brownian nature (white noise) i.e. the time series is self-sufficient and uncorrelated [15] and there is zero correlation, it implies that the roughness of the time series increases [16].

3. RESULTS AND DISCUSSION

3.1 Time Series of Plasma Turbulence

Albeit, the basic objective of the study depended on statistical analysis of plasma turbulence and their nonlinear behavior at Pakistan atmospheric region, this section describes brief explanation of daily and seasonal variation of plasma turbulence during the period 1989. The daily variations of plasma turbulence at Pakistan atmospheric region are shown in Fig. 1.

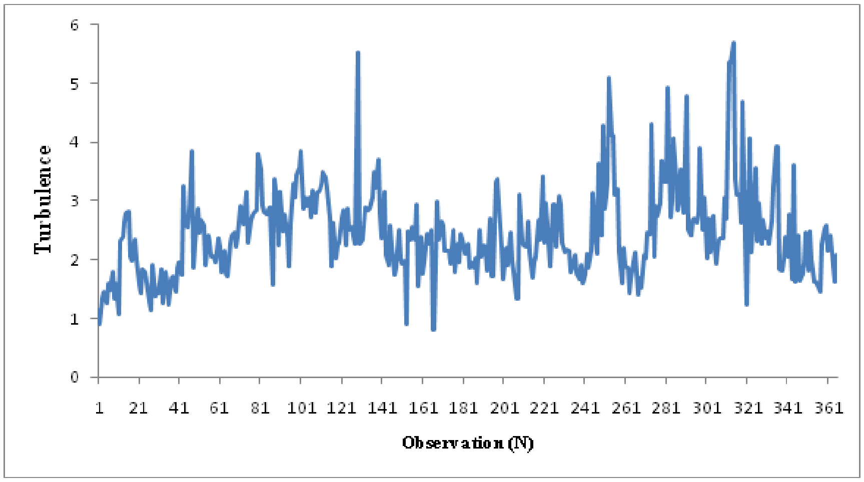

Fig. 1. Daily average time series observations of plasma turbulence at Pakistan atmospheric region for the year 1989.

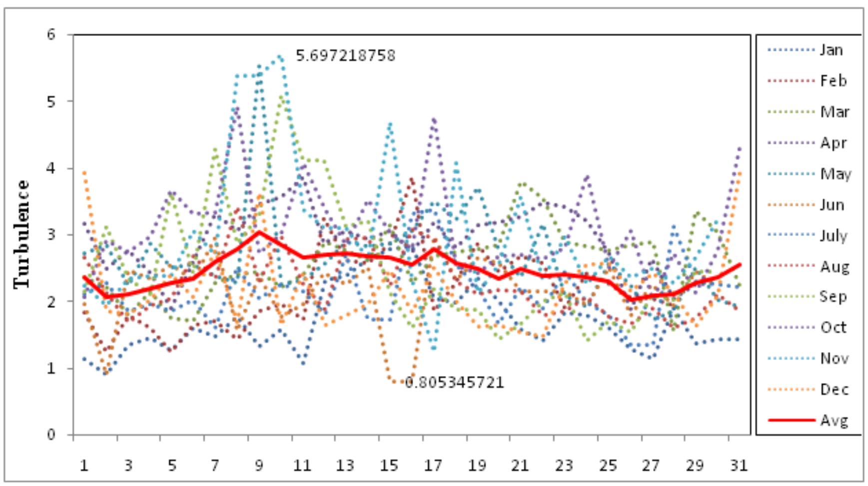

Fig. 2. Monthly average time series plot of plasma turbulence at Pakistan atmospheric region for the year 1989.

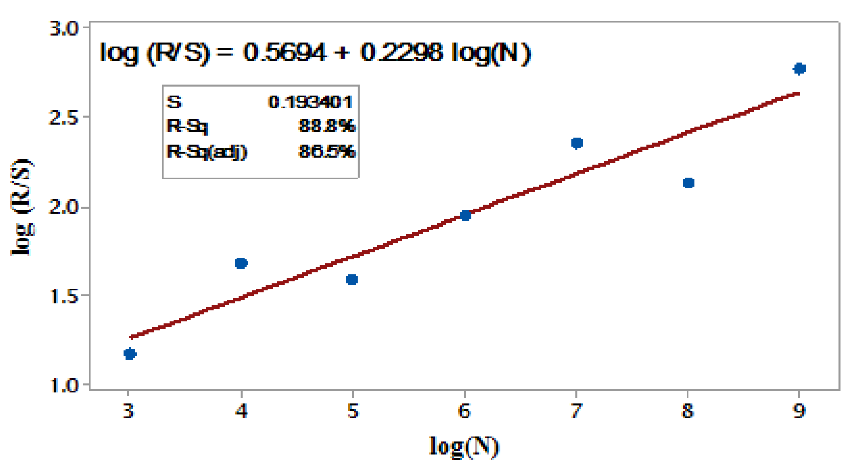

Fig. 3. Log-log fitted interpretation to estimate Hurst exponent for plasma turbulence at Pakistan atmospheric region.

Table 1. Parametric values of the plasma turbulence data series during year1989 in the Pakistan atmospheric region.

Parameters	Value
Sample size	365
Mean	2.445820546
Standard Deviation	0.768061337
Variance	0.589918218
Kurtosis	2.445118638
Skewness	1.099233839
Minimum	0.805345721
Maximum	5.697218758

Table 2. Estimation of rescaled range analysis for ionospheric turbulence over the Pakistan region to estimate Hurst Exponent for year 1989.

Level	Data size (N)	R/S	$\log_2(N)$	$\log_2(R/S)$
1	256	6.8109	9	2.7678
2	128	4.3677	8	2.1268
3	64	5.1171	7	2.3553
4	32	3.8409	6	1.9414
5	16	3.0125	5	1.5909
6	8	3.2037	4	1.6797
7	**4**	**2.2592**	**3**	**1.1758**

It revealed clearly diurnal peaks variation. The uppermost amplitude of the diurnal peak was 5.697219 on 11th November and lowest 0.805346 on 15th June (Fig. 2). This may have resulted from additional formation of ions or instability at this region. The red line shows the average variation in plasma turbulence at studied region. The summary of the fluctuation of plasma turbulence in the atmospheric region of Pakistan is reported in Table 1.

Skewness is an indicator applied in distribution analysis as a sign of irregularity and deviation from normal distribution [14]. Positive skewness is a sign of right skewed allocation of plasma turbulence. It means that mainly of values are resolute on left of the mean with extreme values. Kurtosis illustrates the virtual peakedness or smoothness of a distribution compared with the normal distribution. Positive kurtosis in this case as shown in Table 1 highlights a quite peaked distribution, representing more intermittency in plasma turbulence in Pakistan.

3.2 Fractal Structure of Plasma Turbulence

The second section aimed to investigate the intermittent nature of plasma turbulence. The issue of fractal dimension is of great importance for space plasmas [17] researchers because it described the irregular nature of plasma turbulence in the solar wind [9]. As discussed earlier, turbulence in space plasmas usually include fluctuations of all diversity and sizes, which interact and transmit along the plasma system. For further illustration, consider some detailed fluctuations that have predictable geometrical characteristics in three-dimensional Euclidean space. Due to the intermittent and confined nature of plasma turbulence, it is simple to visualize that they usually cannot fill the complete three-dimensional space that they take up at a particular time. In another words, the space that the fluctuations inhabit is simply a fraction of the complete three-dimensional space. These geometrical characteristics i.e. fractal or fractal geometry were studied by Mandelbrot in 1977 [17, 18]. The method of rescaled range analysis is briefly discussed in methodology and is applied independently on the average daily plasma turbulence of 256 data points over Pakistan atmospheric region for the year 1989. The process of rescaled range analysis for different levels of plasma turbulence is estimated in Table 2.

Using the least square procedure, slope of the best fitted straight line to the curve of $\log_2(R/S)$ between $\log_2 (N)$ illustrated the significant value of Hurst exponent (Fig. 3). From Fig. 3, the observed value of Hurst exponent of plasma turbulence was H=0.193401 (approximately H=0.2). The accuracy of the result was about 88.8 % at 95% confidence interval at $P < 0.05$ which revealed the strong correlation. Using the relation D = 2- H, we obtained the fractal dimension of plasma turbulence is D = 1.8. Since, the value of H is significantly < 0.5 and fractal value D > 1.5 for our data, this indicating that the behavior of plasma turbulence at considered region considerably anti-persistence, i.e., negative correlation. This result indicated randomness and intermittency in plasma turbulence.

4. CONCLUSIONS

In this study, we observed an irregular nature of plasma turbulence in upper atmospheric region of Pakistan. In particular, we established the fluctuation of plasma turbulence on perspective of statistical analysis and fractal dimension analysis to observe their nonlinear dynamics and complex behavior. The results emphasized that the instability and intermittency of plasma turbulence is due to the diurnal peaks' variation and positive character of kurtosis and skewness. In section two, we have demonstrated the fractal dimension with the help of Hurst exponent using rescaled range analysis. There are 256 data points and corresponding value of fractal dimension D= 1.8 for plasma turbulence (i.e., D > 1.5), this means that the behavior of plasma turbulence at atmospheric region of Pakistan shown anti-persistence, i.e., negative correlation implying that randomness and intermittency in plasma turbulence. Finally, to conclude, it is observed that an important advantage of this method is extremely helpful to explore the chaotic behavior of plasma turbulence.

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