ISSN Print: 2518-4245
ISSN Online: 2518-4253
Vol. 53(3), September 2016

PROCEEDINGS
OF THE PAKISTAN ACADEMY OF SCIENCES:
A. Physical and Computational Sciences

PAKISTAN ACADEMY OF SCIENCES
ISLAMABAD, PAKISTAN

Annual Subscription: **Pakistan:** Institutions, Rupees 2000/- ; Individuals, Rupees 1000/-
Other Countries: US$ 100.00 (includes air-lifted overseas delivery)

HEC Recognized, Category X; PM&DC Recognized

Published by **Pakistan Academy of Sciences**, 3 Constitution Avenue, G-5/2, Islamabad, Pakistan
Tel: 92-5 1-920 7140 & 921 5478; **Fax:** 92-51-920 6770; **Website: www.paspk.org**

Printed at **PanGraphics (Pvt) Ltd.,** No. 1, I & T Centre, G-7/l, Islamabad, Pakistan
Tel: 92-51-220 2272, 220 2449 Fax: 92-51-220 2450 E-mail: pangraph@gmail.com

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 239–253 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

An Exploratory Study of Success Factors in Software Integration for Global Software Development Vendors

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Abstract: The trend of software development has changed from local to global software development (GSD) with acceleration in information and communication technologies. GSD offers certain benefits like reduced cost, high quality, availing skilled human resource and latest technology etc. It also faces a lot of challenges like communication coordination issues, cultural, language and temporal challenges along with the technical challenge of software integration. The objective of the current study is to identify success factors for software integration to assist GSD vendors in integrating the software components into a final working product. We have used the systematic literature review (SLR) process by following standard SLR guidelines. We have identified a list of 14 success factors by extracting data from 89 selected papers. Out of these, 9 factors were ranked as critical success factors (CSFs). Some of the top ranked CSFs are "Consistency in Requirements and Architecture Design", "Intra and inter team Communication and Coordination" and "Component/Unit Testing prior to integration". We have also analyzed these CSFs on the basis of period published, project size and methodology used. We identified 9 CSFs which may assist GSD vendors in almost all size of projects at different phases of the software integration process.

Keywords: software integration, systematic literature review, global software development, vendors, success factors

1. INTRODUCTION

The trend of software development has been changed from local to global software development (GSD) with the rapid acceleration in information and communication technologies (ICTs) from the last decade. The ICTs have connected the software development organizations and the development teams to perform software development activities across national boundaries. The reason for the changing trend is to develop high quality software with low cost in minimum time by working 24-hours around the clock. GSD has not only solved the problem of finding skilled human resource but also provide the vendors to avail latest infrastructure and show their presence locally at different locations of the globe [1, 2]. Like any other field GSD will also take its time to grown-up and overcome the challenges like temporal, cultural and language differences, communication and coordination problems and deprived contract, knowledge and relationship management, etc. [3-6]. Alongside the above non-technical issues the GSD teams also face technical issues like problems in the architecture of the software components, their version management, configuration management and integration of the software components developed in isolation by GSD teams with inadequate communication [7, 8]. The integration phase uncover many of the problems that remain hidden in the previous phases of software integration [9].

Almost all types and size of software are composed from more than one software components/ modules. These components may be developed in-house or may be outsourced offshore or onshore.

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Similarly the components may be purchased from the market as a "commercial off the shelf (COTS)" component or from the large pole of open source community as a "off the shelf (OTS)" component. There is a need to identify the positive factors that may ease the integration process and the barriers that hurdle it in the above different scenarios [7, 8].

Integration is one of the most critical phases but its importance is not fully understood all the time. Enough resources and proper integration plan is crucial for this phase because this phase will assemble all the parts developed independently by GSD teams[10]. Regardless the importance of the software integration phase, enough empirical research has not been done for the identification of software integration success factors and their practices/solutions [9, 11]. We have planned to assist GSD vendors in the software integration process by developing a software integration model (SIM) [12].

For bridging the gap and easing the integration process for GSD vendors we have designed the following research questions.

RQ1: What are the success factors, as identified in the literature, to be adopted by GSD vendors software at various stages of the product integration, i.e. before, during and after the integration process in GSD environment?

RQ1-a Do the identified factors vary from decade to decade?

RQ1-b Do the identified factors vary from project to project?

RQ1-c What methodology/study strategy has been used in the selected papers as identified by the SLR?

This research paper is the extended version of our earlier published paper in IEEE SNPD 2015 Conference at Japan [13]. Because of the page limitations of the conference we were unable to report all findings/analysis of our SLR study. In the current paper we have reported findings of RQ1 supplemented by RQ1-a, RQ1-b and RQ1-c.

The remaining paper is structured as follows. In Section 2 we have presented background/related work. In Section 3, the methodology is presented. In Section 4, results of the SLR related to the above mentioned research questions are reported. In section 5 we present summary and discussion about the current study and in Section 6 we present the limitations of the study. The last section, i.e., section 7 present conclusions and future work.

2. BACKGROUND

The integration process require proper attention of the developers because many of the software projects are delayed during testing due to the complexities and incompatibilities found between software components in the integration phase [9]. The vendors need to properly plan the integration strategy while boarding into the software product development before assembling the sub components developed [13].

McConnel [14] describes integration as an activity of software development in which the isolated software components are combined in a single unit. Herbsleb et al. [15], in a case study reported that integration phase is one of the most complicated phases of software projects in GSD environment. They found that the loss of communication and coordination hurdle the integration phase in multisite development. Similarly the components required for assembling the final product may be unavailable according to the schedule expected and the components available may have fault in their interfaces.

In an exploratory study, Van Moll et al. [16] reported that the software integration phases is one of the complex and challenging phase in GSD environment for more than 50% of the software projects. They recommended considering the integration and software testing phase as a separate process. Guimaraes and Silva [17] have proposed a solution for continuous integration that can automatically integrate the committed and uncommitted code in the background. Thus it can automatically detect any conflict that may occur during programming in multiples teams of developers. Stayhl and Bosch [18] also performed a systematic review of literature about the practices of continuous integration. They reported that no

uniform practices exist for continuous integration in all development environments. They further reported that variations exist from environment to environment and different environment may have different set of practices for continuous software integration. Tekumalla [7] from a systematic review reported that some topics of components based software engineering (CBSE) like implementation, selection and components quality are populated while other topics like components integration, testing and their storage need to be empirically researched through experiments and case studies in industry.

Similarly S. Schneider et al [8] from an SLR reported that certain areas of software engineering like requirements engineering and project management are very populated while project monitoring and control and software product integration are narrowly populated indicating the need for research in the area.

It is evident from the literature that there is no study which has considered integration problems deeply on the basis of software project size and types of software products. There is no categorization of factors in literature that the vendors need to consider at all three stages of software integration process, i.e., before integration, during integration and after integration. In the current paper, through the SLR process, we have reported a list of positive factors for each stage of the software integration process. We also examined that how these success factors differs in their significance on the basis of software project size. We have further analyzed how these success factors change from decade to decade (1994-2003, 2004-2013) due to maturity in the integration phase of GSD products or due to technological changes. We have also analyzed the methodologies used in the studies identified through SLR.

3. STUDY DESIGN

We have conducted a systematic literature review (SLR) to obtain the results of the current study by following the SLR standard guidelines [19]. The SLR process is used by many researchers as it is moderately rigorous and fabricate more reliable outcome in contrast to other methods of literature review [7, 8]. The SLR process utilizes a well defined methodology and is repeatable up to some extent. The SLR, which is a secondary study, has three phases i.e planning, conducting and reporting the review [19]. In the planning phase the protocol for the study is developed. The protocol for the current study has already been published [20], implemented and now we are reporting the results and analysis of the SLR process in this paper.

3.1 Search

The data in Table 1 list the libraries that we searched, the search string used for searching and the number of primary studies selected during the process. It is worth to be noted that some of the search engines and digital libraries, e.g., Google scholar have a limit of 256 characters on the length of search string. For this reason we divided our search string for Google scholar into four small sub strings. The selection of the primary studies was performed in two steps. Firstly the primary reviewer made the initial selection of papers by reading the title and abstract of the paper. Thus in the first step we selected 336 papers. After passing through the first step the final selection of the papers was made by reading the full text of the paper by primary reviewer. During the final selection we also followed the inclusion/exclusion and quality assessment criteria as already defined in our protocol for this study [20]. In some situations, regarding uncertainty about the inclusion/exclusion decision, the secondary reviewer was consulted for review. To perform inter-rater reliability test the secondary reviewer randomly extracted data from few papers and compared it with the data extracted by primary reviewer but there were no major discrepancies found between the data extracted. As mentioned in our protocol [20], the primary reviewer extracted 17 pieces of data from each selected paper. The secondary reviewer also extracted those pieces of data from few papers. We applied the Cohen's Kappa test for inter-rater reliability and found that its coefficient value is 0.89 which shows a perfect agreement between the primary and secondary reviewers.

We performed a systematic search in the

Table 1. Search results.

Name of database	Search string used in the corresponding library		No. of publications found	Initial selection	Final selection
Science Direct	[("Software integration" OR "Product integration" OR "Component integration" OR "system integration") AND ("Global software development" OR GSD OR "Global software engineering" OR GSE OR "Distributed software development" OR DSD OR "Distributed software engineering" OR DSE) AND (Challenge OR risk OR problem OR issue OR barrier OR trouble OR "critical factor" OR "key factor" OR "success factor")]		185	13	07
ACM			195	23	11
Springer			205	15	05
IEEE			417	34	16
Wiley Online Library			81	08	02
Google Scholar	**String01**	("Software integration" OR "Product integration" OR "Component integration" OR "System Integration") AND ("Global software development" OR "Global software engineering") AND (Challenge OR risk OR problem OR issue OR barrier OR trouble)	1031	163	88
	String02	("Software integration" OR "Product integration" OR "Component integration" OR "System Integration") AND ("Distributed software development" OR "Distributed software engineering") AND (Challenge OR risk OR problem OR issue OR barrier OR trouble)			
	String03	("Software integration" OR "Product integration" OR "Component integration" OR "System Integration") AND ("Global software development" OR GSD OR "Global software engineering" OR GSE) AND ("critical factor" OR "key factor" OR "success factor")			
	String04	("Software integration" OR "Product integration" OR "Component integration" OR "System Integration") AND ("Distributed software development" OR DSD OR "Distributed software engineering" OR DSE) AND ("critical factor" OR "key factor" OR "success factor")			
Snow Balling	Following the references of the selected papers and searching for the publications of the author of the selected papers		**80**	**80**	**43**
Total			**2194**	**336**	**172**
Duplicate papers					**67**
Finally selected papers					**105**

libraries listed in Table 1 and also performed the snowballing technique [21] by:

- following the references in the papers that were finally selected; and
- and searching by authors name found in the references of the papers that were finally selected.

The snowballing procedure increased the sample size of the finally selected papers from 63 to 105 papers after eliminating the duplicate papers as presented in Table 1. The search process is depicted in Fig. 1. The data for RQ1 and RQ1-a to RQ1-c

Fig. 1 Search process.

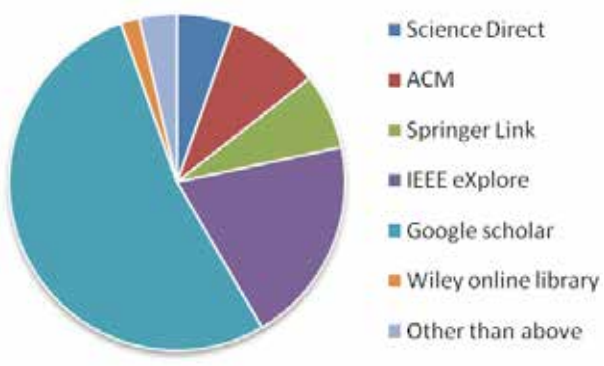

Fig. 2. Publications found in each library.

was extracted from 89 papers among the finally selected 105 papers. The remaining 16 papers were dropped due to quality assessment criteria defined in the protocol designed for this study [20]. The final list of papers found in each library through both searching and snow balling method is shown in Table 2 and graphically depicted in Fig. 2.

3.2 Data Synthesis

We initially extracted the data from 89 finally selected papers and grouped these identified factors into 27 categories/factors. The secondary reviewer, after a thorough review, merged them into 14 categories/factors. The grouping of the factors was carried out in such a way that the similar factors were grouped under the same heading. The final list of 14 success factors is shown in Table 3 and the list of papers from which the data was extracted can be found in appendix-A. More detail can be found in our previously published papers [13, 20]

4. RESULTS

This section discusses the results and examines the identified success factors for each of the Research Questions as stated in Section 1.

Table 2. Final number of publications found in each library (searching + snowballing).

S. No	Library Name	Total public: found	% (n=105)	Public: found for Success Factors	% (n=89)
1	Science Direct	9	8.57%	5	5.60%
2	ACM	15	14.28%	4	4.50%
3	Springer Link	12	11.42%	6	6.70%
4	IEEE eXplore	33	31.42%	27	30.30%
5	Google scholar	88	83.80%	37	41.60%
6	Wiley online library	3	2.85%	2	2.20%
7	Other than above	6	5.71%	7	7.90%
Total Papers found		**156**	**100%**	**89**	**100%**
Duplicate Papers		**51**	**32.69%**	**-**	**-**
Net Total		**105**	**67.31%**	**-**	**-**

Table 3. List of success factors for software integration in GSD.

S. No	Success Factors	Freq (n=89)	%	Reference of papers in appendix A, in which the factor has been found
1	Consistency in Requirements and Architecture Design	35	39	1, 2, 5, 6, 7, 8, 10, 17, 19, 20, 22, 23, 26, 32, 35, 36, 39, 40, 41, 46, 48, 50, 51, 52, 53, 55, 60, 70, 72, 74, 75, 76, 85, 87, 88
2	Intra and inter team Communication and Coordination	34	38	1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 17, 20, 21, 26, 27, 30, 36, 37, 38, 39, 43, 47, 51, 53, 59, 65, 66, 67, 70, 76, 81, 82, 89
3	Component/Unit Testing prior to integration	34	38	1, 2, 3, 4, 8, 11, 14, 15, 17, 19, 22, 23, 24, 26, 27, 28, 29, 34, 35, 38, 39, 43, 44, 58, 62, 64, 66, 68, 69, 73, 76, 77, 79, 82
4	Advance & Uniform Development Environment and Training	33	37	1, 3, 6, 7, 12, 14, 16, 19, 22, 23, 24, 25, 26, 31, 32, 34, 35, 38, 39, 40, 55, 56, 58, 67, 68, 71, 74, 76, 81, 85, 86, 88
5	Efficient Incremental/Continuous integration	31	35	1, 2, 3, 6, 8, 14, 17, 18, 20, 24, 26, 27, 29, 31, 34, 35, 40, 42, 43, 44, 46, 48, 50, 63, 65, 66, 68, 80, 83, 88
6	Efficient specification for Interface Compatibility	31	35	2, 8, 10, 14, 15, 19, 20, 25, 26, 30, 32, 33, 40, 41, 44, 45, 51, 54, 55, 56, 57, 58, 64, 70, 73, 74, 76, 77, 84, 85, 86
7	Proper Documentation & Configuration Management	28	32	2, 6, 7, 8, 13, 14, 15, 17, 19, 22, 23, 26, 27, 31, 33, 38, 39, 47, 48, 55, 67, 70, 73, 76, 81, 86, 88, 89
8	Early Integration Planning and Centralized P3 management	27	30	1, 5, 6, 7, 8, 11, 13, 14, 17, 19, 20, 22, 23, 27, 30, 32, 34, 35, 38, 43, 47, 48, 67, 68, 76, 81, 85, 86
9	Careful evaluation of the COTS/OTS Components	27	30	3, 4, 8, 10, 22, 26, 30, 32, 38, 40, 45, 50, 53, 55, 60, 61, 67, 68, 69, 73, 74, 75, 77, 78, 83, 86, 87
10	Use of Standard Model for Process, Data and Product's Components	7	8	17, 26, 49, 56, 73, 76, 85
11	Use of modular approach	4	5	17, 20, 52, 76
12	Use of Efficient Metrics	2	2	13, 70,
13	Use of Quality assurance	1	1	17
14	Specific Integration Timing	1	1	47

4.1 Software Integration Success Factors

We have identified a list of 14 success factors, using SLR process to answer RQ1, as presented in Table 3. As discussed in Section 3, we have identified a total of 14 success factors in which nine success factors are significant or critical success factors (CSFs). The criterion to decide a factor is critical or not was based on the percentage of frequency with which the factor was appeared and discussed in literature. We used the threshold of 30% for deciding a factor to be a critical one. Thus if the frequency percentage of a factor was more than or equal to 30%, we marked that factor to be a critical one as did by other researchers [3, 6]. The detail about these nine CSFs can be found in our published paper [13].

4.1.1 Comparison of CSFs between Co-located and GSD Projects

The CSFs listed in Table 3 shows that most of the CSFs are also applicable to co-located projects. The CSFs "Component/Unit Testing prior to integration", "Advance & Uniform Development Environment and Training", "Efficient Incremental/ Continuous integration", "Efficient specification for Interface Compatibility", "Proper Documentation & Configuration Management" and "Careful evaluation of the COTS/OTS Components" are the factors that can be considered vital in both co-located and GSD projects for integration process. While the CSFs "Consistency in Requirements and Architecture Design", "Intra and inter team Communication and Coordination" and "Early Integration Planning and Centralized P3 management" are the factors that are important in the GSD environment. It is obvious that all the GSD teams must have a consistent definition of requirements at each site with a consistent architecture design. The GSD teams need a lot of communication and coordination to maintain the above consistency and synchronize their work. Similarly the management of GSD projects needs to properly plan the integration process in advance

and centrally control the project, products and processes.

4.2 Decade-wise Comparison of the Success Factors

To answer RQ1-a, we divided the search period in two decades i.e. from 1994 to 2003 and from 2004 to 2013. It should be made explicit that we have put no date boundaries on our search process but we did not find any relevant paper before 1994. The number of publications found in each decade is presented in Table-4.

Table 4. Decade wise break up of publications.

Decade	Frequency	Percentage
1994-2003	24	27%
2004-2013	65	73%

The data in Table 4 show that integration process has got much more attention of the researchers in the second decade as compared to the first decade. One reason for this may be that GSD and CBSE started recently in the last decade with the advances in ICT technologies. The integration phase has become more critical with GSD development as the finally developed isolated components needs to be integrated into a final product.

To analyze the data of both decades we have performed multiple tests on the data. A comparison of each CSF based on the two decades is presented in Table 5. We have used linear by linear association Chi-square test for finding any significance difference in the success factors between the two decades. A linear by linear association Chi-square test is considered more powerful than Pearson ' s χ 2 test [22].

The data in Table 5 shows that there is a minor difference between the two decade for the success factors CSF1 and CSF4. It means that these factors have been considered important in both decades and have still gained the researcher attention.

While the factors CSF2, CSF5, CSF 7 and CSF 8 have got importance in second decade with the emerging trend of GSD. The factor CSF 6 has gain less researcher attention in the second decade. The possible reason may be the maturity of interface specification in CBSE. The remaining two factors CSF3 and CSF9 have significant difference between the two decades. The importance of the unit/component testing may have been increased with the advent of CBSE in the second decade.

We also performed correlation analysis, by using Spearman's rank correlation method, to find the degree of relationship between the data of two decades as shown in Table 6. We gave the rank value 1 to the highest publication frequency. Similarly the

Table 5. Comparison of critical success factors in each decade.

Critical Success Factor	Decade 1 1994-2003 (n=24)		Decade 2 2004-2013 (n=65)		Chi-square test (linear-by-linear association) $\alpha = 0.05$		
	Freq	%	Freq	%	X^2	Df	P
CSF1 Consistency in Requirements and Architecture Design	10	42	25	38	0.075	1	0.785
CSF2 Intra and inter team Communication and Coordination	6	25	28	43	2.399	1	0.121
CSF3 Component/Unit Testing prior to integration	5	21	29	45	4.152	1	0.042
CSF4 Advance & Uniform Development Environment	8	33	25	38	0.195	1	0.658
CSF5 Efficient Incremental/Continuous integration	5	21	26	40	2.805	1	0.094
CSF6 Efficient specification for Interface Compatibility	**14***	**58**	**17**	**26**	**7.906**	**1**	**0.005**
CSF7 Proper Documentation & Configuration Management	5	21	23	35	1.702	1	0.192
CSF8 Early Integration Planning and Centralized P3 management	5	21	55	85	1.389	1	0.239
CSF9 Careful evaluation of the COTS/OTS Components	**14**	**58**	**13**	**20**	**12.050**	**1**	**0.001**

*The values which have statistical significance difference (P<0.05) have been highlighted as bold.

Table 6. Spearman's rank correlation.

Critical Success Factor	Decade 1 1994-2003 (n=24)		Decade 2 2004-2013 (n=65)	
	Freq	**Rank**	**Freq**	**Rank**
CSF1 Consistency in Requirements and Architecture Design	10	3	25	5.5
CSF2 Intra and inter team Communication and Coordination	6	5	28	3
CSF3 Component/Unit Testing prior to integration	5	7.25	29	2
CSF4 Advance & Uniform Development Environment	8	4	25	5.5
CSF5 Efficient Incremental/Continuous integration	5	7.25	26	4
CSF6 Efficient specification for Interface Compatibility	14	1.5	17	8
CSF7 Proper Documentation & Configuration Management	5	7.25	23	7
CSF8 Early Integration Planning and Centralized P3 management	5	7.25	55	1
CSF9 Careful evaluation of the COTS/OTS Components	14	1.5	13	9

next highest value is given the rank value 2 and so on as shown for the data in the second decade. In some cases whenever two or more values were the same then we calculated the average of their ranks and gave the same average rank to each of them. For example for decade two there were two values of 38 with rank 5 and 6, we assigned the average value of 5.5 to each. The same method was applied for data in decade 1.

The correlation coefficient (ϒ) is –0.67, which shows that there is a strong negative correlation between the factors across the two decades. It means that the researcher priorities have been changed in the second decade. This has been depicted by the scatter graph in Figure 3.

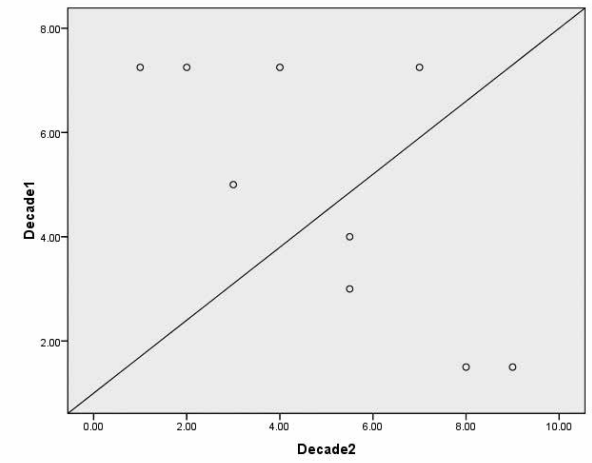

Fig. 3. Scatter graph for correlation.

4.3 Comparison on the basis of project size

To answer RQ1-b we initially divided the projects into three categories (small, medium and large) on the basis of project size as reported in the papers. Table 7 represents the frequency of papers found for each project size. The data in the table shows that most research work has been done on large size projects, while the majority of papers have not explicitly mentioned the project size.

Table 7. Project size break up of publications.

Project Size	Frequency	Percentage
Small	04	05
Medium	01	01
Large	31	35
Mixed/ Not Mentioned	53	60

We have further analyzed each CSF on the basis of project size in Table 8. Due to low frequency of success factors for small and medium projects we have combined their data into one column under the heading "Small and Medium". The data in Table 8 show that the frequency of papers for small and medium projects is five and each success factor frequency is at least two except for the factor CSF9. The factors CSF2, CSF3 and CSF5 can be considered as the most important factors for large projects. The factors CSF1, CSF4, CSF7

Table 8. Comparison of success factors based on the project size.

Critical Success Factor	Total sample size found through SLR study (n=89)						Chi-square test (linear-by-linear association) α = 0.05 Df=1	
	Small and Medium (n=5)		Large (n=31)		Not Mentioned / mixed (n=53)			
	Freq	%	Freq	%	Freq	%	X^2	P
CSF1 Consistency in Requirements and Architecture Design	2	40	11	35	22	42	0.226	0.634
CSF2 Intra and inter team Communication and Coordination	2	40	16	52	16	30	2.834	0.092
CSF3 Component/Unit Testing prior to integration	2	40	16	52	16	30	2.834	0.092
CSF4 Advance & Uniform Development Environment and Training	3	60	10	32	20	38	0.050	0.823
CSF5 Efficient Incremental/Continuous integration	2	40	14	45	15	28	1.716	0.190
CSF6 Efficient specification for Interface Compatibility	2	40	8	26	21	40	0.823	0.364
CSF7 Proper Documentation & Configuration Management	3	60	11	35	14	26	2.236	0.135
CSF8 Early Integration Planning and Centralized P3 management	2	40	12	39	14	26	1.246	0.264
CSF9 Careful evaluation of the COTS/OTS Components	**1**	**20**	**3**	**10**	**23**	**43**	**7.373**	**0.007**

The values which have statistical significance difference (P<0.05) have been highlighted as bold.

and CSF8 can be considered second in the ranked of importance for large projects. The data listed in Table 9 illustrate that there is a significant difference for only the last factor i,e CSF9. This factor has low frequency for small/medium and large size projects and has the highest frequency in the "Not Mentioned/mixed" column. The studies which have not mentioned their project size or which have mentioned all size of projects i.e. mixed category have highest frequency and hence it can be deduce that CSF9 is necessary for all size of projects to be successful in the integration process.

Table 9. Methodology wise break up of publications.

Methodology	Frequency	Percentage
Case study	38	43
Technical reports	12	14
Interview	8	9
Literature review	7	8
SLR	7	8
Experience report	4	5
Questionnaire survey	3	3
Experiment	3	3
Field study	3	3

4.4 Methodology used in the selected papers

As discussed in section 3, the data for CSF was extracted from 89 papers. To answer RQ1-d we have analyzed the methodologies used in these papers in Table 9. The most dominating methodology used in these papers is "Case Study" with 43%. Technical reports have 14%, interview has 8%, while literature review and SLR has 7%. Similarly experience report has 4% while questionnaire survey, experiment and field study has 3% each.

We have further compared and examined each factor on the basis of the study strategy used as shown in Table 10. In this table we have combined experience report, questionnaire survey, experiments and field studies into one group named "Other" because of their low frequencies. We also performed linear-by-linear Chi-square test for each factor based on the methodology used to find any significance difference among the methods if they have. From Table 10 we can see that the only factor which has significant difference is the CSF8. This factor has a 21% frequency for case studies, 25% for technical report and 50% for interview. It has no occurrence in the ordinary literature review while 57% frequency in the SLR. Similarly it has

Table 10. Comparison of success factors based on methodology used.

Critical Success Factor	Total sample size found through SLR study (n=89)												Chi-square test α = 0.05 Df=1	
	Case Studies (n=38)		Technical Reports (n=12)		Interview (n=8)		Ordinary Review (n=7)		SLR (n=7)		Other (n=13)			
	F	%	F	%	F	%	F	%	F	%	F	%	X^2	P
CSF1 Consistency in Requirements and Architecture Design	16	42	5	42	3	38	1	14	3	43	7	54	0.001	0.970
CSF2 Intra and inter team Communication and Coordination	17	45	2	17	4	50	0	0	4	57	7	54	0.172	0.679
CSF3 Component/Unit Testing prior to integration	14	37	2	17	4	50	3	43	4	57	7	54	0.087	0.768
CSF4 Advance & Uniform Development Environment and Training	16	42	4	33	5	63	1	14	3	43	4	31	1.329	0.249
CSF5 Efficient Incremental/Continuous integration	16	42	3	25	2	25	1	14	3	43	6	46	0.242	0.623
CSF6 Efficient specification for Interface Compatibility	13	34	6	50	2	25	4	57	1	14	5	38	0.138	0.711
CSF7 Proper Documentation & Configuration Management	11	29	0	0	3	38	2	29	3	43	9	69	0.826	0.363
CSF8 Early Integration Planning and Centralized P3 management	**8**	**21**	**3**	**25**	**4**	**50**	**0**	**0**	**4**	**57**	**9**	**69**	**5.150**	**0.023**
CSF9 Careful evaluation of the COTS/OTS Components	8	21	6	50	4	50	3	43	3	43	3	23	0.592	0.442
Number of factors found	**9**		**8**		**9**		**7**		**9**		**9**			

The values which have statistical significance difference (P<0.05) have been highlighted as bold.

69% frequency in the "other category". At the end of the table we have also listed the number of critical factors identified by each methodology. We found that case studies, interview and SLR each have identified 09 CSF while technical reports and literature review have identified 8 and 7 CSF respectively. Thus it is evident that nearly all of our identified CSFs have been reported in studies conducted through various methodologies.

The purpose of above analysis is to find out the relative weight of each factor i.e. to find out that a factor is identified by what type of methods. It is more authentic if a factor is identified by both literature and empirical methods like case studies, interviews and questionnaire survey etc. The data in Table 10 shows that out of 89 papers, 38 papers have used the case study method and all the factors are identified using case study methods in addition with other methods. This shows the importance of our identified factors because all the factors have been mentioned in the empirical studies.

5. DISCUSSIONS AND SUMMARY

Through this SLR study we identified a list of success factors for GSD vendors to assist them in successful integration of the software components. We further categorized the critical success factors of software components integration, which needs special attention of GSD vendors. The criterion we used for a success factor to be critical or not was based on its citation in the literature. The factors having a frequency percentage >=30 were marked as critical success factors. The practitioners can, however, define their own criteria for the factors in the list to be critical or not. Thus for answering RQ1 we identified a list of nine critical success factors as shown in Table 3 from serial number 1 to 9.

To answer RQ1-a, we divided the time period of publications into two decades. It was found that the research on software integration has been boosted in the second decade as compared to the first decade. Because in first decade we found 24 publications for success factors while in second decade we found 65 publications. This shows an increase in the importance of the software integration. The data in Table 5 show that all the CSFs are present in both decades i.e these CSFs have been considered important in both decades. We also categorized the above CSF on the basis of project size i.e small,

medium and large for RQ1-b. As large projects are more complicated and face more problems, the literature has cited these CSF in studies conducted on large size projects. Majority of the studies have not explicitly mentioned any specific project size, so these CSF can be considered important for integration process in all size projects.

We further categorized the selected papers on the basis of methodology/ strategy used for RQ1-c. The most dominating methodology used is case study with 43% citation in the selected papers. We also compared the CSFs on the basis of study strategy used and found that case study, interview and SLR, all three method have identified all the CSFs. Technical reports missed one CSF while literature review missed two CSFs.

6. LIMITATIONS

How valid are the results of success factors for achieving successful integration? In some papers the authors have not mentioned explicitly that why they have considered a particular factor to be vital for the integration process. This may be a threat to internal validity. However it was not possible to overcome this problem independently. Similarly some studies contributed to this SLR consisted of self experiences reports and case studies which may be possibly subject to the publication bias.

Due to our limited resources we searched only six digital libraries while conducting the SLR process and may have missed some relevant studies in other libraries. According to SLR criteria this is not a systematic lapse [19].

We also used the snowballing technique for finding the relevant papers, which we may have missed during the search process, and to increase the sample size of papers for our SLR.

7. CONCLUSIONS AND FUTURE WORK

We found that the critical success factors CSF1, CSF3, CSF5, CSF6 and CSF9 can play a vital role before the integration process. It may therefore be helpful that GSD vendors give full consideration to the implementation of these success factors before the integration phase. The factor CSF4 can play a vital role during the integration process since the skilled use of modern and advance technology will ease the integration process. It is also necessary to use the same and uniform technology at all GSD sites to avoid complication during the integration process. Lastly the factors CSF2, CSF7 and CSF8 are the factors that can be considered important for all stages of software integration.

The findings of the this study were obtained from a thorough review of the literature using SLR process, which complete the 1st phase of our proposed software integration model (SIM) [12]. The findings of this study will be further validated empirically in industry in future.

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Appendix-A List of selected papers for data extraction.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 255–266 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

A Student-friendly Framework for Adaptive 3D-Virtual Learning Environments

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Abstract: Three dimensional virtual learning environments (3D-VLEs) with adaptive capability have made the learning process easy and closer to one-to-one tutoring. These systems have the ability to dynamically adapt to the learning capability of students and all the activities they perform, which results in improved learning. In this paper, we present a method for defining the adaptive aspect of 3D-VLEs where student learning is quantitatively measured. The contents of 3D-VLEs are changed according to the learning skill of students. A weak learner is provided more time to complete a given learning module while a good learner can finish his work in less time. As a result students become motivated towards learning. The proposed method is student friendly and it enhances the learning capability of the students by providing them learning materials which they can absorb. The experimental results show the effectiveness of our proposed approach for 3D-VLEs.

Keywords: Virtual reality and education, 3D-educational virtual environments, adaptive 3D-virtual learning environments, computer-aided learning, Fuzzy set theory, student learning evaluation

1. INTRODUCTION

Virtual reality (VR) has greatly changed human perception and working styles of organizations towards better performance [1]. The use of virtual technology is increasing day by day and many fields such as medical rehabilitation, architecture, business, training simulators, gaming, entertainment and education are utilizing virtual reality systems for achieving efficiency in their work processes [2, 3]. In desktop VR, three dimensional virtual learning environments are specially designed to assist students in learning[4]. 3D-VLE is a 3D computer representation of space in which students can easily change their view points and interact directly with the virtual world [5]. They can freely navigate inside the environment, select and manipulate different objects in real time which give them the sense of realism [6].

Virtual reality technology is very much suitable for education and all those systems where the physical alternative is not available, the cost of the actual work is very high or the procedure of the work is too dangerous to perform [7, 8]. In Medical field, it is not possible to provide human body to each student of the class for their experiments; therefore VR simulators are very much cost effective solutions for performing virtual operations and studying parts of human body. Another good example is flight simulators where the safety of pilot is very important, so he is trained in virtual environment about various situations which he may face during the fly.

It is obvious that 3D-VLEs have made learning process easy and cost effective but there are some drawbacks of this technology which need further attention for possible improvements. For example, 3D-VLEs are mostly saturated with different objects. Presenting large information on the screen negatively affect the performance of students in the virtual environment. The new user does not know what to do first or next. He is overwhelmed and easily get lost in the

Received, October 2015; Accepted, June 2016
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environment which results in low learnability [5, 8, 9]. The effectiveness of 3D-VLEs is also low for younger students especially those who have low motivation for learning. The reason is that they spend most of their time in activates which are not very much related to learning and hence results in low performance [10, 11]. One solution to the above problems is to make 3D-VLEs in such a way that dynamically adapt to the learning capability of an individual and all the activities which he performs while interacting with the environment [5]. It may prevent students from being overwhelmed by showing him objects according to has learning goals. Adaptivity could make the distinction between education and entertainment which motivate students for learning. Also it reduces the risk of astray navigation inside the virtual environment due to which students cannot focus on the actual learning materials. All these concerns have been discussed in [8, 12] with great detail.

From the above discussion it is clear that the purpose of 3D-VLEs is to enhance the learning capabilities of students. A very good literature is available regarding the adaptivity of 3D-VLEs but research on designing such environments is still immature [5]. Defining the adaptive aspect of 3D-VLEs is a difficult task because there is no clear strategy for how to generally modify the contents of 3D-VLEs to change a task level for a specific student [12, 13]. In this paper, we use "learning skill" is an adaptive criteria for 3D-VLEs. An attempt is made to quantitatively measure student learning in each level. The contents of 3D-VLEs are changed in the next level according to the learning capability of student measured in the previous level. By using this approach students get motivated towards learning, as a result the learning process is improved.

The remaining paper is organized as follows. Literature review is presented in section 2. In section 3, the proposed model is explained which is followed by experimentation in section 4. Experimental results are discussed in section 5 and discussion is given in section 6. Conclusion and future work are presented in section 7 and 8 respectively.

2. LITERATURE REVIEW

Adaptive learning is a ’promising alternative approach’ for the improvement of students learning outcomes [14]. At the start of 20th century adaptive strategies were used in education for the enhancement of student learning and understanding [15]. Adaptive Hypermedia Architecture (AHA!) is a well known versatile adaptive hypermedia framework for adding adaptive features to different applications such as on-line courses, museum sites and encyclopedia etc. AHA! is used to build and maintain the student model for the purpose of providing specific and personalize learning content [16, 17]. Chittaro and Ranon published many papers regarding the adaptation of virtual environments. In 2000, they used an approach called ADVIRT for introducing adaptation inside VR store [18]. Based on some personalization rules the navigation and layout of the store is customized for different users. In 2002, they presented a software architecture solution called Adaptive Web 3D to customize the contents of 3D website according to the needs of the customers [19]. In 2007, same authors proposed adaptation for navigation and interaction which help user to efficiently utilize the information provided by the application [20]. They also worked on the extension of E-learning platform and introduced the concept of adaptive educational virtual environment (EVE) [12, 21].The environment was adaptive according to the learning style of students and they used AHA! engine to achieve adaptivity inside EVE [22]. Brusilovsky et al. [23] used adaptive hypermedia methods for 3D-E-Commerce applications. The environment support different navigation techniques and is adaptive according to the shopping needs of the customer. In [24], fuzzy set theory is used to update learning model. A pre-test is conducted to compute the learning level of learner which enables him to enter in the first module. There is no systematic way defined to assess the learning capability of the learner in depth. In 2004, Santos and Osorio [25] introduced an approach called AdapTIVE (Adaptive Three-dimensional Intelligent and Virtual Environment) for distance learning systems. The approach was based on some virtual agents which help users during interaction with the virtual environments. Similarly, Baziuke [26] designed and implemented a smart adaptive component for virtual learning environment. An agent oriented approach is used for the creation and upgradation of curriculum according to the needs of students. Giuffra and

Silveria [27] used similar approach to provide adaptability to distributed VLEs by considering and monitoring students performance and study material they accessed. D. Zakrzewska [28] applied clustering techniques to provide appropriate layouts to groups of students with similar preferences. Celentano and Pittarello [29] used software sensors to monitor user behavior for controlling navigation and interaction within virtual environment. The sensors record the data whenever a user interacts with the object. Based on interaction history, the environment is adapted. Dominique et al. [30] used mining techniques for the improvement of adaptive systems. According to them learners' interactions are observed through some parameters which are then used to trigger automatic application of rules that leads to the production of personal learning contents. A very good work is done regarding the adaptation of 3D virtual environments by Troyer and Ewais [5, 8]. They discuss different components of VE and then introduced a set of adaptation types and a set of adaptation strategies for 3D-VLEs. According to these authors adaptation can be applied to a single component as well as multiple components of VE.

As discussed above, different approaches have been used for the adaptation of VLEs. Some of these include personalization rules for customized navigation inside VR stores [18], observing customer behavior for shopping [23], using virtual agents that help users during interaction [25] and the use of software sensors that historically monitor user behavior for controlling navigation and interaction within virtual environment [29]. Similarly, in [24] a pre-test is conducted to compute learning level which is then used to update learner model. All these approaches are effective in their context but no one considered "learning skill" of student as adaptation criteria for changing the contents of 3D-VLEs. We tried to get an insight of student learning capability by quantitatively measuring learning skill of students. The contents of 3D-VLEs are changed according to the learning skill of the students which give them the sense of one-to- one tutor.

3. MODEL DESCRIPTION

In the proposed framework, knowledge is delivered to students in many levels where the number of objects or the amount of teaching material in a given level is dynamically decided and is based on the learning capability of student in the previous level (total number of levels are not fix i.e. less for an efficient student and more for a weak one).

Learning is a qualitative variable which cannot be measured directly but we can use some quantitative variables such as time, no of errors and test score etc to assess the learning skill of students in virtual environments. In general, if a student takes less time to complete an activity in a virtual environment as compared to another student who takes more time, the former is considered as an efficient learner. Similarly, a student who performs small number of errors while interacting with the virtual world and gets high marks in the test at the end of learning module is considered as a good learner. The system should be adaptive in such a way that it must give more time to a slow learner for a given module and at the same time it must be able to cope with the learning capability of good learner to make quick progress.

3.1 The Adaptive Frame Work

In the proposed framework, learning capability of a student is measured using a function called learning decision function (LDF) to update learner model. Based on the performance of the students, the system adapts itself in such way that it fulfills the learning goals of all type of students. More contents are displayed to good learners while weak learners are provided with teaching materials which they can absorb according to their learning skills. The basic diagram of the proposed framework is shown in Fig. 1.

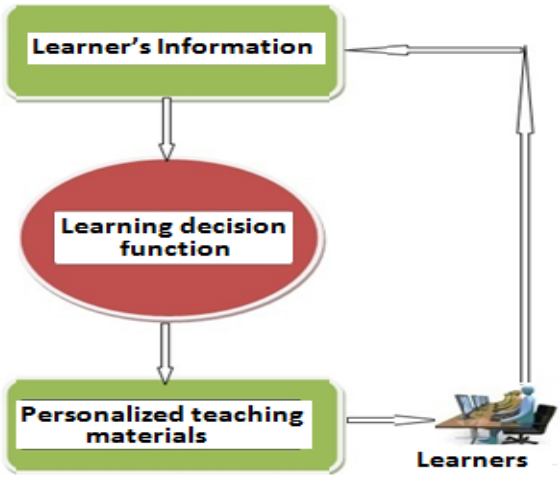

Fig. 1. A student friendly framework for adaptive 3DVLEs.

3.2 Learning Variables

3.2.1. Time

Time is the most important variable for measuring the learning capability of students in general and specially in virtual environments. Slow learner needs more time to understand the given concept while fast learner gets the desired knowledge quickly. In virtual environments, time taken by a student to complete a given module is also related to the way information is presented to the student. But this is a usability problem and is not in the scope of this research. A lot of work has been done in this regard to provide user friendly interface in order to enhance the learning capability of student [31, 32]. For this study, we assume that taking long time to complete a given module means that the student learning capability is low and vice versa.

To properly model time we must consider both total time taken by a student to complete a given module in a virtual environment and also the time to complete an activity within the module. The latter can also be used to identify how the student responds to a complex question. Let t_i presents the time to complete an activity A_i within learning module M in the virtual environment. Then the total time T_m to complete the learning module M is given by Eq.1.

$$T_m = \sum_{i=1}^{n} t_i \quad (1)$$

For more appropriate assessment we can take the arithmetic mean of the time of all previous modules to get an average learning time for the student.

3.2.2. Number of Errors

Counting the number of errors for a student also shows his efficiency. While interacting with the virtual environment; a good learner will perform less number of errors as compared to a weak learner. Errors may be divided into two types i.e. technical and non technical errors. Technical errors show that the student is not familiar with the virtual environment and needs more time to perform the given activity. A list of some most common technical errors is given below.

1. The student performs astray navigation and is lost in the virtual environment. The student becomes confused and deviates from the right path as a result he performs some undesirable actions.
2. The student may make a mistake during object selection and release. Precision and accuracy in object selection and releasing shows the performance of the student in the virtual environment.
3. The student may try to perform some incompatible manipulation on the selected object. e.g. try to move objects which are fixed in the given coordinates of the virtual environment.
4. Interaction devices also play an important role in the virtual environment. Some students are very good to use mouse and keyboard. Other will feel comfortable by using some advance devices such as wimote and leap motion etc. The student will perform more errors in term of precision and accuracy if he is new to the interactive device.

Non-technical errors occur because of poor knowledge of student in the given domain. Some of these types of errors are listed below.

1. Student selects two incompatible objects for manipulation.
2. Students try to perform an activity before doing its prerequisites.
3. Student is unable to map correctly the virtual objects with the real world objects. The reason is that the student does not know anything about the actual object in the real world so he is not able to identify it in the virtual environment.

If e_t represents technical errors and e_n represent non technical errors, then the total number of errors which a student performs in the given learning module M is given by Eq.2

$$E_m = e_t + e_n. \quad (2)$$

High value of Em shows that the student is weak while low value shows that the student is good and can learn quickly.

3.2.3. Test Score

The more appropriate way to assess the learning capability of a student is to give him a test after completing a given learning module. If a student gets high marks it means that he is a good learner otherwise the student has little ability to learn. In the latter case, student needs more time for learning the given module. If q_i represents the

marks obtained by a student for solving i^{th} question. Then the total marks M_t obtained by the student in test after completing the learning module M is given by Eq. 3:

$$M_t = \sum_{i=1}^{n} q_i \quad (3)$$

High value of Mt is desirable and it shows the efficiency of a student.

3.3 Testing Hypothesis

The proposed framework is based on three hypotheses.

H1: A student who takes less time to complete a given module is good learner.

H2: A good learner performs fewer errors while learning a given module.

H3: A good learner takes high score in test as compared to weak learner.

These hypotheses are the backbone for our research. In order to test these hypotheses, we conducted a survey using questionnaire to collect data from teachers. Total of 44 questionnaires was distributed among senior teachers from different schools, colleges and universities. The results are summarized in Fig. 2.

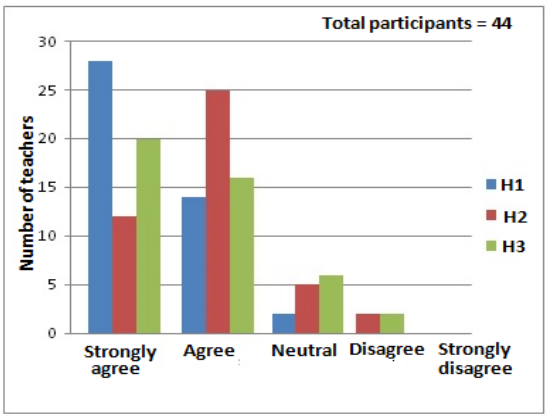

Fig. 2. Survey results for H1, H2 and H3.

The results show that H1 is strongly supported by the teachers i.e. 63% teachers were strongly agreed, 32% were agreed, 5% were neutral and we did not get any negative feedback for it. For H2, 27% were strongly agreed, 57% were agreed, 11% were neutral and 5% were disagreed. Similarly for H3, 45% were strongly agreed, 36% were agreed, 14 were neutral and 5% results were negative. The above results show that the three hypotheses H1,H2 and H3 are correct and can be used to quantitatively measure the learning skill of students.

To know the relative importance of these variables, teachers were also asked to rank these variables on the scale of 0 to 1. The results are summarized in Fig. 3.

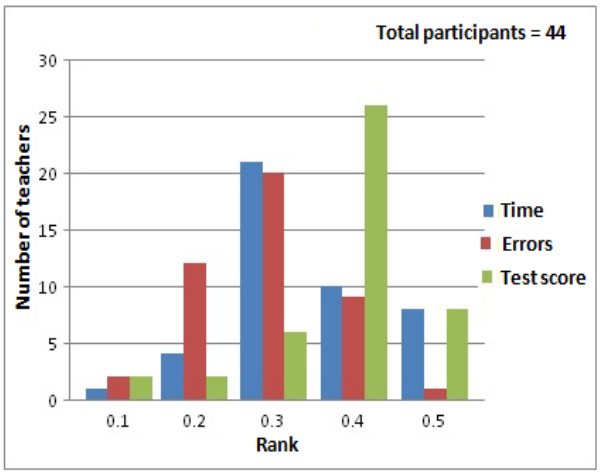

Fig. 3. Relative importance's of time, errors and test score.

The graph shows that total time to complete a given module and no of errors should be given 0.3 weightage each while the test score is given high weight of 0.4 on the scale of 0 to 1. These values show the relative importance of mentioned variables for measuring the learning skill of students.

3.4 Learning Decision Function

We defined a mathematical function called Leaning Decision Function (LDF) which quantitatively measures students learning. The function accepts time; errors and test score as an input, calculates the learning skill of the student in the range of 0 to 1 and displays it as an output.

The LDF can be calculated by using Eq. 4.

$$LDF = f1+f2+f3 \quad (4)$$

Where f1, f2 and f3 are functions which are used to calculate the time spent, no of errors and score of a student in the given module respectively.

Calculation of f1, f2 and f3

f1: f1 is a function which calculates total time spent by a student in completing a given module.

A fast learner requires less time to complete the work while slow learner needs more time for learning. Small value of f1 is desirable which shows that the student is fast learner. Therefore,

$$f1 \propto \frac{1}{Tm}$$

$$f1 = \frac{k1}{Tm} \quad (5)$$

Where Tm is the total time spend by a student to complete a given module and k1 is the constant of proportionality.

f2: f2 calculate the number of errors during the learning process. Again small value for f2 is desirable as it shows the efficiency of student.

$$f2 \propto \frac{1}{Em}$$

$$f2 = \frac{k2}{Em} \quad (6)$$

Where Em represents total number of errors and k2 represents constant of proportionality for f2.

f3: f3 is used to measure the score of a student in the test at the end of a learning module. High score means that the student is efficient and vice versa. Therefore, high value of f3 is desirable for good learner.

$$f3 \propto Ts$$

$$f3 = k3Ts \quad (7)$$

In Eq. 7, Ts represents test score of a student at the end of a learning module and K3 is the constant of proportionality.

Now the LDF function can be written as

$$LDF = \sum_i^3 fi \quad (8)$$

This function can also be used in generalized form as shown in Eq. 9 to consider more variables for learning which will give more insight to measure the learning skill of a student.

$$LDF = \sum_i^n fi \quad (9)$$

3.5 Fuzzy Logic Decision Making

For the purpose of implementation and experiments, we used the results of survey and implement the learning decision function defined in Eq. 8 in such a way that it successfully obey the following two conditions for t ε [15,300], e ε [1,10] and m ε (0,5].

1. The function gives maximum value i.e. 1.00, when a student takes minimum time; perform minimum errors and gets maximum score. For example if a student takes minimum time of 15 seconds, makes a single error and gets maximum score of 5 marks. Then from Eq.8, LDF=1.00
2. The function gives minimum value i.e. 0+ε, when a student takes maximum time, performs maximum errors and gets minimum marks. Where ε is a Greek word, greater than zero, however small no matter. For example if a student takes maximum time of 300 seconds, makes 10 errors and gets minimum score of 0.01 marks. Then from Eq. 8, LDF=0+ ε. The LDF function returns a value in the range of [0, 1] that represent learning skill of a student. Learning is a qualitative variable and due to vagueness in knowledge acquisition, an efficient tool like fuzzy logic is needed to model the learning skill of students [24, 33, 34].

Let x be the be the linguist variable "Learning skill", then the terms weak learner, average learner, and good learner can be constructed as shown in Fig. 4.

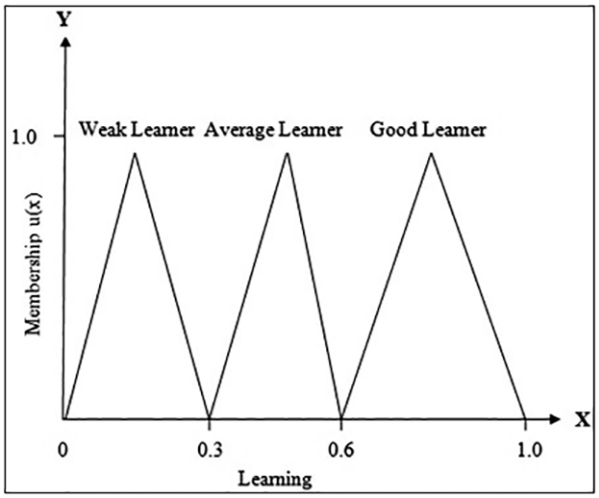

Fig. 4. Fuzzy based approach for measuring learning skill of a student.

Using fuzzy decision making, now we can easily model the learning skills of students on a scale of 0 to 1. All students for which the LDF returns a value in the range of [0, 0.3] are considered as weak learners and they will be provided same amount of teaching materials in the next teaching level. Similarly, students for which the LDF return values in the range of [0.3+ ε, 0.6] are considered as average learners by the system and they will be shown equal amount of teaching materials in the next learning module. The last

range of student learning skill is [0.6+ ε, 0.1]. The students in this range are called good learners and they will be treated equally by the system.

The proposed frame work first calculates student learning skill and then use the fuzzy decision making process to provide appropriate teaching materials. Weak learners are provided small amount of information in the next level. Intermediate learners get the knowledge comparatively quickly as compared to weak students, so they are provided little more material in the next teaching level. Similarly, the system is also able to cope with the learning needs of good learners. They are provided more teaching material as compared to average learners in the next learning level. The beauty of our approach is that system treats weak, average and good learners differently and they are provided teaching material according to their actual learning skills. By nature weak learners are slow learners therefore; they will take more time to complete a given learning module. Similarly, good learners are fast learner. The system is providing them an opportunity to finish their work quickly. The detail description of the proposed system architecture is given in Fig. 5 as below.

4. EXPERIMENTATION

4.1 Experimental Setup

The proposed solution was implemented in MS Visual Studio 2010 using OpenGL Graphics Library installed on HP Corei5 Laptop having 2.4GHz processor, 4GB of RAM, ATI Mobility Graphics Card with 64- bit operating system. Mouse and keyboard both were used for interaction with objects within the environment.

For the purposes of comparison and evaluation of the proposed framework for 3D-VLEs, we also used the traditional system for learning in our experiments. Traditional system treats all students equally. Same amount of teaching material are shown to students when they enter the next learning level. The amount of teaching material in each level is predefined and constant. The proposed system evaluate learning capability of the student by considering total time to complete the learning module, total no of errors and test score at the end of each learning level. On the basis of these variables, LDF function quantitatively measure learning skill of a student and display teaching material in the next learning level according to his learning capability. The system dynamically decides how much information is to be displayed in the next teaching level which the student can easily absorb. Both systems were installed on two different laptops. The simulated environment of the proposed framework is shown in Fig. 6.

4.2 Experimental Protocol

We randomly selected 44 students of class 10 from different schools for the evaluation of the proposed system. Both systems i.e. traditional and proposed were introduced to the students and they

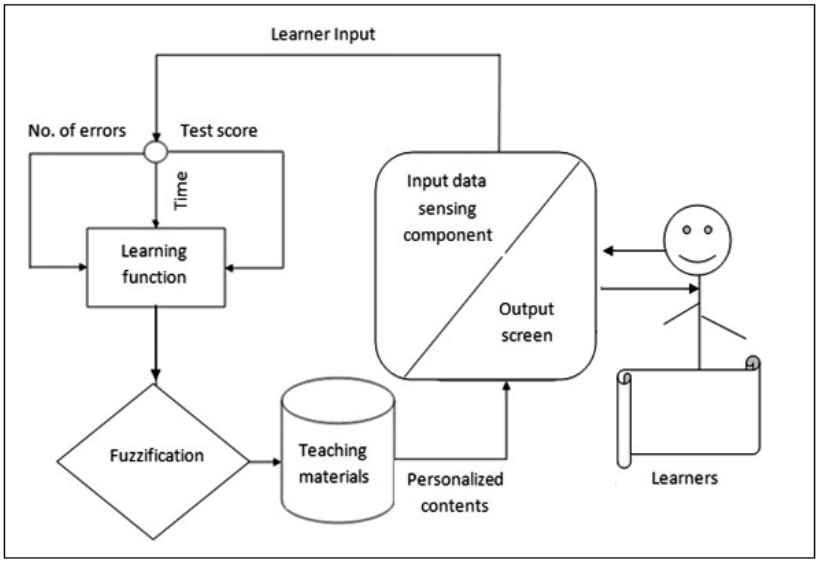

Fig. 5. The proposd system architecture.

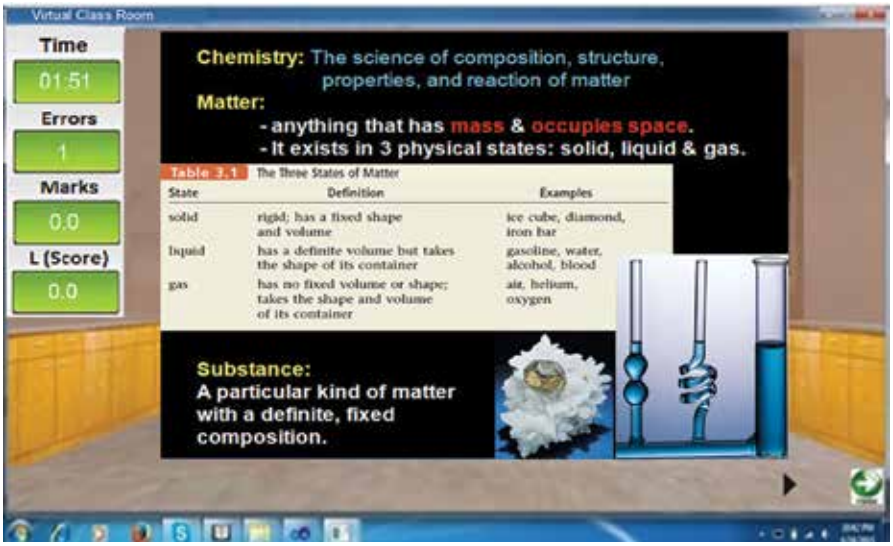

Fig. 6 Virtual class room.

were thought how to select, navigate and manipulate objects within the environment.

For the purpose of experimentation, we selected two different topics from the subject of chemistry of class 10. Participants were randomly divided into two groups i.e. G1 and G2 of 22 students each. G1 used traditional system for learning topic one and then used the proposed system to learn topic 2. Similarly, G2 used the traditional system for learning topic two and then used the proposed system to learn topic one. After the completion of a topic on either system, students were asked to appear in a test containing questions about the topic they covered. The purpose of the test was to check the overall learning of the students on either systems and then to compare the results for the evaluation of the proposed system. In Fig. 7 two students were shown, who are performing their experiments.

Fig. 7. Two students performing experiments.

4.3 Experimental Results

4.3.1. Student Learning

Overall performance of 44 students using the traditional and proposed systems is summarized in Fig. 8. At the end of each learning module, it was observed that most of the students got high marks when they were using the proposed system for learning. Statistically, 68% students showed positive results for the proposed system i.e. they got more marks when they were using the proposed system. 18% results were neutral and 14% results were negative i.e. they got less marks in the proposed system as compared to the traditional system.

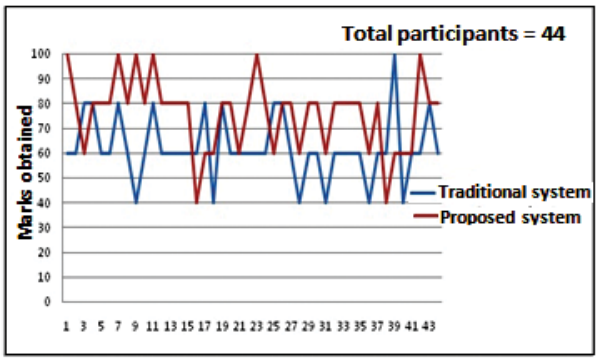

Fig. 8. Overall performances of students on both systems.

We performed ANOVA test on the overall performance of students on both systems based on their marks they obtained at the end of each learning module. The ANOVA ($F(1, 43) = 23.72$,

$p < 0.00001$) is significant. Comparing students' marks, we got, Mean of 61.89 and Standard Deviation of 14.19 for the traditional system while for the proposed system, Mean and Std. Deviation were 76.36 and 13.82 respectively. The statistical data show the effectiveness of our proposed framework for enhancing the learning capability of students in 3D-VLEs.

4.3.2. Subjective Evaluation

In this section we analyze the responses of students regarding the proposed adaptive framework. A questionnaire, consisting of six questions was distributed among the 44 tested students. The questions along with student's responses are given below. Table 1 contains list of five subjective questions about the traditional system and proposed system.

For Q1 to Q3, student's responses were recorded on scale of 5 points as shown in Table 2, while for Q4 and Q5 students were simply asked to give their opinion about the traditional and proposed system.

Table 1. List of subjective questions about the traditional and proposed systems.

Question No	Questions
(1)	The proposed system provided you learning materials in the next teaching level according to your learning skill.
(2)	Your concentration on the actual learning materials was high in the proposed system.
(3)	Did you feel that proposed system overwhelmed you with learning materails at any teaching level?
(4)	Which system was comparatively more saturated with learning materials?
(5)	Which framework do you prefer for 3D -VLEs?

Table 2 Student's responses for Q1, Q2 and Q3 about the proposed system (total participants = 44).

Question No	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
(1)	(54%)	(23%)	(14%)	(9%)	(0.00)
(2)	(39%)	(36%)	(11%)	(14%)	(0.00)
(3)	(5%)	(11%)	(25%)	(32%)	(27%)

Table 3. Traditional system versus proposed system (comparison) (total participants = 44).

S. no.	System Attribute	Traditional System	Proposed System
1	Over all Simplicity of the system	43%	57%
2	Motivation towards learning	22%	78%
3	Memorability of teaching materials	14%	86%
4	Amount of irrelevant / unnecessary information	69%	31%
5	Ease of navigation inside the Virtual Environment	45%	55%
6	Student friendly	30%	70%

Table 4. No. of levels and completion time (total participants = 44).

S. no.	No. of students in percent	No. of levels	Average completion time	
			Minutes	Seconds
1	14%	4	10	22
2	50%	5	13	37
3	32%	6	16	34
4	4%	7	22	10

Most of the students showed positive response for question Q1 i.e. 54% students marked strongly agreed, 23% were agreed and 14% were neutral. The remaining 9% showed negative response and marked it disagree. For Q2, 39% students were strongly agreed, 36% were agreed, 11% remained neutral and 14% were disagreed. Similarly, for Q3, 5% students marked strongly agree, 11% were agreed, 25% remained neutral, 32% were disagreed and the remaining 27% strongly rejected the opinion.

From students' responses, it was observed that 77% marked the traditional system while 23% marked the proposed system for Q4. Similarly with response to Q5, 14% students marked the traditional system while the remaining 86% were in favor of using the proposed system.

In the questionnaire, students were also asked to compare different attributes of the traditional and proposed systems. The system attributes along with students' feedback are summarized in Table 3.

5. DISCUSSION

The purpose of the proposed framework is to enhance the learning capability of students in 3D-VLEs. In teaching practice, it has been observed that a teacher give more to weak students for understanding a given concept while the same topic is delivered in less time if students are sharp and intelligent. This attitude towards teaching fulfills the needs of all students and they get the desired time for understanding a given concept. Weak learners are not overwhelmed with teaching materials and at the same time good learners do not get bored because of listening the same thing again and again for a long time from teacher. In the proposed framework an attempt is made to implement this behavior in 3D-VLEs in order to make it student friendly. Knowledge is delivered to students in many levels where the number of objects/teaching material in a given level is dynamically decided and is based on the learning capability of the student in the previous level. Here the total number of levels is not fixed i.e. less for efficient and more for weak students. The competent students can complete all the work in less time and can utilize it in some other useful activities while weak learner is provided more time to get the desired knowledge. The experimental result shown in Table 4 confirms this opinion.

Using the proposed system, 14% students complete the given learning module in 4 levels by taking an average time of 10 minutes and 22 seconds. Similarly, 50% students finish the work in 5 levels with average time of 13 minutes and 37 seconds. 32% students spent 16 minutes and 34 seconds to complete the work in 6 levels. The remaining 4% students pass through 7 levels in 22 minutes and 10 seconds to get the desired knowledge. It is clear from Table 4, that behavior of proposed system is smart and clever. The same amount of knowledge is delivered intelligently. The first 14% students were treated as good learner and they were provided more teaching material in each level, therefore they finished quickly. The second and third serials students were considered as average learners, therefore intermediate amount of information were displayed to them in each levels. The last 4% students were slow learners and they completed the learning module in 7 levels. These students were provided more time for learning as compared to average and good learners.

6. CONCLUSIONS

In this paper, we presented a new approach regarding the adaptation of 3D-VLEs which is effective, efficient and student friendly. The proposed approach quantitatively measure student learning skill and use it as adaptation criteria for changing the contents of 3-DVLEs which has many advantages. It enhances the learning capability of the students by providing him learning materials which he can absorb at a given time. The student with little learning capability is provided more time to acquire the desired knowledge and at the same time it can cope with the learning capability of fast learners to make quick progress. Secondly, it prevents students from being overwhelmed with teaching material. The proposed solution also has the ability to handle the technical weakness of a student in 3D-VLEs. In initial stages, a student may perform many errors during interaction with the virtual environment. This increases total time for the student in the next learning level which gives him an opportunity to stay more in VE and becomes familiar with the technical aspects of the system. Finally, the proposed solution motivates students towards learning by showing their progress in each

level. As a result the overall performance of students is increased in adaptive 3D-VLEs.

7. FUTURE WORK

Although, the proposed system is student friendly and enhances the learning capability of students in 3D-VLEs but there are still some limitations which need further attention for possible improvements. For measuring learning skill of a student, the LDF function considers only three variables i.e. time, errors and test score. More variable such as student initial profile, student GPA etc. shall be included in function definition to get more insight of student learning capability. Also, the teaching material displayed in the next level depends on student performance in the previous level. A good student may show low performance in next level because of some mental or physical stress. The proposed system does not handle this situation. The solution must be modified in such a way that if a good student shows low performance in some level, the system should be so smart to treat him as a good learner rather than weak one. Further, improvements can be made if the proposed solution provides different learning paths for three types of learners. Weak learners will go through more teaching materials and examples for understanding a given concept by following the appropriate path. This will enhance the efficiency of the proposed work.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 267–280 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Critical Barriers in Project Management Faced by Offshore Software Multi-Sourcing Vendors: A Detailed Study

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Abstract: Multi-sourcing is a modern business strategy in Global Software Development (GSD), adopted by many organizations in developed areas for producing fast and improved quality software products with minimum developmental cost by contracting the project with copious vendor organizations at low prices countries. The objective of this research work is to find out all possible critical barriers (CBs) in software project management faced by multi-sourcing vendor organizations. This will assist in planning for mitigation and avoidance of these risks/barriers for successfully achieving project goals in earlier planning phase. Systematic Literature Review (SLR) was used for identification of these critical barriers that can be faced by vendor organizations at various stages of SDLC for multi-sourced software projects development. Our research reveals that most of the challenges are relevant to planning phase of SDLC.

Keywords: Global software development, multi-sourcing, software project management, vendors, SLR, offshore software development outsourcing, barriers

1. INTRODUCTION

Global software development (GSD) is a phenomenon of development of software by teams dispersed throughout the different geographical locations, also called development sites. This approach has been adopted by many software development organizations for the last two decades in order to increase their business incomes. In GSD, the software projects are distributed and developed in different firms normally located in different countries of the world. Basic purpose of this strategy is to develop software 24/7 hours which reduce the product availability time to market. In GSD, offshore software development outsourcing (OSDO) or software outsourcing, is the modern approach to software development in which software products development projects are contracted to firms from low cost countries for developing high quality software products and decreasing the developmental costs and time [1, 2].

The knowledge behind GSD paradigm is that software engineers from geographical locations around the globe, with different languages, cultures and temporal backgrounds, collaborate and work together for development of software projects, communally; it is termed as "global distance" [32, 33, 34, 35, 36]. In GSD environment, geographical remoteness creates physical separation between software engineers/team members and team management [3]; temporal distance minimizes the opportunities of direct contacts and communication and cooperation between team members [31], and cultural differences undesirably affect the understanding and appreciation of the work progress of the team members and remote colleagues [9, 36]. The difference in native languages, also called "linguistic distance", creates multiple critical

Received, October 2015; Accepted, July 2016
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barriers to communication [8, 33, 35].

Global software development has been found fruitful by many organizations, because through GSD the vendor organization can take benefits in terms of cost and time by accessing global multi-skilled workforce, antagonism, easy access to global markets and end users/consumers [3]. Global software development is a broad category providing a platform to different dynamic development environments in different formats like outsourcing, freelancing, partnership, crowd sourcing, etc. [4]. Here, the discussion will be limited to outsourcing paradigm of GSD, which is the most popular and rapidly growing feature of GSD. Most software development organizations around the world are adapting outsourcing and moving towards GSD model due to its various benefits.

Outsourcing is a contractual association between vendor and client organization based upon written agreement. This strategy has been adopted and implemented by many software development firms throughout the world gives new prophecy to business process that play key role in making business decision [5]. Based upon nature of geographical separation, outsourcing relationship can be further divided into three major types such as include onshore outsourcing, nearshore outsourcing and offshore outsourcing [6]. The offshore outsourcing can be further divided into uni-sourcing (one-one relationship between client and vendor) and multi-sourcing (one to many or many-to-many relationship(s) between clients and vendors). In each type of outsourcing, an agreement is made between vendor and client located apart from one another in different countries.

The management of software projects in case of outsourcing is more complicated than software projects executed and developed indoor because parties involved in outsourcing belong to different cultures, time zones and languages etc. Another type of outsourcing called Multi-sourcing or multi-vendor sourcing or multiple outsourcing in which one client or vendor organization is associated with multiple clients or vendors located in different locations/sites. There is one-to-many relationship in this type of outsourcing. The client organization is dependent on more than one vendors as well as there is high dependency between involved vendors. The project management in this type become toughest than outsourcing because of additional work done for controlling communication and coordination between parties, product integration from involved parties etc.

Cohen [7] has defined the multi-sourcing relationship as "the organizing and establishing of IT infrastructure and business services in regimented way for achieving targeted project and business goals successfully by contracting the project with optimum external and internal service supplier".

The main reason behind increasing trend of multi-sourcing strategy is that each firm tries their best to reduce product development cost, time and increase the efficiency, flexibility and quality of product in fast changing global market [9]. Multi-souring strategy is fruitful in many aspects and lead to incredible benefits as discussed [26], but on the other hand, it challenge the existing administrative competencies and approved operational models of the organization [7, 27].

Challenging the managerial capabilities of an organization is the key limitation of multi-sourcing paradigm in IT projects; otherwise it is the most suitable strategy for successful completion of software projects with reduced risks [28].

1.1 Why Study SPM in the Context of Multi-Sourcing?

The main idea behind this study is to facilitate and empower vendor organizations to successfully execute multisource software development projects effectively and efficiently by following a set of processes and procedures (a model). This study will assist the vendor organization in decision making either to outsource the project by considering different constraints and critical barriers/challenges that can be faced in multi-sourcing environment or not.

To manage and handle multiple suppliers in parallel from different backgrounds like different cultures, languages and time zones is not an easy task and require strong internal management at vendor organization. Project management in software multi-sourcing should be based upon

strong procedures, processes and models, so that vendor organization can take positive and bold decisions for improving their business [9]. Ebert [10] and Prikladnicki [39] have argued that proper risk management and risk handling are most critical challenges, which project managers are facing in GSD software projects. These risks become more critical in multi-sourced software projects.

2. BACKGROUND

Software project management is a process of planning, monitoring, controlling, budgeting, scheduling, project resource allocation/de-allocation, communication, collaboration, documentation and change and risks management. Good project management contributes significantly in success of any project/program. In case of software projects, the project management have a vital role in successful completion of development activities. The software project failure comes in different forms like budget limit exceed than allocate one, project not completed in the given time frame or not functioning according to SRS, etc.

According to the CHAOS report only 37% of all software and Information Technology (IT) projects are successfully completed and succeeded in the sense that they were delivered within given constraints of time frame, within calculated budget, with all the required specification, features and functions. The research conducted by IBM showed that 54% of software projects failures were due to poor management of software projects.

Lago et al. [2] argues that one of the main reasons behind software project failure is the lack of project management knowledge of project managers as they do not know how to deal with uncertainties occurs during the project execution. Current research and literature [43] has acknowledged different reasons due to which software and IT projects failed, like unclear requirement specifications, weak project escalation, lack of risk management knowledge, high user expectations, inadequate software development or project management model and processes, or no record track knowledge of previous failed or succeeded projects.

A report in IT Weekly magazine reveals that Eighty percent of software organizations who have outsourced and contracted there software projects to vendors in offshore development environment faced many critical problems because both vendors and client's inexperience, unawareness with outsourcing and poor software projects management in GSD [37]. One of the main problem is that most of the client organisations make global contracts with their vendors before proper judgment of their potentials and efficiencies required for successful project management in GSD [38].

Multi-sourcing in offshore software development outsourcing (OSDO) has many benefits; but there are several critical challenges faced by vendor organizations. According to the literature, these are as under:

- In OSDO relationships, Khan et al. [11], pointed out number of critical challenges like lack of communication and coordination between vendors organizations, lack of project management, delayed responses etc. which can leads the project toward failure if proper attention is not given for mitigating these challenges in start of project execution.
- In OSDO, the rich Communication and strong coordination are the two key factors and plays vital role in success of software outsourcing projects. These are badly affected by geographical separation between vendors, time zone, cultural differences and language differences [12].
- In a study, Verner et al. [13] pointed out different barriers faced in OSDO like selection of appropriate vendors, project management, selection of appropriate development process and design, requirement engineering, architectural design, software integration and component management, training, coordination/collaboration and communication and planning for risks control.
- Due to geographical dispersion among vendors/stakeholders in the GSD, some of the challenges like complexity in communication and collaboration process, language, culture and time difference, knowledge sharing and management are automatically created [14].

Geographical separation is the biggest challenge in GSD because some projects activities like coding and testing are carried out by one sub-vendor located at one geographical location while other activities like planning, requirement specification and analysis, integration, implantation and testing are carried out at some other place. The basic of strong and effective software project management in GSD is coordination and control. But geographical separation and long distances introduce complications which directly influences the command, control and coordination through its effect on communication and cooperation [29, 30].

Many researchers and author pointed out difference risk factors and barriers in context of outsourcing. All these factors and barriers also faced by vendor's organizations working on multi-sourcing in OSDO. The most critical barriers in multi-sourcing environment are high dependency among stakeholders, lack of collaboration and coordination and delayed responses. These make the software project management for vendor organization working in multi-sourcing environment tougher and needs additional work done.

Different process models and procedure have been developed and defined for successful completion of software projects within given constraints of time budget, time, quality standard etc [1, 19-25].

3. RESEARCH QUESTIONS

Following research questions were formulated keeping in view the project management challenges in context of multi sourcing in OSDO relationships from vendor's perspective:

RQ1: What are critical challenges, as identified in literature, in software project management in the context of software multi-sourcing?

RQ2: What are critical challenges, as identified in real World, in software project management in the context of software multi-sourcing?

RQ3: What are the real-world practices for software project management in the context of software multi-sourcing?

4. RESEARCH METHODOLOGY

To achieve our expected goals and objectives as outcome from this research accurately, we have adopted systematic literature reviews (SLR), questionnaire survey and case study techniques to gain maximum from existing literature and for validation of these results. This research approach has been adopted by other researchers also [15, 16]. A SLR is a new approach in software research field for identification, evaluation and interpretation of all relevant research for a Specific research query/ question, or topic area, or phenomenon of Interest [18]. In first steps, to identify software project management critical barriers/challenges faced by vendors in OSDO multi-sourcing relationships, the existing literature has been reviewed through SLR. In addition to barriers, some practices/solutions for addressing these challenges have been noted. In 2nd step, an empirical study will be conducted in industry working on OSDO multi-sourcing approach for validation of the identified challenges of the SLRs. In this step, experts will point out more challenges or practices in addition to the identified ones. In third step the practices will be identified using SLR and empirical study for the purpose to avoid/mitigate the identified challenges. In step fourth the various levels will be defined on the basis of the identified critical challenges and practices. Finally, the developed model of PMMSM will be validated through case studies and changes will be made if required. The detail of search strategy is given in our developed protocol which is in pipeline for publishing.

5. DATA SYNTHESIS

In this phase, the barriers were extracted from 45 research papers and have been categorized in 23 different groups with specific name, their frequency and percentage. The percentage will help in decision making regarding criticality of a barrier. This categorization was done by author and co-author as shown in Table 1.

We have extracted different challenges from different research papers and articles through SLR. 'Lack of Communication and Collaboration between Stakeholders' is most critical challenge identified in our study i.e. 96% as shown in Table

Table 1. List of Identified Barriers through SLR.

S. No.	Barrier Class	Frequency Out of 45	Percentage
01	Lack of Communication and Collaboration between Stakeholders	43	96%
02	Difference in Language, Culture, Time and Geographical Distance	38	84%
03	Complex Relationship Between Vendors	16	36%
04	Lack of Experience in Multi-sourcing Projects	20	44%
05	Lack of Technical Skills	28	62%
06	Delayed Feedback	08	18%
07	Volatile Customers Requirements	16	36%
08	Poor planning and estimation	19	42%
09	Ambiguous software design	14	31%
10	Issues in software coding and testing	13	29%
11	Lack of Trust	29	64%
12	Software Integration Problems	02	4%
13	Lack of Client and Top Management Involvement	16	36%
14	Security and Privacy Issues	10	22%
15	Weak monitoring and Control	23	51%
16	Lack of Standard PM Practices and Processes in Multi-sourcing	19	42%
17	Organizational Politics	22	49%
18	Complexity in Multi-sourcing contracting	08	16%
19	Lack of Training	22	49%
20	Change in Roles and Responsibilities	08	18%
21	Lack of Team Spirit	06	13%
22	Hidden Costs	14	31%
23	Lack of Knowledge Sharing	20	44%

1. It means that vendor organizations have to do more work for improvement of communication and collaboration between involved stakeholders (sub-vendors, clients etc.) when managing software multi-sourced project in OSDO. The 2nd most critical barrier is 'Difference in Language, Culture, Time and Geographical Distance' with frequency 84%. Geographical separation of stakeholders means that different team members may be involved and working on same project e.g. some teams and team members are from USA, some from Asia and some from Europe etc. Geographical separation provide base for language, time and cultural differences. Now, it becomes very difficult to manage the involved teams and members together at the same time due to time differences. This time difference directly effects the quality of collaboration, communication and coordination in offshore multi-sourcing. The difference in cultural means difference in societies, religious factors and rules, public and custom holidays, values, languages, procedures, and thoughts [44]. Khan S.U and Azeem [44] have also discussed in their research that culture difference is most critical challenge for vendors working in OSDO relationship. Difference in language mean that as people involved in OSDO relationship belongs to different regions and speak different native languages, which leads to problems like unknown expressions, gestures, ideas and directions. Some authors [e.g., 45] argued that language and cultural differences can contribute a lot in lacking the communication, coordination and collaboration processes between clients and vendors in OSDO relationships. 'Lack of Technical Skills' (62%) is another barrier faced by vendor organizations. By this we mean the shortage of technological experiences, absence of matured processes, procedures and outdated technologies possess by a vendor organization working in OSDO. Technology difference between vendors and clients can create serious problems.

'Lack of trust' is another critical barrier faced by vendor organizations in management

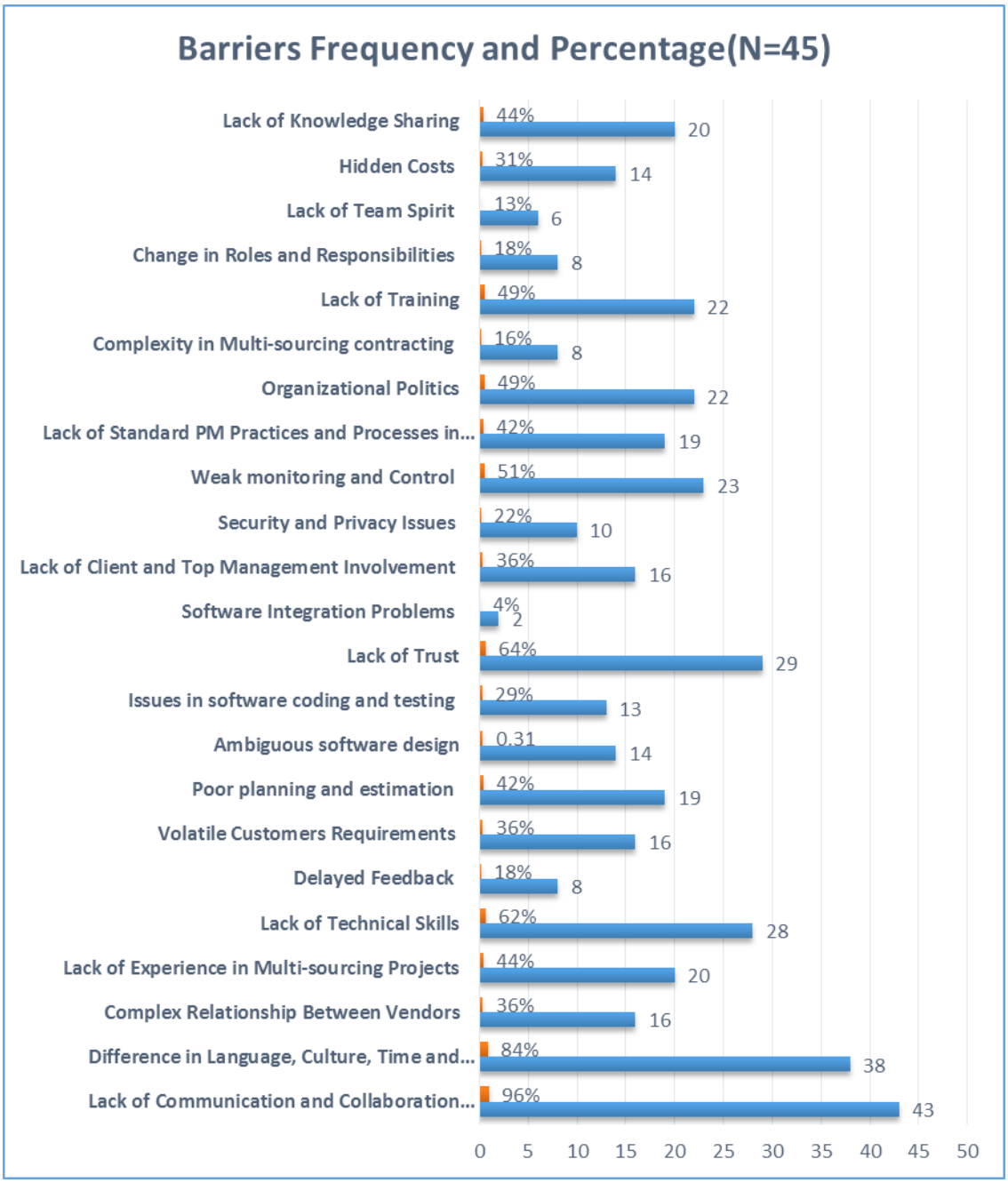

Fig. 1. List of Barriers Identified Through SLR.

of software multi-sourced projects. N. B. Moe and D. Smite [46] in their research identify some of the important factors that creates lack of trust in teams and a there members in GSD. These are lack of face-to-face communication, cultural and social mismatch, language difference, poor conflict handling, absence of cognitive-based trust, unnecessary observations, inconsistency in development processes and procedures. In case of trust absence, the employees waste much of their energy in self-protecting, individual goals become more important as compare to group goals and doubt negative feedback from management.

'Lack of experience in multi-sourcing projects' is another critical barrier. According to this, the vendor organizations, before going to contract multi-sourced software projects in GSD, should measure and examine their capabilities for managing and executing such projects i.e. technical and managerial capabilities of their managers,

potentiality of their employees, technologies, processes and resources currently in use, effective risks management etc. The lack of experience for handling such projects become difficult and may lead to failure.

'Poor planning and estimations (42%)' are other barriers that need special attention before execution of project because if proper planning, scheduling, resource allocation and budgeting are not performed, the project will be definitely delayed or will exceed allocated budget. Other barriers like 'weak monitoring and Control (51%)', 'Organizational Politics (49%)', 'Lack of Training (49%)', 'Lack of Knowledge Sharing (44%)' should be kept in mind by vendor organization in managing multi-sourced projects in GSD.

Fig. 1 shows detailed distribution of barriers with frequency and percentage identified through SLR.

6. RESULTS

In this section, detailed discussion and analysis will be done of each barrier from different angles. We have used SPSS for finding out facts and figures. The identified barriers that have frequency >= 30% will be considered critical barriers. According to this criterion there are 16 critical barriers out of 23 identified barriers as shown in Table 1.

6.1 Database Wise Detail of Research Papers and Their Percentage

Detail of the research papers selected through SLR across the various search engines and data bases (directories) are shown in Table 2. The research papers selected through snow balling having higher frequency are shown in Fig. 2. We have also used snowballing technique to find out most related

Table 2. List of selected papers across various search engines and libraries (directories).

Data Base	Frequency	Percentage
IEEE Explore	11	24.4
ACM	2	4.4
Science Direct	11	24.4
Google Scholar	2	4.4
Snow Balling	19	42.2
Total	45	100.0

Table 3. List of papers across the various continents.

Continent	Frequency	Percent
Asia	13	28.9
Europe	21	46.7
N. America	6	13.3
Mixed	5	11.1
Total	45	100.0

Table 4. List of papers across the two decades.

Period	Frequency	Percent
2000-2010	20	44.4
2011-2015	25	55.6
Total	45	100.0

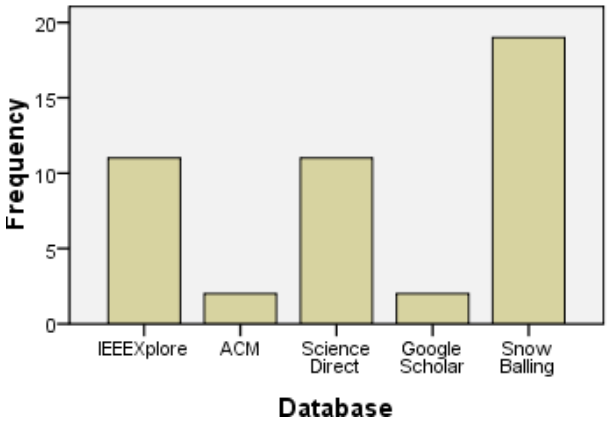

Fig. 2. Database wise distribution of research papers.

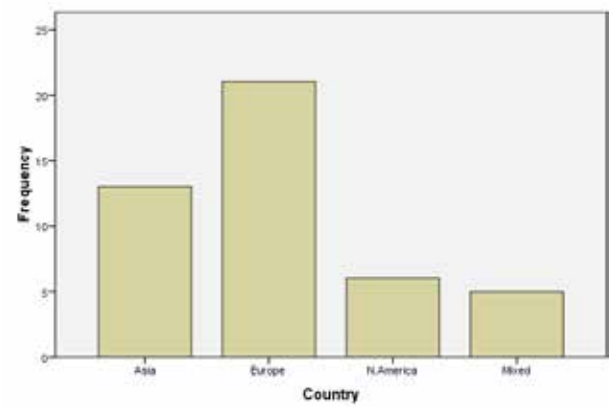

Fig. 3. Continent wise research papers detail and their percentage.

papers that have been missed out through formal search. The results shows that limited number of researchers have worked in this area.

Table 3 shows research papers distribution across the various continents. Results shows that 46.7% of all selected research papers are from Europe which means that researchers from this area gives more attention and interested in exploring the hazards faced in management of software projects in offshore environment. The results also show that researchers from Asia are also working in this area.

6.2 Decade-wise Detail of Selected Publications

We have divided search periods into two decades, the first decade is from 2000-2010 and second decade is from 2011 to 2015. In our trail search, we have found that no research paper has been found before 2000 which discussed the project management in offshore multi-sourcing from vendors' prospective. Our search and results also show that this is new area of software project management maturing since 2000 and limited numbers of researchers has contributed to this area. The search results also show that this area require more attentions of researchers to dig out the practices/solutions of barriers in second decade because many of the software organizations from all over the world adopting GSD strategy for secured widening of their business and making positive decisions [40].

The analysis of data in each decade has been made by using multiple tests as indicated in Table 5. Each critical barrier has been compared decade wise. Linear by linear association Chi-Square test has been used for finding any significance difference in the critical barriers across the two decades. The reason behind using linear by linear association Chi-Square test in our analysis is that it is more powerful than Pearson's χ 2 test [41]. The highlighted values having statistical significance difference ($P<0.05$). The below Table 5 shows that there is a minor difference between the two decades for the critical barriers 'Lack of Communication and Collaboration between Stakeholders', 'Difference in Language Culture Time and Geographical Distance', 'Ambiguous software design', 'Lack of Experience in Multi-sourcing Projects',' Volatile Customers Requirements', 'Lack of Trust', 'Lack of Standard PM Practices and Processes in Multi-sourcing '. This means that these factors have been considered most important in both decades and still the focus points for researchers of this field.

While the factors like 'Complex Relationship Between Vendors', 'Organizational Politics' , 'Lack of Training', 'Hidden Costs', 'Issues in software coding and testing' got attention and importance in second decade because of increasing trend of GSD. 'Delayed Feedback', 'Poor planning and estimation', 'Change in Roles and Responsibilities' has gained less researcher attention in 2nd decade because of new approaches to software development like RAD, COTS reuse, Extreme programming and agile techniques etc.

The three components 'Lack of Client and Top Management Involvement', 'weak monitoring and Control' and 'Lack of Team Spirit' have big difference between two decades.

In Table 6, we further compared each identified barriers from each study strategy. In below Table 6, we have combined Case study, Interview and Literature review. The linear-by-linear Chi-square test has been performed also for each identified barriers on the basis of study methodology used for finding any major difference between study strategies if any. The Table shows that there is no significance between barriers on the basis of study strategy. The Table also shows the detail of each barrier in each study strategy.

6.3 Continent-wise Comparison of Critical Barriers

The data comparisons between different continents, i.e., Asia, Europe and N. America are shown in Table 7. The data from other continents has been ignored because of low sample size. The objective of this analysis is to find out any differences in these continents with respect to the identified critical barriers. We have used linear-by-linear association chi-square test to find any significant difference between barriers throughout the continents. There are 22 barriers in Asia, 23 in Europe and 17 in N. America. The Table 7 shows three major variations of identified factors for all three continents that are 'Lack of Communication and Collaboration between Stakeholders', 'Difference in Language

Table 5. Comparative analysis of critical barriers in each decade.

Critical Barriers Lack of Communication and Collaboration between Stakeholders	**Decade-1st 2000-2010 (N = 20)**		**Decade-2nd 2011=2015 (N = 25)**		**χ2 test (linear-by-linear association)α =0.05**		
	Frequency	**%**	**Frequency**	**%**	**χ2**	**df**	**P**
Lack of Communication and Collaboration between Stakeholders	18	90	24	96	.629	1	.428
Difference in Language Culture Time and Geographical Distance	17	85	20	80	.186	1	.666
Complex Relationship between Vendors	5	25	12	48	2.445	1	.118
Lack of Experience in Multi-sourcing Projects	8	40	13	52	.629	1	.428
Lack of Technical Skills	11	55	17	68	.781	1	.377
Delayed Feedback	6	30	3	12	2.200	1	.138
Volatile Customers Requirements	5	25	9	36	.613	1	.434
Poor Planning and Estimation	9	45	9	36	.367	1	.545
Ambiguous software design	5	25	6	24	.006	1	.939
Issues in Software Coding and Testing	3	25	9	36	2.450	1	.118
Lack of Trust	13	65	14	56	.367	1	.545
Software Integration Problems	1	5	1	4	.026	1	.873
Lack of Client and Top Management Involvement	**3**	**15**	**13**	**52**	**6.491**	**1**	**.011**
Security and Privacy Issues	4	20	6	24	.101	1	.751
Weak Monitoring and Control	**6**	**30**	**15**	**60**	**3.929**	**1**	**.047**
Lack of Standard PM Practices and Processes in Multisourcing	8	40	11	44	.071	1	.790
Organizational Politics	6	30	13	52	2.155	1	.142
Complexity in Multi-sourcing contracting	3	15	5	20	.186	1	.666
Lack of Training	8	40	15	60	1.739	1	.187
Change in Roles and Responsibilities	4	20	4	16	.119	1	.730
Hidden Costs	4	20	8	32	.800	1	.371
Lack of Knowledge Sharing	7	35	11	44	.367	1	.545
Lack of Team Spirit	**5**	**25**	**1**	**4**	**4.146**	**1**	**.042**

Culture Time and Geographical Distance', 'Lack of Training'. First two barriers have highest frequencies for Asia and Europe and low frequencies for N. America. It means that these two factors are more critical in Asia and Europe as compared to N. America. Lack of training has the highest frequency in Asia and Europe i.e. 62% and lowest in N. America, i.e., 33%.

'Complex Relationship Between Vendors', 'Lack of Experience in Multi-sourcing Projects', 'Lack of Technical Skills', 'Lack of Trust', 'Lack of Client and Top Management Involvement', 'Organizational Politics', 'Lack of Knowledge Sharing' are considered as most important and most critical barriers in all the three continents because of their higher frequencies as shown in Table 7. These factors are given more attention in each continent. Similarly, the Delayed Feedback and Lack of Team

Table 6. Comparison of barriers based on methodology used.

Critical Barriers Lack of Communication and Collaboration between Stakeholders	**Case Study (N=15)**	**Interview (N=14)**	**Literature Review (N=16)**	**χ2 test (linear-by-linear association) α=0.05**		
	Frequency %	**Frequency %**	**Frequency %**	**χ2**	**df**	**P**
Lack of Communication and Collaboration between Stakeholders	14	12	16	.577	1	.447
Difference in Language Culture Time and Geographical Distance	11	10	16	3.769	1	.052
Complex Relationship between Vendors	5	3	9	1.762	1	.184
Lack of Experience in Multi-sourcing Projects	8	7	6	.772	1	.380
Lack of Technical Skills	12	8	8	2.869	1	.090
Delayed Feedback	4	1	4	.008	1	.929
Volatile Customers Requirements	4	7	3	.253	1	.615
Poor planning and estimation	7	6	5	.758	1	.384
Ambiguous software design	3	4	4	.098	1	.755
Issues in software coding and testing	3	3	6	1.206	1	.272
Lack of Trust	8	6	13	2.546	1	.111
Software Integration Problems	2	0	0	3.106	1	.078
Lack of Client and Top Management Involvement	4	10	2	.764	1	.382
Security and Privacy Issues	5	1	4	.273	1	.601
Weak monitoring and Control	7	7	7	.028	1	.868
Lack of Standard PM Practices and Processes in Multi-sourcing	6	6	7	.043	1	.835
Organizational Politics	8	5	6	.759	1	.384
Complexity in Multi-sourcing contracting	3	1	4	.146	1	.702
Lack of Training	6	7	10	1.538	1	.215
Change in Roles and Responsibilities	4	1	3	.300	1	.584
Hidden Costs	4	2	6	.485	1	.486
Lack of Knowledge Sharing	5	4	9	1.704	1	.192
Lack of Team Spirit	2	0	4	.952	1	.329

Spirit have higher frequencies in Europe than Asia and N. America, which means that vendor's organizations and researchers in Europe give more attention to these factors as compared to others continents. The Volatile Customers Requirements and Lack of Standard PM Practices and Processes in Multi-sourcing have higher frequencies in Asia and Europe than N.America. Its means that less attention has been given to these factors by researchers and vendor organizations in N. America.

Table 7. List of CBs across the various continents.

Critical Barriers Lack of Communication and Collaboration between Stakeholders	Asia N=13		Europe N=21		N. America N=6		$\chi 2$ test (linear-by-linear association) $\alpha = 0.05$		
	Frequency	%	Frequency	%	Frequency	%	X2	df	P
Lack of Communication and Collaboration between Stakeholders	**13**	**100**	**21**	**100**	**5**	**84**	**9.331**	**1**	**0.002**
Difference in Language Culture Time and Geographical Distance	**12**	**92**	**19**	**90**	**5**	**84**	**9.612**	**1**	**0.002**
Complex Relationship Between Vendors	3	23	10	48	3	50	.081	1	0.777
Lack of Experience in Multisourcing Projects	5	38	12	57	2	33	.016	1	0.899
Lack of Technical Skills	6	46	16	76	4	67	.002	1	0.965
Delayed Feedback	1	7	7	44	0	0	.025	1	0.874
Volatile Customers Requirements	4	31	6	29	0	0	1.106	1	0.293
Poor planning and estimation	3	23	9	43	3	50	2.419	1	0.120
Ambiguous software design	2	15	6	29	1	17	.701	1	0.402
Issues in software coding and testing	4	31	5	24	2	33	.082	1	0.774
Lack of Trust	7	54	16	76	2	33	.823	1	0.364
Software Integration Problems	0	0	2	9	0	0	.011	1	0.918
Lack of Client and Top Management Involvement	5	38	7	33	2	33	.000	1	0.982
Security and Privacy Issues	3	23	3	14	3	50	.259	1	0.611
Weak monitoring and Control	6	46	10	48	2	33	.036	1	0.849
Lack of Standard PM Practices and Processes in Multisourcing	6	46	10	48	0	0	.166	1	0.684
Organizational Politics	6	46	7	33	3	50	.310	1	0.577
Complexity in Multi-sourcing contracting	2	15	3	14	2	33	.371	1	0.543
Lack of Training	**8**	**62**	**13**	**62**	**2**	**33**	**5.723**	**1**	**0.017**
Change in Roles and Responsibilities	2	15	5	23	0	0	.049	1	0.825
Hidden Costs	5	38	4	19	2	33	.418	1	0.518
Lack of Knowledge Sharing	6	46	9	43	3	50	1.852	1	0.174
Lack of Team Spirit	1	8	5	24	0	0	.427	1	0.513

7. SUMMARY

Through SLR, we have identified different barriers classes that should be addressed and keep in view by vendor organizations when managing multi-sourced software projects in offshore software development environment. In our study, the defined criteria for criticality of barriers is 30%, the barriers which have frequency greater than defined frequency, it will be considered as critical. According to this criterion there are 16 critical barriers out of 23 identified barriers as shown in Table 1, Our research reveals some of the barriers need special attention because their occurrence creates serious threats to management of software projects in GSD environment and may result in the projects failure. We also found out the impact of different barriers through different decades and continents for vendor guidance because some barriers were most critical in one region while less critical in other region. Similarly, some of the factors were more critical in previous decade but less critical presently because of different improvements in software processes and technologies.

8. LIMITATIONS

There are 45 research papers selected through SLR for conducting this research. In these research papers, maximum have been written and published

by scholars, academics and faculty members of the universities. Most of these researchers may not have the practical experience of managing the multi-sourced software projects in GSD. Theoretical work has been done by most of the researchers. We have used these research papers for finding the barriers in faced in managing multisourcing projects from vendors prospective in GSD. Now problem is that up to what extant our research findings are valid? To prove these findings correct and to the point, we plan to conduct questionnaire survey and empirical study in Software industry and take feedback of practitioners who practically working in multi-sourced software projects in GSD and to find other Factors apart from identified one which has been skipped in this study.

9. CONCLUSIONS AND FUTURE WORK

Through SLR, we have identify 23 different barriers faced by vendors in multi-sourced software projects in GSD as shown in Table 1, in which 16 were critical barriers according to our set criteria. These identified barriers may help the vendor organizations for successful completion of OSDO project by keeping these barriers in mind at the time of project start and during execution. These barriers are also analyzed from different angles like decade wise and continent wise for vendor guidance. In our study we have identified following goals that we will follow in future:

- The Validation of identified barriers by using the technique of questionnaire survey and empirical study with the help of experts and practitioners working in OSDO environment.
- Additional critical barriers will be identified from experts and practitioners through empirical study if any.
- Finding the practices and solutions against identified barriers through SLR and empirical study.
- Development of PMMSM.
- Validation and verification of PMMSM from practitioners working in OSDO.

10. ACKNOWLEDGEMENTS

The authors are grateful to Higher Education Commission Pakistan for encouraging and granting permission to their employee for higher education. Moreover, Muhammad Imran Khan is also thankful to Dr. Siffatullah Khan and Mr. Abdul Wahid Khan, UOM for supervising this research work as their valuable comments and quick responses contributed a lot to the successful completion of this research.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 281–291 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Text Clusters Labeling using WordNet and Term Frequency-Inverse Document Frequency

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Abstract: Cluster Labeling is the process of assigning appropriate and well descriptive titles to text documents. The most suitable label not only explains the central theme of a particular cluster but also provides a means to differentiate it from other clusters in an efficient way. In this paper we proposed a technique for cluster labeling which assigns a generic label to a cluster that may or may not be a part of the text document cluster. It finds the theme of a document and designates it as its label. We used Term Frequency and Inverse Document frequency at baseline for tf-idf, with the Term Frequency calculation refined by using a thesaurus. WordNet was used as an external resource for hypernym generation of the terms having the K-Highest tf-idf. The hypernyms with the highest frequency are then taken as the label of the cluster. The details of the datasets used for experimentation and the comparative results with existing methods are provided in the paper, and clearly reflects the meaningful outcome of our technique.

Keywords: clustering, cluster labeling, WordNet, thesaurus

1. INTRODUCTION

One of the well-known features of text mining is Document Clustering, or the breaking of large text documents into clusters. Users are then able to use these groups of text for analysis and other purposes. The well-defined clusters have high similarity in its inner group items and low similarity with outer group items. Such clustering chunks may be shaped in a more useful way by assigning each of them an appropriate label through a process called Cluster Labeling. Cluster labeling resolves the problem of weak readability [1] and helps users to understand the theme of the cluster. It also assists in checking whether a particular cluster contains the information relating to a particular interest.

This paper presents an automatic approach for cluster labelling. The concept of term frequency dictates that the word which appears the most in a cluster is the one assigned as the label. In cases where this is not possible, our approach allows a generic label that describes the theme of the cluster to be assigned. Objective of assigning label to a cluster is achieved through:

- Finding tf-idf of words appearing in document cluster
- Refinement of words through thesaurus using WORDNET
- Selection of Cluster label using hypernym frequency

The rest of the paper is organized into four

Received, February 2016; Accepted, August 2016
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sections. Section 2 includes a review of existing clustering and cluster labelling research. Section 3 is about our proposed methodology while Section 4 discusses the results of our experiment. Section 5 gives our conclusion based on the results and future work directions.

2. LITERATURE REVIEW

Text clustering has been a major area of attention when researching text mining techniques. However, due to the need for and importance of cluster text labeling, researchers have also started to explore techniques related to it. Carmel et al. [2] enhanced the labeling method by using Wikipedia .In this technique documents are initially indexed and then clustered using the well-known clustering techniques. Important terms are extracted from each cluster by using the technique described in Cutting et al. [3].For each important term related Wikipedia pages are extracted. Final labels are selected by the use of pointwise mutual information [4] and statistical co-occurrence [5]. Use of Wikipedia for assigning label to clusters may not be very useful as information provided in Wikipedia pages is not accurate. Any person at their own may provide information without any authenticity whose use for labeling purpose would lead to poor results. Authors didn't mention the cases where Wikipedia cases are not available. Ahmad and Khanum [6] described an algorithm called EROCK (Enhanced Robust Clustering Algorithm for Categorical Attributes)which can make and label clusters. They initially arranged into documents (i.e. clusters) then established the link between each through cosine similarity. Ahmad and Khanum then assigned the word that most frequently appeared as the label. Use of frequent term for the purpose of cluster labeling may not produce the accurate result as a term would not be able to give the central theme of the cluster. Moreover authors have not provided any detail for the cases where more than one terms having same frequency would occur. There is no criteria defined to select amongst them. A document with diverse theme would not be handled accurately using technique under discussion.

Tseng et al. [7] presented a hypernym search algorithm for labeling cluster. The proposed technique creates a generic title based on WordNet. By using correlation coefficients (CC), specific words related to cluster are extracted, while hypernym search algorithm determined the final labels and maps it into WordNet. They used WordNet as an external resource for finding labels of the clusters. They have used hypernyms of all the keywords obtained through CC technique. Refinements of the words are not done. Authors provide no mechanism for selection of label amongst different themes obtained through hypernym search algorithm. Review of some more relevant techniques is provided as well.

Pantel and Ravichandran [8] described a method that automatically assigns labels to clusters based on semantic relationships. They then selected the terms that most describe the clusters as the representative words. Bouras and Tsogkas [9] proposed enhancements in the k-means algorithm which used WordNet before clustering and then labelled the cluster. The authors improved the efficiency as compared to k-means clustering and the quality of labels is improved as well.Carmel et al. [10] presented term extraction in the domain of word cloud generation. They used tag-boost method which boost the terms occasionally used by people to tag the content. They claimed to achieve robustness in compared to other techniques. Mehrotra et al.[11] used unmodified Latent Dirichlet Allocation (LDA) to topic model for short text. They have used Twitter dataset and by using hashtag pooling with LDA achieved improvement compared to unchanged LDA. Morik et al. [12] produced structures for navigating social websites.They considered this as an optimization problem and solved it by using Genetic Algorithm. Jiang [13] provided a survey about information extraction. Majorly survey is about, named relation extraction and entity recognition.

Sun [14] proposed a technique for short text classification by using a non-parametric approach. They selected a small set of words based on their defined criteria and matched it with query words. Authors achieved better classification through this approach. Roitman et al. [15] labelled the clusters using the fusion method. They argued that the label of the cluster should be stable even if there are missing data in the clusters. They tested their technique on different datasets and achieved better performance. Alfred et al. [16] used hierarchical agglomerative clustering for document clustering. The agglomerative clustering was used to counter the fact of other clustering techniques that in most

of the cases in prior number of clusters are unknown. They applied different distance measures to investigate the quality of different clusters. They applied different distance measures to investigate the quality of different clusters. Nayak et al. [17] presented a clustering and cluster labelling method for Wikipedia documents. They took Wikipedia as a subset of the whole web. They tested their technique with 1000, 10000 and 50000 clusters. Xu et al. [18] proposed an approach which achieved Chinese word similarity by using hybrid hierarchical structure through HowNet. They performed their experiment on a SemEval 2012 dataset. Matthias et al. [19] used queries for cluster labeling. They combined internal and differential cluster labeling techniques for acquiring desired results. Daoudet al. [20] presented cluster labeling technique which over the clusters of Arabic tweets. They used key terms as candidate labels and through the web enriched them. They used Bayesian network to find semantic relation between the enriched terms. Hurtado et al. [21] proposed methodology for finding topics from collection of documents. The used association rule based pattern mining for their proposed research. They presented a forecasting technique as well which predicted the recognition of a topic in coming future. Diogo and Jonice [22] used topic labeling technique to detect innovative knowledge from scholar data. They completed their research by performing operations of candidate selected, score ranking and label selection. In [23], Alicante et al. used semantic labeling technique for medical data clusters. They constructed word embedding feature dictionary from Wikipedia pages which was later on used for feature creation and cluster labeling. The clusters were formed using k-means clustering algorithm.

3. PROPOSED METHODOLOGY

The proposed technique selects labels for text clusters in two phases: 1) the pre-labeling phase (may also be called as pre-processing phase) and 2) the labeling phase. In each phase, various steps are performed on text data.

3.1 Pre-Labeling Phase

The pre-labeling phase may also be called as labeling pre-processing phase. Clustering, stemming and term extraction are part of pre-labeling phase. The aim of these steps is to make data clean and eligible for accurate labeling.

3.1.1 Clustering

In this step a given dataset is partitioned into a number of similar homogeneous groups. The clusters are formed using a well-known hierarchical clustering method which combines

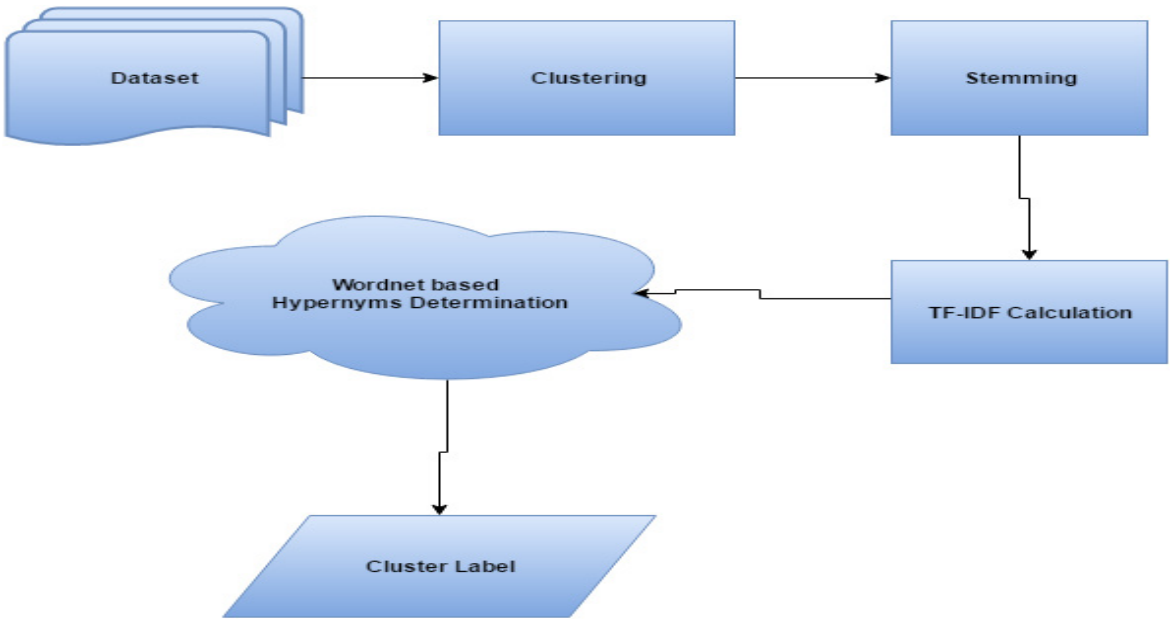

Fig. 1. Flow diagram of proposed technique.

similar observations called agglomerative K-Mean Clustering [10]. Given the document collection D, clustering results in converting D into a set of clusters i.e., C= C_1, C_2,..,C_k. A cluster is represented by its centroid and documents member of each cluster.

3.1.2 Stemming

After the formation of clusters the next phase is stemming. The main aim of this step is to eliminate common words like full stop, commas, articles and other irrelevant words. In order to eliminate common word we make some modification in standard Porter Stemmer algorithm [24] to eliminate not only postfix and prefix but also more and more common words.

Algorithm 1: Stemming

INPUT: Source Blocks SB_X,SB_Y

OUTPUT: Cluster with stemmed words, SC

1: Match $W_i \in C, \forall 1 \leq i \leq n$:n is total no of words in C(Cluster) with $W_p \in CWL, \exists 1 \leq p \leq m$:m is total no of words in CWL(Common Word list)
2: **if** Match is true **then**
3: Remove $W_i$from C
4: **else**
5: Keep $W_i$in C
6: **end if**
7: SC:=C

3.1.3 Term Extraction

In this phase we automatically extract a list of significant terms t(C) = (t_1, t_2, t^1,...,t_n) by calculating the term frequency and inverse document frequency for each word in each given cluster $C_i \in C$ and i= 1,2,3,...k. Term frequency is defined in this situation as the number of times a term appears in a document. Inverse document frequency, on the other hand, is the measure of the general importance of the term based on this formula:

$$idf(t) = \log \frac{\|D\|}{d: t \in d} \quad (1)$$

$$tfidf = tf(t) * idf(t) \quad (2)$$

The selected words are set as candidate words and will be used in the labelling phase (see section 3.2 and Algorithm 3).

Algorithm 2: Term Frequency Calculation using Thesaurus(TFC)

INPUT: SC

OUTPUT: Term Frequency List (TFL)

1: Create a Two dimensional dynamic list TFL initially with size=q×2
2: Initialize p=1
3: Get first word fw from SC and assign
4: TFL[p][1]:=fw
5: TFL[p][2]:=1
6: for each word $W_i \in$ SC
7: **if** W_i matches TFL[j][1] , ∃ j **then**
8: TFL[j][2]:= TFL[j][2]+1
9: **else**
10: p:=p+1
11: TFL[p][1]:=W_i
12: TFL[p][2]:=1
13: **end if**
14: for each list item k in TFL
15: Find thesaurus of TFL[k][1] through WordNet
16: **if** thesaurus(TFL[k][1]) matches some TFL[m][1] **then**
17: TFL[k][2]:= TFL[k][2]+TFL[m][2]
18: Remove TFL[m][ ] from list
19: **end if**

3.2 Labeling Phase

Once the terms are extracted, the Labelling Phase commences. This phase is what we consider as the main step of the proposed technique in which a final label for a particular cluster is assigned or generated. We do this by obtaining the hypernyms of each candidate using WordNet. The hypernyms with the highest frequency is selected as the label of the cluster. The label selected may not

necessarily be a word that can be found in the cluster but a generic label based on the WordNet hypernyms.

Algorithm 3: Cluster Labeling

INPUT: Thresholded Highest TF-IDF Term List (MFTL: Most Frequent Term List)

OUTPUT: Cluster Label (CL)

1: Create a dynamic hypernym list, HL
2: for each potential candidate word,$pcw_j$from MFTL
3: find pcw_j in WordNet
4: **if** match found **then**
5: Add hypernym of pcw_jin HL
6: **else**
7: Add pcw_jin HL
8: **end if**
9: find frequency of each hypernym in HL
10: CL:=hypernym with highest frequency

4. EXPERIMENTAL SETUP, RESULTS AND DISCUSSION

This section describes our experimental setup and the results we obtained after applying our proposed technique. Manual labeling from experts are taken as the perfect labels and their accuracy is bench marked as 100%.Below are the details of each of step.

Data sets: Four different types of text data sets were chosen for the experiment. Table 1 shows details about datasets used for experiments.

Table 1. Dataset details.

Data Set	No of Clusters	Text Chunks
Daily Jang Newspaper	1	2000 Text words/cluster
ODP	6	2000 Text words/cluster
20-News Group	20	2000 Text word/Cluster
Reuter	6	2000 Text words/cluster

In order to evaluate the proposed technique, we have performed experiments over the four datasetsi.e., Daily Jang Newspaper, ODP, Reuter and 20-News Group. Daily Jang Newspaper dataset contains 1 Cluster, ODP has 6, Reuter has 6 and 20-News Group has 20 categories. Each of the dataset has 2000 words per cluster.

Experiment I

In this first data from the Daily Jang newspaper, the document is considered as one cluster containing information about different types of games. It is reduced by applying a modified stemmer algorithm and a term extraction step to pick up top words. Using a thesaurus, the strength of the top words is reduced to greater or equal to threshold level. All of the top words are then mapped using WordNet so that accuracy may be achieved in generating the final cluster label. Results of this data set are shown in Table 2.

In Table 2 details of Experiment I are provided.

Table 2. Detailed Results for Experiment I.

Category/ Cluster	Words having highest tf-idf	Refined Words using thesaurus	Cluster labels through hypernyms
SPORT	1. Football	1. Football	1. A type of sport.
	2. Player	2. Player	2. Sports Man.
	3. Hockey	3. Hockey	3. A type of sport.
	4. Ground	4. Cricket	4. A type of sport.
	5. Cricket	5. Match	
	6. Captain		
	7. Match		

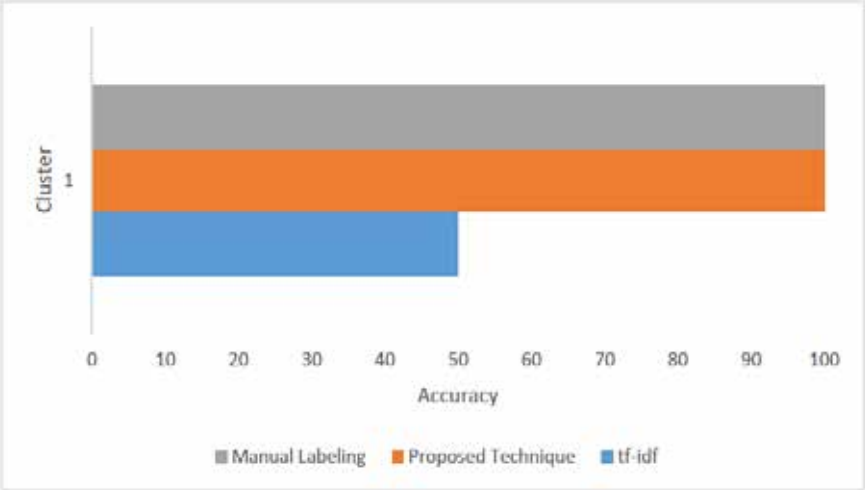

Fig. 2. Comparative results of proposed technique over Jang dataset: Experiment I.

Input dataset contains documents of sports topic. All of these documents are treated as one cluster. Initially Words having highest tf-idf are selected. Important keywords are given as input to algorithm TFC using thesarus where refined words are obtained. Lastly Hypernyms of refined words are obtained through Cluster Labeling algorithm which gave hypernyms,a generic word, of refined words.

Fig. 2 shows comparative results Manual Labeling, tf-idf and the proposed technique in graphical form. Results show that proposed technique achieved the same accuracy as of manual labeling. Performance of proposed technique for Experiment-I is double to tf-idf technique for cluster labeling.

Experiment II

The second collection was gathered by downloading pages from the Open Directory Project (ODP). For this purpose, we randomly

Table 3. Detailed Results for Experiment II.

ID	Category/ Cluster	Words having highest tf-idf	Refined Words using thesaurus	Cluster labels through hypernyms
1	Animals	Rabbit, John, Horse, Cluster, Name, Dog	Rabbit, Horse, Cluster, Dog	1. Herbivorous/Animal 2. Herbivorous/ Animal 3. Group of similar things 4. Carnivores/ Animal
2	Automobile Information		CNG, Fuel, Truck, Car, Automobile, Road	1. A substance 2. A vehicle 3. A vehicle
3	Air Line Information	Column, Scan, New, John, Code	Code, New	1. Unfamiliar, Unknown 2. Rules, Principle, Law
4	Language	John, Claim, Enough, Cluster, Germany	Claim, Enough, Cluster	1. Demand for something 2. Sufficient for something 3. Grouping of similar thing.
5	Male Expectation	Life, Age, Year, Africa, Expectation	Life, Age, Year, Expectation	1. Mode of Living 2. How long something exists. 3. Period of time Expectation
6	Protein Amount	Fat, Protein, Beef, Amount, Calcium	Fat, Protein, Beef, Calcium	1. Bodily Property 2. Substance of Egg 3. Beef Cattle 4. Metallic Item

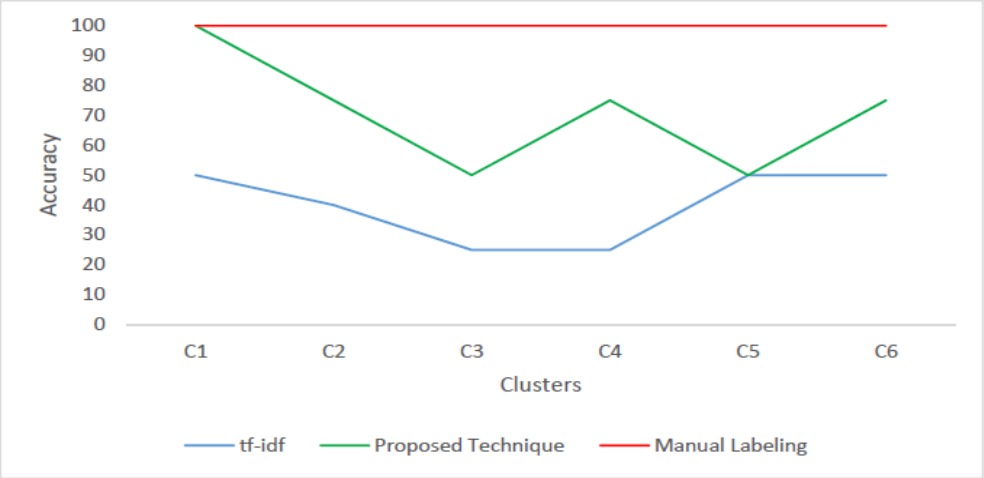

Fig. 3. Comparative results of proposed technique over ODP dataset: Experiment II.

selected six different categories from the ODP hierarchy. For each category we then randomly selected up to 100 documents, resulting in a collection size of about 10,000 documents. We then manually labelled the categories in both collections. These ground truth ”correct” labels were later used to evaluate our labeling system.

Table 3 gives detailed results of Experiment-II performed over proposed technique. 5 highest tf-idf words are selected as label candidate which are subject to TFC using thesaurus algorithm for refinement. Lastly hypernyms are taken as final cluster labels using Cluster Label algorithm.

Fig. 3 depicts comparative results of proposed technique with tf-idf and manual labeling over ODP dataset. For the category of animals our proposed technique produced the same result as by expert human resulting 100% accuracy in comparison to 50% of tf-idf. For the category of automobile information our technique produced A Vehicle as a label whilst tf-idf produced CNG, a less appropriate word. For all the remaining four categories, although proposed technique didn't produce the same label as human expert yet it was more appropriate than produced by tf-idf. Overall proposed technique attained 70% accuracy as compared to 40% through tf-idf over ODP dataset.

Experiment III

Experiment III is performed over dataset having twenty clusters that contain different news stories. We initially stemmed the data set, extracted the candidate terms and then further refined it using a

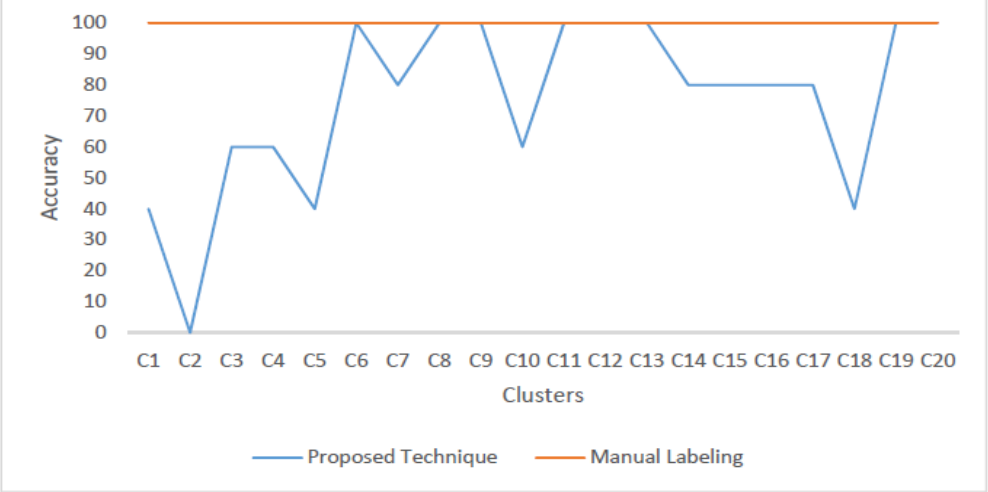

Fig. 4. Comparative results of proposed technique with manual labeling:Experiment III.

thesaurus. In the final step hypernyms of refined words are extracted and mapped to final label. Experiment was performed over 20-News Group dataset which contained approximately 20000 documents from 20 different newspapers. First 5 categories are closely related to each other i.e., computer related documents, for categories 6-9 subject matter is sports related, in categories 11-13 documents contained different topics related to scientific information, whilst category 14 contained documents having forsale topics, documents in categories 15-17 have political talks on three different areas i.e., misc., guns, mildeast. Categories 18-20 have documents containing religious topics in three different areas.

Fig. 4 is the graphical representation of comparative results achieved by proposed technique to expert labeling. Results achieved for first five categories having computer related documents accuracy of the proposed technique is just average. For a particular cluster two having miscellaneous MS windows documents our technique was failed to provide any suitable label. Proposed technique achieved an average 40% accuracy for clusters through 1-5. However for the

Table 4. Detailed results for Experiment IV.

Category/ Cluster	Words having highest tf-idf	Refined Words using thesaurus	Cluster labels through hypernyms
Earn	1. NET 2. QTR 3. Shr 4. Cts 5. Net 6. Revs	1. NET 2. QTR 3. Net 4. Revs	1:goal 2:trap 3:income 4:income
Acquire	1. Acquire 2. Acquisition 3. Stake 4. Company 5. Share	1. Acquire 2. Stake 3. Share	1:device 2:stock certificate, stock 3:wedge
Money	1. Currency 2. Money 3. Market 4. Central banks 5. The Bank 6. Yen	1. Currency 2. Money 3. Market 4. Yen	1:currency 2:currency 3:marketplace, mart 4:China Currency
Grain	1. Wheat 2. Grain 3. Tones 4. Agriculture 5. Corn	1. Wheat 2. Grain 3. Corn	1:seed/ eating food 2:cereal, cereal grass 3:foodstuff, 4:food product
Crude/fuel	1. bpd 2. Crude oil 3. OPEC, 4. mln barrels 5. Petroleum	1. Crude oil 2. OPEC, 3. Petroleum	1:lipid, lipide, lipoid 2:fuel/oil 3:fossil fuel
Trade	1. Trade 2. Tariffs 3. Trading 4. Surplus 5. Deficit 6. Gatt	1. Trade 2. Tariffs 3. Trading 4. Surplus	1:business 2:UN business agency 3:prevailing wind 4:Business rule

clusters 6-9 (i.e., sports related categories)the proposed technique achieved 95% accurate labels. Scientific topics are contained in clusters 10-13 and the proposed technique resulted promisingly by achieving 90% accuracy in labeling clusters. Cluster 14 was about documents having concept of forsale and it is accurately been labeled. Clusters 15-17 are labeled with 80% accuracy whilst labeling accuracy for clusters 18-20 remained 80% as well through proposed technique. Overall accuracy achieved for 20-Newsgroup dataset is 75%.

Experiment IV

In this experiment we have used Reuter-21578 dataset. As originally Reuter dataset has 21578 text documents and multiple categories, we have used largest six categories amongst them for our experimentation i.e., Earn, Acquire, Money, Grain, Fuel and Trade.

Table 4 shows the results obtained through different steps of proposed technique over Reuter dataset. Top five words are selected on the basis of tf-idf and where more than one words have same frequency, all are selected.Results achieved through proposed TFC using thesaurus algorithm and cluster labeling algorithm are presented as well.

Fig. 4 shows comparative results of proposed technique Tseng [7], tf-idf and Manual labeling. For cluster 1 with Earn category, accuracy in cluster labeling for proposed technique matches with accuracy of manual labeling and leading Tseng and tf-idf accuracy. Proposed technique was unable to select appropriate label for cluster 2 and accuracy of proposed technique remained at bottom. For cluster 3 proposed technique achieved same accurate label as tf-idf better than Tseng label and lower than Manual label. As far as cluster 4-6, proposed technique achieved 100% accurate label and leading both tf-idf and Tseng labels. Overall the proposed technique 90% accuracy in labeling clusters of Reuter dataset as compared to 66% accuracy of tf-idf and 72% accurate labels of Tseng.

The results obtained by applying the proposed technique on various text data sets reflect that the performance of the proposed technique is better in terms of its accuracy. After the comparison of the proposed technique with other existing techniques, it is clear that the performance of proposed technique is quite improved. This is also evident in the graphical representation of each experiment. However, the technique proposed do have some constraints. WordNet doesn't cover all the terms extracted from text clusters. Some of the WordNet-generated titles may not also reflect the theme of a particular cluster accurately.

5. CONCLUSION AND FUTURE DIRECTIONS

Cluster labeling is the process of allocating appropriate title to a particular cluster. In our approach, we labelled clusters using an external resource called WordNet. To achieve the task three algorithms are presented Modified Stemming algorithm, Term Frequency Calculation using

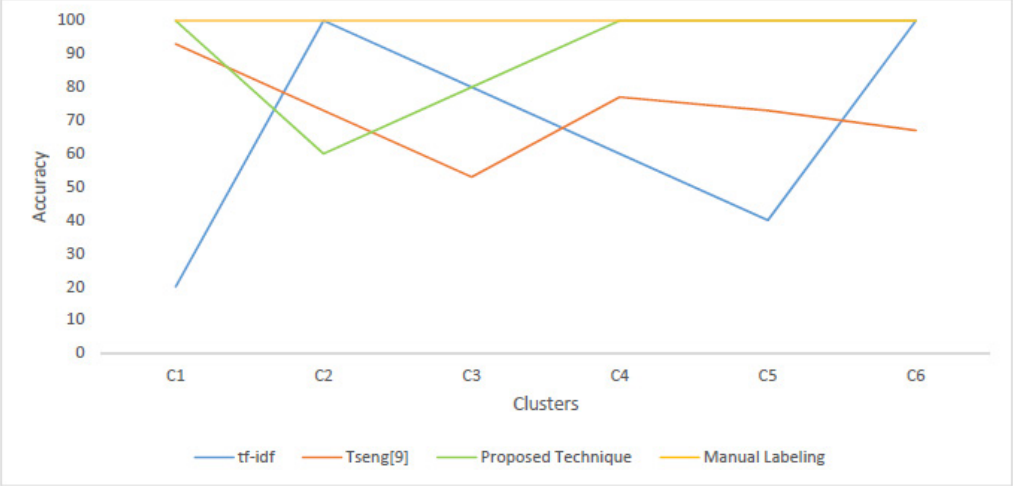

Fig. 5. Comparative results of proposed technique over Reuter dataset.

Thesaurus and Cluster labeling algorithm. TFC and Cluster labeling algorithms use WordNet as an external resource to get Thesaurus and hypernyms. We have performed experiments using four datasets Daily Jang Newspaper, ODP, 20-Newsgroup and Reuter. Experimental results achieved through proposed technique are quite encouraging by attaining 100%, 70%, 75% and 90% accuracy in labeling the clusters for Daily Jang Newspaper, ODP, 20-Newsgroup and Reuter datasets respectively. In different experiments comparative results of proposed technique along with tf-idf, manual labeling and Tseng are presented as well. Comparisons results have clear reflection of achieving better results than Tseng and tf-idf techniques whilst achieved comparable results against manual labeling. Although experimental results reflect that Cluster labeling using WordNet has shown promising results but the performance may be affected by topics whose WordNet hierarchy is not available. We also observed this in those that require multi topic labels. Unfortunately, the proposed system may be unable to generate multi topic label for a particular system. For such collections, there is a need to take an intelligent decision regarding multi topic labeling with the use of WordNet and thesaurus. This could be taken as future direction to improve our proposed technique.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 293–307 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

A Novel Light Weight and Automatic Authentication based on Centralized Approach for Pervasive Environment

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Abstract: Today is the age of hyper-connectivity, no standalone system ever exist. Now each device has the processing and communicating capabilities. Pervasive computing brings all these devices into a uniform layer for ease of use and for providing on fly services. In pervasive environments, smart devices communicate each other to provide pervasive services according to the user modes and contexts. To construct an ad hoc and un-structure network of heterogeneous entities and to standardize all different technologies into a uniform solution, many technical and functional challenges are need to be addressed. With dissimilar nature and distributed control over the resources in unfriendly situation the pervasive environments always in trouble due to lack of proper security system. For consistent dynamic flow of services in an ad hoc pervasive network, the authentication of users, devices, services and process are critical. Here in this research work we proposed "A novel light weight and automatic authentication scheme based on centralized approach for pervasive environment". In this approach a central base station is responsible for providing resources and implementing security policy for all entities. Public Keys, Public key Certificates, Nonce, IDs and time stamps are parameters used in the proposed scheme. The new scheme is validated and analyzed in a simulator in the presence of attacker. The proposed model is designed to prevent most sophisticated DoS attacks and man in middle attacks.

Keywords: Pervasive computing, context awareness, ambient intelligence, middle ware, embedded devices, ad hoc network, man in middle, DoS.

1. INTRODUCTION

Pervasive or ubiquitous computing means that all objects in our surrounding are become so intelligent that they understand the context and behaves according to the situations. Physical spaces, mobile devices and building infrastructures interact with each other and provide services according to the context. The omnipresence of chip-based smart devices with hybrid network enables us to interact and use all the available services in a uniform way. Billions of smart devices make out environment more interactive, attractive and user friendly. Pervasive is now termed as Internet of Things (IoTs) because services are available for everyone, everywhere and any time without knowing the underlying infrastructure [1, 2]. The Mark Weiser was the pioneer in giving the idea of pervasive computing in 1991 [3, 4] and now with the emergence of the miniaturization in devices and sensor technology enable us to construct pervasive environments, where services are available at everywhere, at any time for any one. The interactive spaces and smart dust make our environment more accessible and convenient. Services are available anywhere, anytime for everyone with a zero-click. Users even did not know about the nature of the software, platform and services while achieve the service as he wished.

Pervasive computing declines time and space by providing on fly services which ultimately lead to reduce the cost for the offered services. User can use other user resources without hesitation if the service is reachable in the same premises. Pervasive computing changed the traditional dull computing into more interactive computing. Now every device is embedded with microprocessor, memory and with communicating facility. These

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smart devices reflect the current circumstances according to the user moods and conditions. It can remember crucial moments because they have memory, they show context sensitive behavior because they have sensors and they are responsive because they have communications links. It provides a new apparition of computing where computing will be disappear into specialized invisible computers. In simple words these ubiquitous personal assistance will be the integral part of human environment. As stated by Moor law [5] , that after every eighteen months the processing and storage capacities shall be increased in double. The figure.1 shows the trend that how technology makes available pervasive computing.

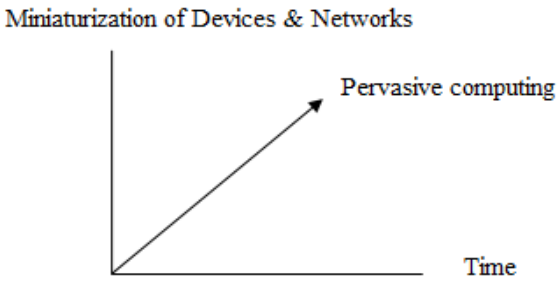

Fig. 1. Trend in pervasive computing.

Traditional computing approach is dull and passive where all burdens on a user with single machine for single user as in system centric approach. While pervasive environments are user centric where all burdens is on the surrounding smart devices. In user centric approach, the portable and embedded devices communicate each other and behave according to the context. Hardware and software resources are arranged into a resource channel in cascading. Resources are arranged in such a uniform layer that they operate themselves automatically according to the context. The framework bring all the nearby resources (hard ware & software) to a platform where services are available everywhere, for everyone, any time. The pervasive computing will subsist in our lives everywhere and that's why MIT called its pervasive project "Oxygen" [6] .When devices came under such framework, a single system can use a bunch of resources at a time which cannot possible for standalone system. Figure 2; show the basic of pervasive computing paradigm.

Fig. 2. Pervasive environment paradigm.

For real pervasive environment many technical and functional challenges need to be resolved. But the impromptu and diverse nature of pervasive network with partial resources and frosty environment leads to the inherent weakness of security. Traditional wired and wireless networks are secured with strong administrative policies with most dedicated devices like firewalling and Intrusion Detection Systems. But a network with different channels and with different devices having no defined infrastructure, these mechanisms are infeasible. For better accessibility and good availability of resources, authentication of all entities always desired. A new light weight and portable scheme is always needed to fulfill the requirements of the authentication of all entities including users, devices, services and process.

2. RELATED WORK

Here the first scheme is discussed about providing end to end authentication of users, devices and services [7]. The proposed system is based on Globus Grid Security Infrastructure (GGSI) [8] in which users are authenticated but machines and related services do not. In GGSI extensive use of proxies while here the system works without proxies. PKI based certificates and CA play main roles in authentication. Commonly used key is 256-bits. The Fat browser use APIs with WS protocols for security purpose [7]. The SAML credentials are uses for authorization of entities and XML provided exchange of information. It cans prevention of many variants of Man-in-the-Middle (MITM) attacks. The service based architecture, bilateral authentication and cascading process in authentication are the main steps in this

model. Authentication for every entity creates latency and overhead in the system. The browser compatibility on user side is another problem [1].

Another scheme is based on critically analysis of Turkanovi et al. scheme [9]. In which first the scheme is investigated for man in the middle and stolen smart card attacks. Much vulnerability is found out and based on the same weakness; the scheme is updated for resistance to such types of attacks. The scheme is useful because it used only symmetric techniques of cryptography. It uses XOR operation and hashes which is very simple and light weight for such low energy network. All entities are authenticated by a systematic method of protection of passwords, many choices for new passwords, dynamic addition of other entities and quickly changing passwords policy. The performance analysis in BAN-Logic and with AVISPA simulation tool of the improved scheme is better and more efficient to its successor [10].

A lightweight and low power authentication scheme has proposed for authentication of devices and services [11]. They deploy "Generic Bootstrapping Architecture (GBA)" [12] of "3rd Generation partnership Project (3GPP) of mobile technology for authentication of participating entities. The scheme is useful for global mobility of such pervasive devices but it produces extra overhead in terms of IP address pool and calculating of hashes for IPs. The authors claim analysis of scheme is energy efficient but scheme have narrow scope in low energy devices [11].

Low level authentication scheme "Aggregated Proof based Hierarchical Authentication scheme (APHA)" is based on U2IoT architecture in hierarchical method. The two protocols used light weight methods for calculating dynamic hashed for authentication, confidentiality and integrity. It uses chaotic maps and direct paths methods for ensuring authentication of data [13]. APHA mainly described in three phases; first, aggregated proofs are collected for unidentified network traffic in both directions. Second, for mutual authentication of chaotic maps, homomorphism and path descriptor apply combine. Third, different levels of trust are defined and named for hierarchical authentication of data and devices. APHA is good for data integrity and confidentiality but the scheme is not recommended for authentication of entities including devices, services, channels and users in pervasive environment [13].

Another light weight authentication scheme server-client based architecture has proposed [14]. Physical objects are authenticated and verified its status "Constrained Application Protocol". In first phase, the requester and provider entities are authenticated each other. In second phase, only those services are provide that only specified certain conditions in the request. The first phase authenticates heterogeneous devices with different specification, architecture and data rates. The second phase minimizes resources usage by only fulfilling the specific request. The scheme is looking good against some specific attacks like eaves-dropping and key fabrication but not recommended for resource exhaustion and denial of service attacks [14].

Table 1. The symbols used in system.

BS	Base Station
N_A	Node-A
N_B	Node-B
N_N	Node-N (any node/entity)
Cer_A	Certificate of Node-A
Cer_B	Certificate of Node-B
$RrgreqSer_S$	Registration request for service
PU_{BS}	Public key of base station
E	Encryption
D	Decryption
$Time_{1...n}$	Specific time for message request
$Time_{expire}$	Time on which certificate expired
$Time_{stamp}$	Time when certificate issued
ID_A	Identity of Node-A
ID_B	Identity of Node-B
PU_A	Public key of Node-A
PU_B	Public key of Node-B
$Services_N$	Some specific service
Req	Request for a service
Specs	Specification of the request type (network & supporting technologies)
N_1	Nonce at time T1
N_2	Nonce at time T2
Dev_i	Device List having (i) number of devices
$Serv_i$	Services List having (i) number of service
$Task_i$	Any task
Req_i	Request for resource
Dev_s	Specific Device
$Serv_s$	Specific service
MiM	Main In Middle

An automatic authentication scheme has proposed based isolated zero knowledge approach [15]. Sending messages are used for authentication between legitimate nodes based on secret session keys. The bulky data are shared between authenticated nodes in broadcast manner. For public key exchange many case scenarios are discussed. For analysis, the system is implemented in "Android Open Source Project" reveals that it is light weight authentication with minimum utilization of resources with high level of security.

The scheme is useful for abrupt and dynamic networks but prior knowledge for communicating entities make it hurdle for pervasive environments [15].

Very faster and more efficient authentication mechanism which works on short encrypted and authentic messages is explained in [3]. A short random string is appended to the plaintext message before encryption. A single one time key is used and many other schemes have discussed based on Message Authentication Codes (MACs) [16] and radio frequency identification (RFIDs) [17]. The Lima and three theorems are nicely is explained. The small message size and small modular further minimize the amount of processing data, which increase system efficiency. Due to the light-weighted modular multiplication, the hardware implementation is efficient as compared sophisticated cryptographic operations [17]. The proposed scheme is recommended for short messages are used but not for bulk data [17]. Here another system based master key proposed for novel authentication. This master combined all digital keys for authentication. The same master key is responsible for starting authentication and for the selection of other keys on the basis of code words with locks. The taxonomy and choices for master key creation nicely explain for achieving good usability, authentication and security of the users. Exchange code in master key support key locks automatically without user interventions. The master key maintains the security by applying key locks interaction and keeps the authentication secret. Master key scheme does not sustain multiple groups of key owners [18].

A new approach for authentication in ad hoc and wireless environment are studied and analyzed with assessment [19]. Only authenticated devices are the part of network, therefore, if user password and other credentials are stolen, network resources are still confined [19]. Threat model for physical device authentication include Bluetooth specification, Radio Frequency Identification (RFID) tags based attacks and the vulnerabilities of key based sensors network. The IEEE 802.1X framework [20], 802.16 Case [21], Trusted Computing Solution (TCG) [22] and many other authentication schemes [23] are discussed. The main focus here is on correct identification of devices without revealing user's credential [19]. The paper also spotlights some points about 4 G pervasive environments [19]. In pervasive and ad hoc network system, the devices are resource restricted in processing power, memory, communication and software support [24]. Most of these devices are portable, hand held and light weight. Robustness and dependability are difficult in such heterogeneous and multiplatform environment [24]. The system is divided into network security and system security. Field Programmable Gate Arrays [25] and Suggested Application specific integrated circuits [26] with their low-cost, low-power and easily deployment, are better option for light-weighted cryptographic algorithms. Reducing the input key, number of rounds and processing bit for specific system does not increase system efficiency [24].

One of the schemes suggested for light-weight authentication key agreement protocol for authentication based on of a user behavior. The Elliptic Curve-based Secure Authenticated Key Agreement protocol (EC-SAKA) [27] provides basis and Diffi-Hellman key based protocol make the system more resistant against malicious users. The 3-pass scheme for authentication generates a common secret key with collaboration to an elliptic curve-based digital signature [28]. The demand and expiry approach is used for minimum resource utilization. The system is focus on metric values rather than the underlying network. For judgment the human behavior, a technical approach is used based on number of control messages exchanged and the total number of actions for specific events [28].

Another scheme for Privacy-Preserving Location proof Updating System (APPLAUS) [29] is suggested, in which mobile entities correlate each other by using Bluetooth. Mobile entities verify their location by updating pseudonyms periodically with location servers. The APPLAUS structure with different entities (prover, witness, Location Proof Server, CA) in the environment and their interaction with each other are also explain [29]. The simulation results also show power consumption and the proof exchange latency for APPLAUS. The performance evaluation has been done with three metrics (overhead ratio, proof delivery ratio and average delay). The location is verified from both parties with updated pseudonyms for avoid intruders [29].

A scheme known as "trustworthy authentication" based on trustworthy behavior of the genuine entity is defined [30]. The typical procedure consists of eight steps, main

components are trustworthiness record and local trustworthy certificate with mentioned parameters and higher level trustworthy certificate with pre-define requirements. This approach is better for environment where most of the nodes are transportable and network links are effervescent. For clear autonomy the mobile users develop a trust for resources and vice versa [30].

Here in this paper, suggested two-step QR-Auth, 2D barcode authentication for entities with minimum user interaction [31]. The system consumes the visual QR-Codes in arbitrary alphanumeric data. Authorization Delegation and One-Time Password Generation are explained at packet level. In systematic and sequential way the protocol collects the sample images, transforms these images into bits and uses it as a proof of authentication [31]. The use of the system is easy due to visual rather strict and complicated password schemes. The proposed scheme shows resistance for main in middle and denial of service attacks. The visual channel is considered to be suspect for intruders, other credentials are considered to be secure [31].

A scheme known as "Secure Ubiquitous Authentication Protocols (SUAP) [32]" for efficient authentication is suggested. SUAP is a hybrid of "low-cost authentication protocol (LCAP) [33]" and "one-way hash-based LCAP (OHLCAP) [34]". The new scheme removes major drawbacks and combines the advantages of both the schemes. The random numbers and hashing value is used for encrypting the key for the protection RFID system. The threat model for RFID system consist of information leakage, traceability and location privacy, impersonation and replay attack and denial of service (DoS) attack [32]. The LCAP is explained in six steps while in OHLCAP the static identifier, a secret and one way hashed function is used. These protocols work on challenge–response method with low cost, hashed address indexing and one way functions [32].

It is self organizing scheme based on audio sampling. Which authenticate devices when they are in a specific acoustic area [35]. Recording phase, feature extraction phase, feature exchange phase and verification phase are discussed. To avoid and prevent the attacks, the feature extraction is not an arbitrary process. Because in those cases, the attacker can records the environmental sound samples and can analyzes about the auditory skin tone for a specific region [35]. The system also analyzes other relevant things which improve the system efficiency like computation cost and energy cost.

This paper argued some challenges about authentication based on formal and graphical system [35]. In the first part, cryptographic protocols or graphical language is used while in the second part, the logic is used for reasoning about the authentication protocols [35]. The payload consists with the potential identities rather than sender or receiver identities. Protocol Derivation Logic (PDL) [36] is actually the new description of Compositional Protocol Logic (CPL) [37]. In proximity authentication, a fresh nonce is used to prevent replay attacks. Proximity verification is done with the help of time channels, time channel response, specifying timed channels in PDL and with security goals of proximity authentication [35]. PDL to distance bounding is explained with a detail description of Distance bounding protocols and with Brands-Chaum Protocol [38].

Another authentication scheme is based on enhancement of the work of Lee, Batina and Verbauwhede [39] [40]. On the basis of this analysis, two of the protocols show strong privacy and third one has weak privacy preserving. A searching protocol is also offered in which a server querying a specific tag with efficiency. In first part, three previous approaches (Lee, Batina and Verbauwhede) are re-new for authentication for privacy preserving. In second part, a searching protocol has discussed based on a novel approach and working on querying a particular tag [39].

The Revised Elliptic Curve Based Randomized Access Control (EC-RAC) protocol is also explained with Randomized Schnorr Protocol [41]. Hui and their co-workers suggest new scheme for distributed authentication [42]. A trust pervasive model illustrates trust relationship among hosts based on distributed applications. Instead of the trusted third party (TTP) for reliability and security, here they distribute the services based on threshold cryptography [43]. The agent owner create signature by signing task. The agent is dispatch into pervasive network and search for a specific offer. When find an acceptable offer, it sign with TTP. Two proxy keys (prA and skA) are used for signing process [42].

A proposal is used for the protection of original digital content authentication from copyright

infringement. It protects intellectual property from modification or fraudulent use of digital contents. It limits the access privileges by setting the scope of content usage [44]. The mechanism proves the authenticity for extraction and by comparison original and targeted contents. System efficiently used in offline mode for verification of the contents. Some implementations are pixel based and histogram based comparison, entropy Based Comparison Mechanism and comparison based which focus on border icon in a specific area. Also include a brief overview of frame similarity extraction algorithm. Proposed scheme is properly analyzed by applying different techniques [44].

3. CENTRALIZED APPROACH: A NEW SCHEME

We proposed a centralized approach for real time authentication in pervasive environments. Instead of mesh and dull pervasive environment, the centralized approach is more useful and efficient. Centralized approach provides a central point for connection establishment and central policy implementation. In previous approaches, the device first search for a specific service (hardware/ software) in dull passion, where every device works an independent entity in a passive manner. So if more than a dozen devices in environment and all of them are in requesting phase. Then the environment becomes interlocked and the system performance would be degraded.

4. PROBLEMS WITH EARLIER APPROACHES

4.1 Unintelligent Network

In previous approaches, most of the time, devices connect each other in a dull passion. If the required service is available with required specification then the service is availed. But if the requested entity have not compatible with provider entity then all the process is worthless. Such type of system has no idea that how to provide services on a uniform layer for hybrid network of different devices.

4.2 Latency and Delay

Devices in earlier approaches are communicated to each other concurrently for same or different services. If the multiple devices need one request which is already occupied by another one or many request generate for many services on same time, then the network experience delay in response. This delay leads to create latency in the network and finally packet results.

4.3 Hybrid Network

Earlier approaches have no support for heterogeneity between networks. Different devices have different network support. So if the requester belongs to one network and provider belongs to another, they cannot communicate. For smooth communication between different network devices, a mechanism is needed.

4.4 Binding and Resuming of a Service

What will happen if the service provider is down or fail during service consumption? From where the requester get the same service and from which point the service need to be resumed? Another scenario, if the requester needs two service and these services located on different location with devices. Then who will bind both of them for requester? For binding and resuming, a mechanism is always desired.

5. THREAT MODEL

Pervasive computing is, in fact, an ad hoc and unstructured network of different device with different network support. Compared to its predecessor ad hoc network, it has more security threats. And as newer area the pervasive networks is not so mature to prevent all attacks deployed on ad hoc network. Here we discuss the most sophisticated denial of services (DoS) and man in middle (MiM) attacks. We designed our scheme keeping in view the structure of ad hoc pervasive network with respect to these attacks.

5.1 Denial of Services (DoS)

In this attack, the attacker makes an attempt to prevent the legitimate users from availing the services. In our model, when a single user sends too many request to the base station for serving their requirements. The base station verifies the node identity and signs a certificate for it and sends it back. All this process tack time and too many requests can cause the base station for denial of service for another user. Another form of this attack is when a user occupies a service for all time and another user waiting for it.

5.2 Man in Middle (MiM)

In MiM attack, a malicious node intercepts the traffic between two communicating entities without their intensions. The man in the middle

captured the packets, open it and may be changed or not and resend it for destination. Sometime the packets are captured and resend it again and again. This is variation in MiM and known as replay attack.

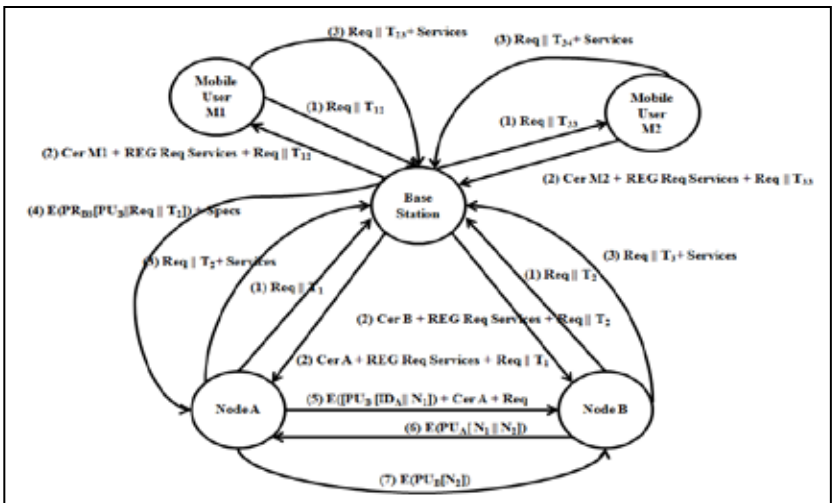

Fig. 3. The system architecture.

6. SYSTEM REQUIREMENTS

Central point (Base Station): Our proposed system is mainly focused on a central Base Station (BS) where all services are registered and policies are implemented. The Base station play a vital role in system performance and it shall increase system efficiency.

6.1 Hybrid Topology

Our system implementation need hybrid of both static and mobile entities with heterogeneous network supports. The central point has support of all the networks. All entities communicate to the base station. The base station provides all network support.

6.2 Two Types of Node

Our system implementation needs two types of nodes. The blind node which is only provides services and well defines nodes which provide and use the services.

6.3 Two Type of ID are Defined

If node is dull node (with very little memory & processing), the base station is responsible for all activities including key creation, distribution etc. While the well defines nodes can react in more intelligent way for key creation etc.

6.4 Two and More Networks

A base station would be providing the connecting point for different entities belongs to different networks.

7. THE BASE STATION: A CENTRAL POSITION

The base station is a central position where the entire all the devices are registered with their services with their privilege. The BS works like a central server where all devices first to authenticate itself and register their services. The BS also central place where all decisions are made and security policy are implemented. When new user came for appetites its need for a specific service, it makes a connection to the BS. The BS provides a list of services including network services. In figure-3, the overall structure with BS at the center and the basic architecture.

8. SYSTEM ARCHITECTURE

In the proposed architecture, all devices in pervasive environment should enroll with the BS and its services. The BS registers the devices and its services for controlling and accessing the registered services. The BS allows the requesting devices for using these registered services. In figure-4, all the static and incoming mobile entities registered its services with the BS.

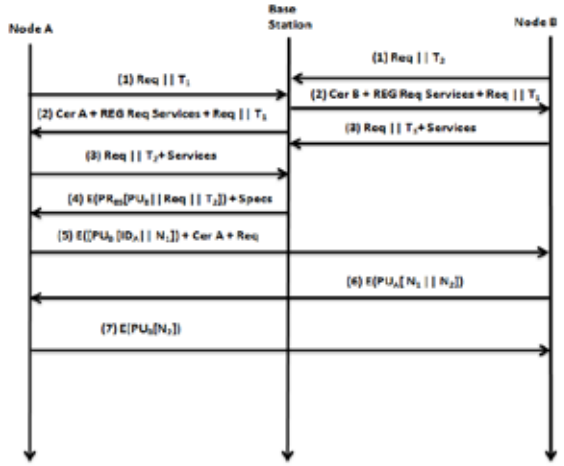

Fig. 4. The BS registered the devices with the services.

In figure.4, the node "A" required a specific service; it shall request the Base Station (BS). The node "A" wants a service for example it need a high resolution screen. The BS provides a list of devices having resolution screens. The BS fist search for the screen and on the basis of ontology's, a list of devices having this services are authenticated and registered with list of instructions. Node "A" sends a request in time T1 for the BS.

$$N_A \rightarrow BS\ (Time_1 \,||\, Request_A)$$

The Base Station receives the message and time of request and types of request. As the BS have already a list of services with list of privileges and the level of authenticity.

Now the BS provides the identity certificate for Node-A, and also ask for registration of services plus sending its original request to avoid main in meddle and replay attacks. The receiving of the original request, enable Node-A, to match this corresponding earlier request and to verify that the original request was not altered before reception by the BS. This guaranteed the message integrity and prevents replay attack.

$$BS \rightarrow N_A\ (Cer_A + RrgreqSer_S + Request_A \,||\, Time_1)$$

$$:: Cer_A = E\ (PR_{BS},\ [Time_{stamp} \,||\, ID_A \,||\, PU_A \,||\, Time_{expire}]\ )$$

A then pass this certificate to any other, who reads and verifies:

$$D(PU_{BS}, Cer_A) = D(PU_{BS}, E\ (PR_{BS}, [Time_{stamp} \,||\, ID_A \,||\, PU_A \,||\, Time_{expire}]))$$

$$= [Time_{stamp} \,||\, ID_A \,||\, PU_A \,||\, Time_{expire}\ ]$$

If Node-A have any service for which it welling to provide, they first enlist it with the BS before used other device service.

$$N_A \rightarrow BS\ (Services_N + Time_2 \,||\, Request_A)$$

In the same way other entities also register its services with BS. And from the same the BS also knows that which entity required which service and on which device that service available.

$$N_B \rightarrow BS\ (Time_2 \,||\, Request_B)$$

$$BS \rightarrow N_B\ (Cer_B + RrgreqSer_S + Request_B \,||\, Time_1)$$

$$N_B \rightarrow BS\ (Services_N + Time_3 \,||\, Request_B)$$

……

……

$$N_N \rightarrow BS\ (Time_n \,||\, Request_N)$$

$$BS \rightarrow N_N\ (Cer_N + RrgreqSer_S + Request_N \,||\, Time_n)$$

$$N_N \rightarrow BS\ (Services_{N+1} + Time_{n+1} \,||\, Request_N)$$

The BS knows about all the nodes with their services and also about their request for the specific services.BS is the only place where traffic been diverted from one place to another. After passing some initial important messages for basic trust, the BS leave the communication between the nodes and remain un-active for a while.

Now after these three initial important messages with the BS, the BS responding with

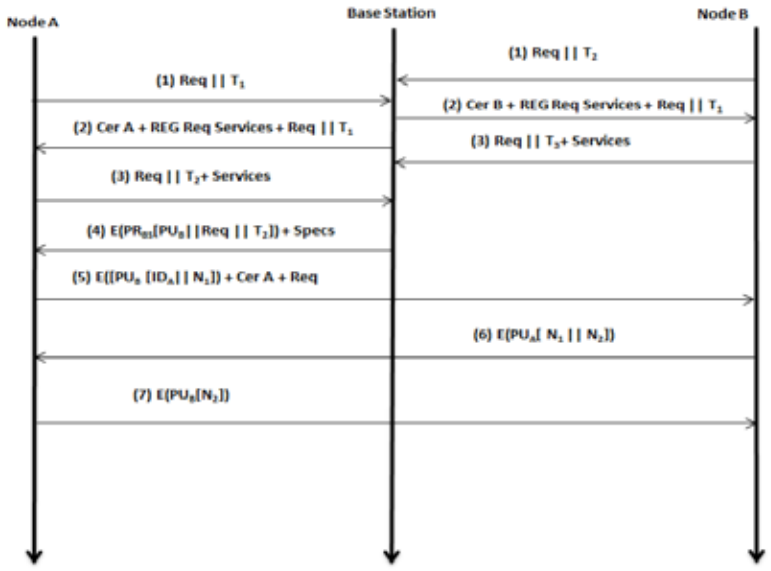

Fig. 5. Node-A and Node-B authenticate with BS.

properly identified entities for that specific service. Consider the Figure-5, in which two entities Node-A and Node-B develop a trust and they authenticate with BS.

After the third message from the Node-A to BS, the BS responds with the encrypted same request and public key of Node-B, having the requested service and also included the specifications of the same device. Specification tells the requested node about the system description including network and platform and also tells about the policy and access rights.

$$BS \rightarrow N_A\ (E\ (PR_{BS}[PU_B||Req\ ||\ T_2]) + Specs)$$

After receiving this message the Node-A, know the identity of the Node-B (PU_B), the type of request and all this encrypted by the "PR_{BS}" (Private Key) of BS. So the Node-A decrypt it with "PU_{BS}". Now Node-A knows about the other entities having the service and also clarify the access of these resources by the BS.

Now Node-A sends a message directly for Node-B, after analyzing and decrypting above message. The message moves directly from Node-A to Node-B without the involvement of BS.

$$N_A \rightarrow N_B\ \ (E([PU_B\ [ID_A||\ N_1]) + Cer\ A + Req)$$

Now Node-B receives the message having "Cer_A" (Certificate of Node-A), request type and "ID_A" identity of Node-A, who is requesting for a service and unique nonce "N_1". The ID_A and N_1 are encrypted by "PU_B" (Public Key of Node-B). Nonces are used to identify this transaction uniquely. The message prove many things including the identity of the requesting node and certificate of Node-A, which prove the authenticity of the node and can verify from BS. The nonce avoids the main in the middle (replay attack) attacks and encryption with public key ensures the encryption of the message. The Node-B can verify the identity or authenticity of the Node-A, from the BS.

Now the public keys have been securely delivered to both node "A" and "B". At this point, the identities (public keys) of both are delivered and verify and now able to start secure exchange. However, some additional steps are required. Now node "B" responds against the request of node "A" with nonce (N_1) and new generated nonce (N_2), encrypted with "A" public key (PU_A). Nonce (N_1) assured node "A" that response is come from "B".

Now the Node-B responds with a verify message for completing the trust level. The Node-B sends an encrypted message having nonce "N_1" and nonce "N_2" encrypted with the "PU_A" (Public key of Node-A).

$$N_B \rightarrow N_A\ \ (E(PU_A[\ N_1\ ||\ N_2]))$$

The encrypted message verifies the authentication as well as encryption and only the corresponding Node-A can decrypt it with its private key. The nonce verifies that messages are not replay or duplicate.

The Node-A, sends the last message before the actual use of the service on the Node-B. Now Node-A return nonce (N_2) to Node-B, is encrypted by public key of "B" (PU_B) which assured that corresponding is Node-A.

$$N_A \rightarrow N_B\ \ (E\ (PU_B\ [N_2]))$$

Hence, seven messages are required for complete understanding and conformation of messages between them. The initial four messages are used rarely because when these messages are received, the nodes save the public keys for future use and the technique is known as "caching".

9. SECURITY ANALYSIS

Here we discuss the proposed system with proper prevention of some most dedicated attacks. We analyze the scheme in the presence of some attacks.

9.1 Base Station has Overall Policy

The base station is a central point where all decisions are made and all policies are implemented. It is the base station which periodically checks the status of the resource. If resource is remains for more than one entity a specific time, the BS disconnected the session and updates the status of the device in device list (Dev_i). So if another device waiting for the same resource, it should be made available for them.

9.2 Unauthorized Access of Resources

The Base station implements the security policy and the whole network of different devices follow the same policy for using and offering the resources. Every notation and symbol has a proper meaning with proper resistance against some attack. The unauthorized accesses to resources are

prevented by valid certificates (Cer_N). Any one wants to use the resource; it first goes for BS to gain the attention for resources. The BS authenticates the requesting node by issuing proper certificate. Those nodes having no certificate should not authorize to use the resource. In the diagram 3, the first three messages between requesting node and base station ensure the identity for authorization.

9.3 No one can Spoof

Every entity has a pre-define certificate and a proper identity before it becomes a part of pervasive network. The identity and certificate of a node is properly defined and this mechanism prevents all type of spoofing attacks. The messages are communicated between entities in a proper mechanism and messages are communicate securely between base station and nodes. No one can pretend the identities of the other nodes for impersonation.

9.4 No Replay Attack

The time stamp and nonce in every request ensure time sequences and unique transaction for each message respectively. These parameters (time stamp &nonce) prevent any attempt for impersonation and base station know the time and sequence of the messages. The base station can discarded those messages which are sent by node again and again for illegal operation at the base station.

9.5 Main in the Middle Attacks (MiM)

The MiM attack is prevented by using IDs, nonce, certificate and keys related in each transaction. The encrypted messages create secure transactions of message between base station and nodes and between nodes to nodes.

10. SERVICES SELECTION ALGORITHM

The following algorithm determines available devices and also resolves the required service request if available.

Algorithm 1: Devices discovery and services selection: DevSelServSel (Dev_i, $Serv_i$, $Task_i$,Req_i,)

1: Base Station (BS) has a list of Devices (Dev_i) with list of Services ($Serv_i$).
Let BS: ={Dev_1 + $Serv_1$ * Dev_2 + $Serv_1$ + $Serv_2$* Dev_3 + $Serv_4$* Dev_5 + $Serv_5$* Dev_6 + $Serv_8$*……*Dev_n + $Serv_{n+1}$}

2: Sort List of Devices (Dev_i) according to time and frequent use (quality of device)
Let sorted List: = {Dev_2 + $Serv_1$ + $Serv_2$* Dev5 + $Serv_5$ *……}

3: Request(Req_i) for a specific Device (Dev_i) or Service ($Serv_i$)
Let the (Req_s):= {Dev_s + $Serv_{s1}$ + $Serv_{s2}$}

3: Checkforeach device in the List(Dev_i)
For i: =Dev_1 to Dev_nDo

4: Check for each service in the List ($Serv_i$)
For j: = ($Serv_1$) to ($Serv_n$)Do

5: Check if the request is fulfill for specific (Req_s)
IFfit {(Req_i)== (Dev_i)AND (Req_i)== ($Serv_i$)}
Else Go to Exit

5: Check the status of the device (Dev_i) and service ($Serv_i$)
IF {(Dev_i)|| ($Serv_i$)== Buzzy}
Else Go to Exit

6: Selection of specific (Dev_s) or ($Serv_s$)
Select {(Req_s) ➔ (Dev_s)AND (Req_s)➔($Serv_s$)}

7: Check the Authenticity of devices (Dev_s)
For each selected Device (Dev_s)and Service ($Serv_s$)
IF (Authentic == Successful)
Else Go to Exit

8: Check the Authorization
For each Device (Dev_i)and Service ($Serv_i$)
IF (Authorized == Successful)
Else Go to Exit

9: Connection Granted

10: End IF

11: End For

12: End IF

13: End For

14: End IF

15: End IF

16: End IF

11. THE DATA FLOW DIAGRAM

The algorithm clearly mentions the main step of the overall system. In following data flow diagram, the device makes a request for connection with the other nodes for services. The

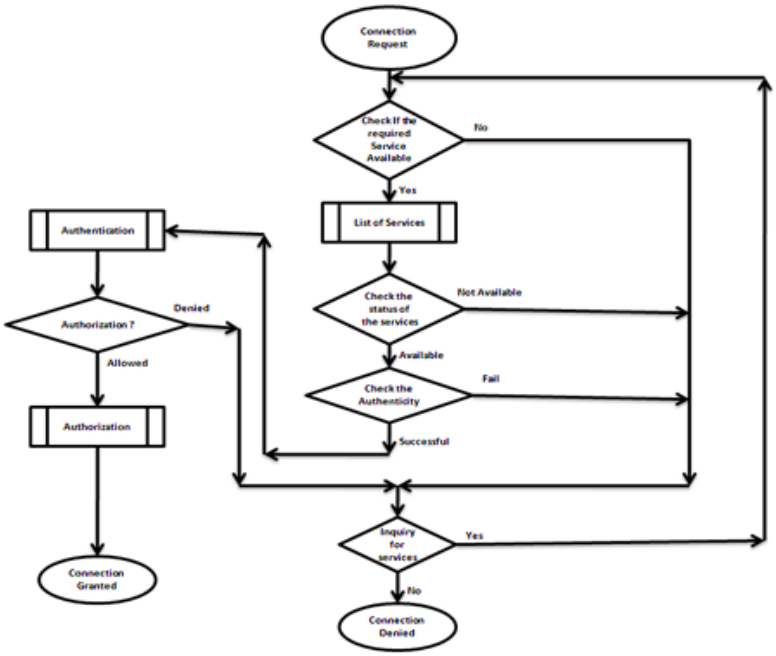

Fig. 6. The data flow diagram.

base station checks the policy implemented on the network entities, if satisfied, the connection is granted. In figure-6, show the overall flow of control from requesting phase to granted phase. When the requester is make a request for connection to the network for required services. The base station checks the required service in the service list with its specification and availability. If the service is not available then request is satisfied with no availability of service. And if the service is available, it is provided a list of that service. The list tells the requester about the number of availability and specifications. After that the status of the service is checked, either it busy or not and also its priority on user level. If the service is available then check it authenticity. And if the authenticity is satisfied then the algorithm checks the authorization of the requester for the specific request. If the authorization is also satisfied then the connection for a request is granted. Figure-6, show the overall data flow model.

12. ANALYSIS OF THE SYSTEM

For performance evaluation of our proposed novel light weight and centralized scheme, we use NS-2 (v-2.35) network simulator. The terrain is about (600 x 600) meters with randomly deployed nodes and a dedicated BS. The transmission range of the BS is 400 meters while other nodes have the range of 250 meters only. Some nodes are stationary while some in motion. The environment is check with different number of nodes from 5 to 50. Moving nodes move randomly in the same topological space with a speed 1,5,10,15,20,25 and 30 m/s with simulation pause time is fixed to 25 seconds. The network is established with IEEE 802.11 at data link and physical layers. The AODV (Ad-hoc On-Demand Distance Vector) protocol is implemented on network layer and with CBR (continuous bit rate) traffic over UDP link on transport layer. With 0.2 Mbps packets transmission rate, 512 bytes packet size and 200 seconds is simulation time with average transmission for flow is 2 bytes per second. The key size is 512 bit and the same model is used for uniformity in simulation. The cbrgen is used for constructing linking patterns while setdest is used for creating mobility model.

Average end to end delay: The time experienced by data packets when transmitted by CBR source for its corresponding CBR receiver. Average end to end delay includes all types of delay in the network like delay in buffering, acquisition delay and even processing delay at nodes. From simulation results in figure-7, indicates the end to end delay between two nodes without base station. End to end delay increases in the presence of a malicious node. When the malicious node working as in middle to gain the messages access and also gain authorization.

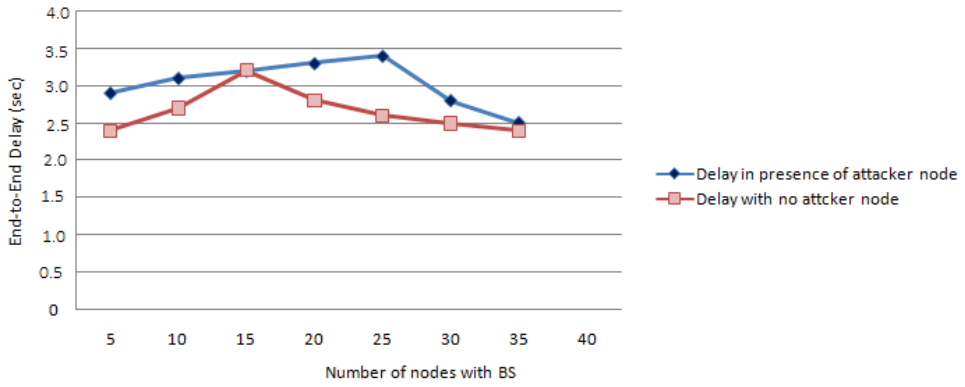

Fig. 7. End to end delay.

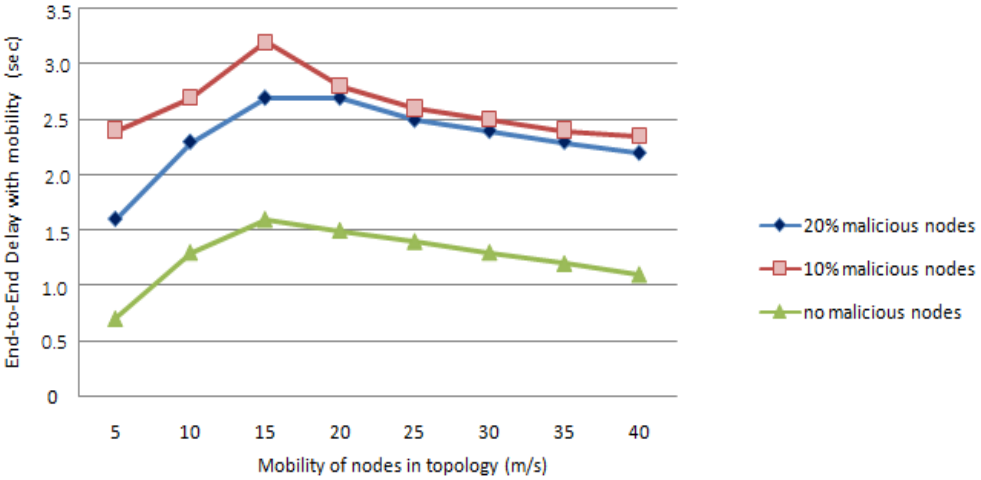

Fig. 8. End to end delay with mobility.

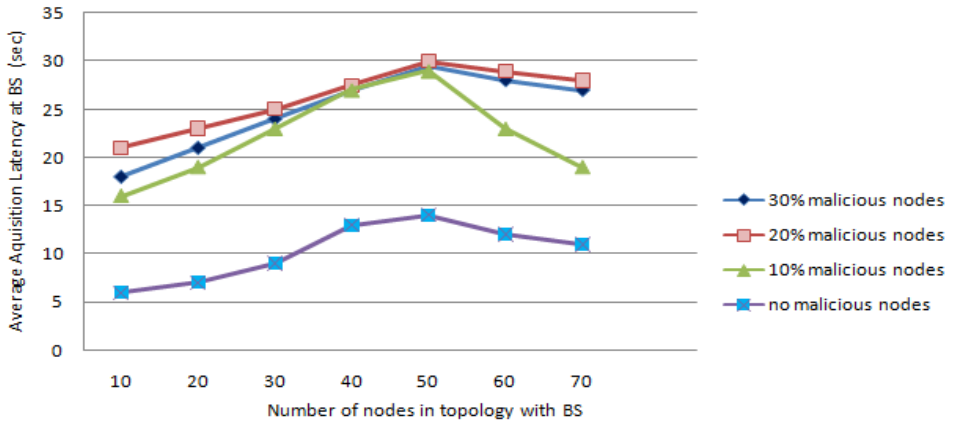

Fig. 9. The average acquisition delay.

Average end to end delay with mobile nodes: We also analyzed the proposed scheme with mobile and static nodes. The figure is, in fact, the results of the static nodes network analysis. In figure-8, the scheme managed mobile nodes with the presence of malicious nodes. The results clarify when the number of malicious nodes increase (15%), the system will experienced more delay as compared to the network having less amount (5%) of malicious nodes. The 5% malicious nodes have delay at certain level but the carve 10% and 15% of malicious nodes overlap each other, show that after a certain level the system performance degrade when the malicious nodes increases.

Average acquisition latency: This is the amount of time experienced when a node first request for a service, search it, content to base station and find the service on a specific device. In fact, it is the average delay experienced by node when request for a service available in the same premises. In figure-9, the average acquisition delay increases moderately when the number of malicious nodes increases. With increasing the number of genuine and malicious nodes, the base station feels more load for serving certificate and related security policies. But after certain level (30 nodes), carve tends to lower because the congestion feels at the base station been resolved.

13. CONCLUSION

Traditional computing have changed by real time and embedded devices by providing on the fly services. These embedded and smart devices have greater impact on our daily lives. Now services are available everywhere, every time and for everyone. These devices use its processing and communicating capabilities and conduct itself according to the user mood and circumstances. These devices communicate each other and construct an ad hoc and unstructured network of heterogonous devices. For better quality of services, these devices should communicate in such a way to provide services to end user in a uniform way. In such hybrid network, the authentications of all entities are important including devices, services, users and process. Here we proposed a novel lightweight, portable and centralized scheme based on symmetric security approach. The communicating entities ensure its authentication with the base station before the availability of a service. The base station is a central point with controlling the overall network entities. The base station checks the level of authenticity and specification of both devices before they make the connection. In our research we implement a security policy on ad hoc and hybrid network in NS-2. We analyzed our scheme in the presence of malicious nodes and we conclude that the system is properly worked and it is securing the resources from an attacker.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 309–317 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Evaluation of Losses and Life of Distribution Transformer under Non-linear Load using Wavelet Transform

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Abstract: In this paper, impact of harmonics on losses and life of distribution transformer is discussed. Wavelet Transform is used to analyze voltage and current signatures of distribution transformer operating under non-linear load. Power loss is calculated using wavelet based current and voltage profiles. Reduction in life time of the transformer has also been calculated which results due to harmonics in power system. Results show that power loss of a transformer increases under non-linear load as compared to its usage under linear loading, increase in power loss causes decay in transformer's operational life.

Keywords: Harmonics, wavelet transform, 3-phase distribution transformer, transformer loss, life of transformer

1. INTRODUCTION

In today's competitive market, power quality, to drive various linear and non-linear loads, is a major concern. Devices and relays in control systems are so sensitive that distorted signal at input leads to the inaccurate results. Harmonics are the major factor which causes distortion in wave form of the signal. Distribution transformer is highly influenced by harmonics. Distribution transformers losses in European Union were assessed to 33 TW.h/yr. Increase in the loss due to harmonics was found to be 15% [1]. Harmonics bring a no of challenges for a distribution transformer including increase in heating loss, insulation degradation, rise in temperature, decrease in efficiency, higher hot-spot temperature, and reduced life time [2-3].

Harmonics, derived by a word Acoustic, are basically the integral multiple frequency components of fundamental frequency. Harmonics are injected where linear behavior between voltage and current loses like power electronic switches

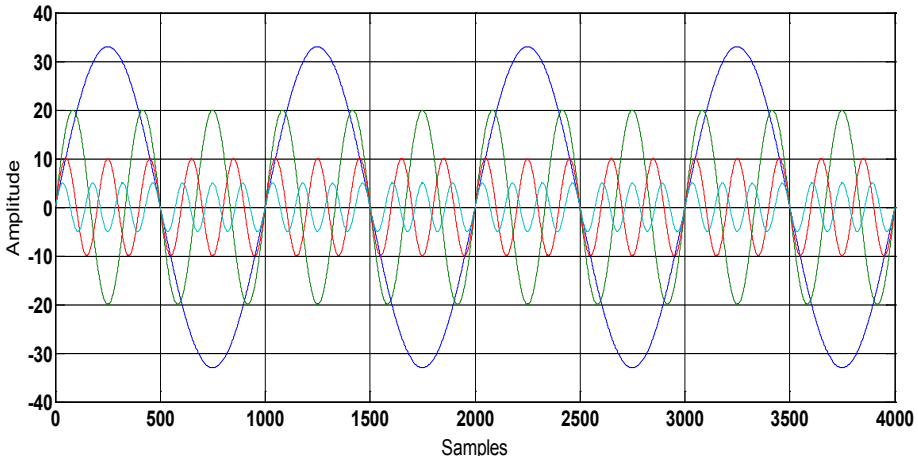

Fig. 1. Fundamental component and its multiple components.

Received, April 2016; Accepted, August 2016
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[4]. For some signal with 50 Hz as its fundamental frequency, 3rd, 5th, 7th, harmonic will be at 150 Hz, 250 Hz, and 350 Hz respectively. Fig. 1 shows the fundamental component and its various multiple frequency components which are commonly known as the distortion of a signal.

Quantitative measure of all the multiple frequencies present in a signal is known as Total Harmonic Distortion (THD) which is given as [4]:

$$THDv = \frac{\sqrt{\sum_{h=2}^{\infty} V_h^2}}{V_1} \quad (1)$$

THD Total harmonic distortion

V_h Voltage under harmonic

V_1 Fundamental component of voltage

A complete analysis of harmonics behavior in time domain as well as in frequency domain is very important. Time domain gives information of signal at every instant of time. In a similar fashion, analysis of signal in frequency domain gives the information of all frequencies present in it. Various techniques are used to examine the harmonics behavior in both time and frequency domain. Laplace transform, Fourier Transform, Short Time Fourier Transform and Wavelet Transform are used to analyze harmonics.

Root Mean Square (rms) values of voltage and current profiles with a time period "T" are given with the help of following mathematical expressions in analog form [21]:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} \quad (2)$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \quad (3)$$

Same can be written for digitized nature of both the signals [21]

$$V_{rms} = \sqrt{\frac{1}{2^N}\sum_{i=0}^{2^N-1} v_i^2} \quad (4)$$

$$I_{rms} = \sqrt{\frac{1}{2^N}\sum_{i=0}^{2^N-1} i_i^2} \quad (5)$$

Voltage harmonics are very harmful for distribution transformer just like current harmonics. Voltage harmonics causes winding insulation degradation however current harmonics are major cause of increase in power loss of transformer [5]. In distorted signal of a distribution transformer, voltage harmonics not only exist at some particular instants of the voltage signature rather they exist throughout the signal. In this paper, behavior of harmonics on voltage signal is studied at every instant of time. Further more, impact of harmonics on winding insulation deterioration and transformer losses is to be studied at every instant of time throughout the signal to check whether harmonics are more harmful near peaks of the signal or near zero crossings of current and voltage profiles. Fig. 2 shows the actual voltage signature of a distribution transformer where harmonics exists near peak value and Fig. 3 makes the idea clear that harmonics exists near zero crossings.

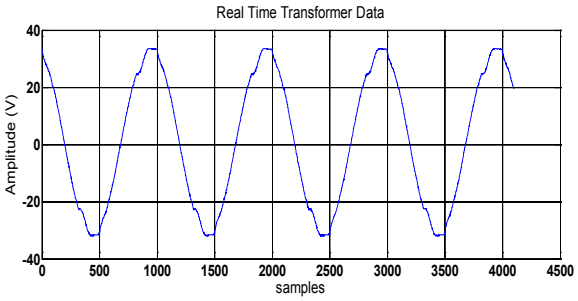

Fig. 2. Transformer's voltage profile.

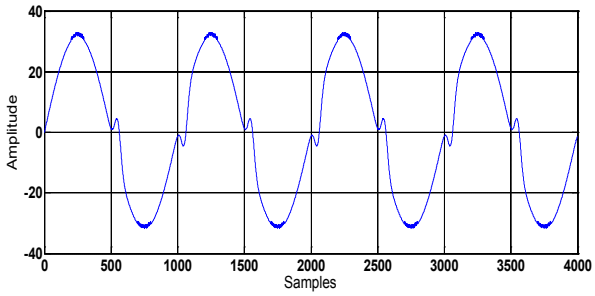

Fig. 3. Voltage harmonics near zero.

2. WAVELET TRANSFORM

Different transformation tools are used to deal with both time and frequency domains with various constraints. Fourier Transform gives information about all the frequencies present in a signal. It also aids to determine the order of frequencies and their magnitudes present. However it does not give any info of varying frequency in a signal. Fourier Transform of a signal with multiple frequencies and that of having continuously changing frequency is same. It does not distinguish between stationary and non-stationary signals and gives the same result for both however there is a difference in the behavior of such signals. Fourier Transform determines only the number of frequencies present in non-stationary signal but does not give any information

where (at which instant of time) frequencies exist [6].

Fig. 4 and Fig. 5 show a stationary signal with different frequencies in it and its Fourier transform which shows different frequencies at different points on frequency scale. Fig. 6 & 7 show a signal with varying frequency and its Fourier transform respectively. Fig. 5 and Fig. 7 make the idea clear that Fourier transform of both Stationary and Non-Stationary signal is same. Moreover, it can be used only in one domain in one time. Fourier Transform ignores positive and negative peaks, it only gives information of frequencies present in a signal. On the other hand, Short Time Fourier Transform (STFT) is capable to work in both domains but it works like Heisenberg Uncertainty Principle which states that speed and position of any atomic particle cannot be measured accurately at a same time [6]. In case of electronic signals, it is not possible to simultaneously know the frequency of a signal and the time instant where it exists and vice versa. Detailed analysis of signal is impossible in both time and frequency domains simultaneously using STFT. If signal is studied in frequency domain, its time resolution becomes inaccurate and vice versa. Resolution problem occurs because of fixed window height in STFT. However, it is capable of distinguishing between stationary and non-stationary signals unlike Fourier transform. Fig. 8 shows STFT of Non-Stationary signal and it is obvious that STFT is capable of working in both time and frequency domain simultaneously. Fig. 8 shows STFT of a signal which gives detailed information of frequency domain but poor resolution for time domain. In Fig. 9, with the variation in window size, time domain resolution becomes better but frequency is not accurate because it show frequency spectrum on frequency axis. In Fig. 10, frequency resolution is much accurate and time domain resolution is very poor.

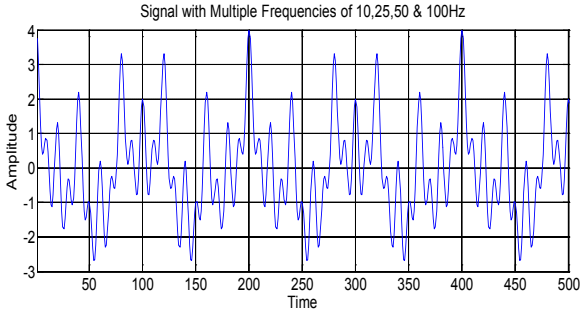

Fig. 4. Stationary signal with multiple frequencies.

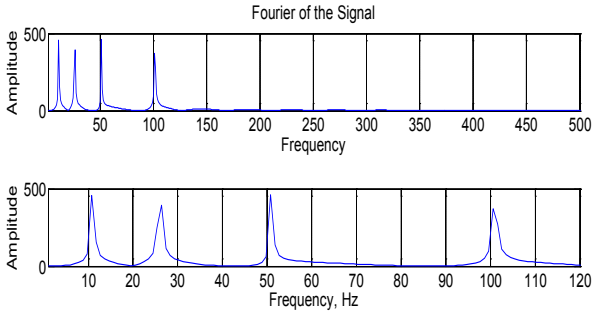

Fig. 5. Fourier of stationary signal.

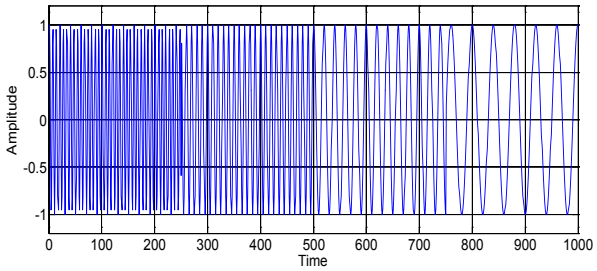

Fig. 6. Non-stationary signal.

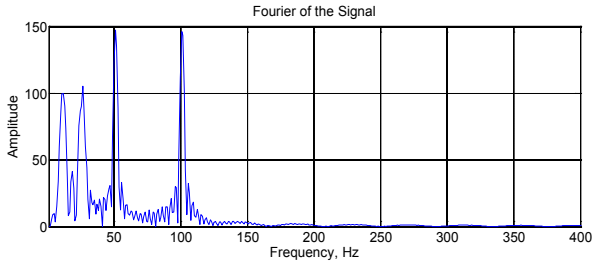

Fig. 7. Fourier of non-stationary signal.

Wavelet transform is most suitable technique to analyze harmonics both in time as well as frequency domains simultaneously. It does not show resolution problem in either time or frequency domain unlike STFT [6]. In Wavelet Transform, correlation of signal is found with the help of Mother Wavelet. Wavelet Transform has a series of mother wavelet functions like Haar, Meyer, DMeyer, Shannon, Mexican Hat, Daubechies, Symlets, Coiflets, and Gaussian etc. Unlike STFT, Wavelet transform is taken at every slice of the signal in time domain. It gives information about the behavior of a signal for both positive and negative peaks. Mathematically, wavelet transform is represented as follows [6]:

$$W.T(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \,.\, \Psi^* \left(\frac{t-\tau}{s}\right) dt \qquad (6)$$

s Scale

τ Translation

WT Wavelet transform

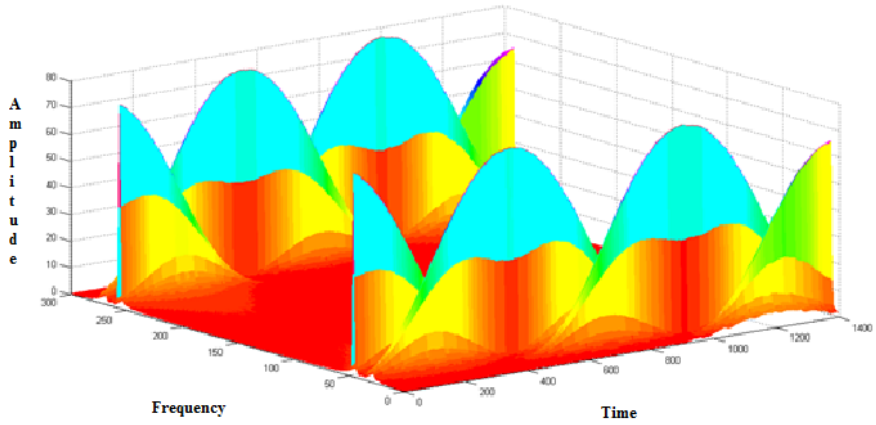

Fig. 8. STFT of a signal.

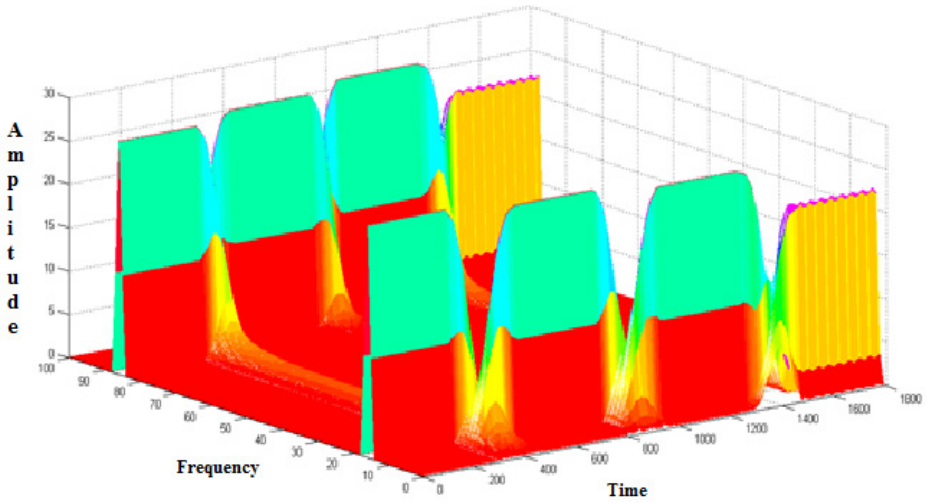

Fig. 9. STFT of same signal with new window size 1.

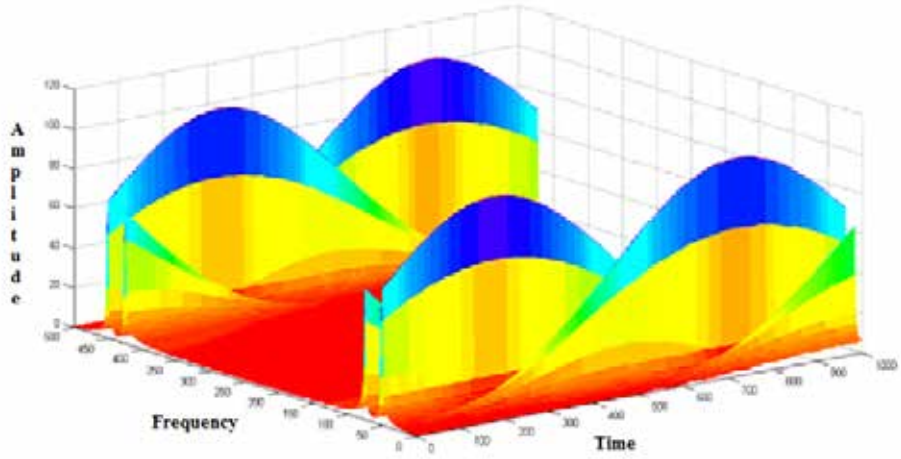

Fig. 10. STFT of same signal with new window size 2.

Equation (6) shows a function x(t) whose wavelet is to be taken. Ψ(t)is a mother wavelet which has translated versions of a signal delayed or advanced by "τ" and scaled by "s". Fig. 11 shows variable window size pattern in Wavelet Transform. Variable window size is actually the translated and scaled versions of mother wavelet which is to be multiplied with original signal. Translation is basically the location of mother wavelet and scale is 1/f. in order to compute the wavelet of a signal, translated and scaled versions of mother wavelet are multiplied at every segment of signal in time domain and also with every spectral component in frequency domain [6].

Fig. 11 gives the idea of variable window size in Wavelet Transform. Each box in Fig. 11 has same area with different lengths and widths and each box represents one coefficient of Wavelet Transform. Region "A" has small length along "Scale axis" and relatively wider along "Translation". This region deals with low frequencies. Region "B" deals with the signal at lower scale as compared to "A". Region "C" gives the detailed information of higher frequency components because higher frequencies lie at lower scale in Wavelet transform. In fact, this variable scale is responsible for complete behavior of higher frequencies in a signal or harmonics of that signal. Analysis of every frequency in frequency spectrum can be performed in wavelet environment due to different values of scale. Wavelet transform is also capable to analyze a signal in time domain because mother wavelet has time shifted versions at various instants in time domain. It can be observed in Fig.11.

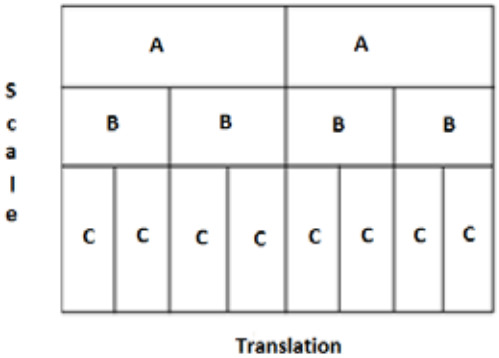

Fig . 11. Variable window size.

Mathematically, Wavelet of digitized values of current and voltage in equations (4) and (5) can be expressed in the following manner [21]:

$$V_{rms} = \sqrt{\frac{1}{2^N}\sum_{i=0}^{2^{N-j-1}}\sum_{k=0}^{2^j-1}(d_{j,k}^{*j})^2} \quad (7)$$

$$I_{rms} = \sqrt{\frac{1}{2^N}\sum_{i=0}^{2^{N-j-1}}\sum_{k=0}^{2^j-1}(d_{j,k}^{j})^2} \quad (8)$$

Where $d_{j,k}$ and $d^*_{j,k}$ are coefficients of Wavelet transform at i^{th} node, j^{th} level and k^{th} sample.

3. POWER LOSS IN DISTRIBUTION TRANSFORMER

Distribution Transformer's losses can be divided into two types as No-load Loss and Load Loss.

3.1 No-load Loss

No-load Loss is also studied as core loss of distribution transformer which is result of core excitation i.e. such type of loss is due to the hysteresis and eddy currents which are highly dependent upon frequency and flux density. Due to their high dependency on frequency, they are influenced by harmonics. No-load loss can be determined by performing Open Circuit Test on distribution transformer.

3.2 Load Loss

Load loss is the resultant of different losses like Cu- loss, winding eddy current loss and other stray loss. Other stray loss is due to the structural parts of distribution transformer like clamps, tanks, etc. Winding eddy current loss and other stray loss are lumped together and known as Total Stray Loss [7].

$$P_T = P_{C.L} + P_L \quad (9)$$

$$P_L = P_{ohmic} + P_{T.S.L} \quad (10)$$

$$P_{T.S.L} = P_{E.C} + P_{O.S.L} \quad (11)$$

P_T Total power loss

P_{CL} Core loss

P_L Load loss

P_{TSL} Total stray loss

P_{OSL} Other stray loss

P_{EC} Eddy current loss for winding

All the above mentioned losses are dependent upon harmonics according to the following mathematical expressions [8]:

$$P_{ohmic} = \sum_{h=1}^{h_{max}} I_h^2 * R \quad (12)$$

$$P_{E.C} = P_{E.C.R} * \sum_{h=1}^{h_{max}} [\frac{I_h}{I}]^2 \, h^2 \quad (13)$$

$$P_{O.S.L} = P_{O.S.L.R} * \sum_{h=1}^{h_{max}} [\frac{I_h}{I}]^2 \, h^{0.8} \quad (14)$$

I_h Current under Harmonic

I Fundamental current component

4. LIFE ESTIMATION

Rated losses of a distribution transformer are increased when it is used to work under non-linear load. This rise in loss results an increase in transformer's temperature, hotspot temperature and ultimately decrial in life time of the machine. Equations (15-17) show temperature of top oil of a transformer and hotspot temperature [9, 10]. Equation (18) shows life of a transformer working under non-linear load in per unit [10,11]. Equation (19) represents the actual life of a transformer [9, 12].

$$\Delta T = T * (\frac{P_{L.H}}{P_L})^{0.8} \quad {}^0\text{C} \quad (15)$$

$$\Delta T_{T.O} = T_{T.O} * (\frac{P_{L.H} + P_{C.L}}{P_L + P_{C.L}})^{0.8} \quad {}^0\text{C} \quad (16)$$

$$T_{H.S} = T_A + \Delta T + \Delta T_{T.O} \quad {}^0\text{C} \quad (17)$$

$$L = 9.8 * 10^{-18} * \exp\left(\frac{1.5*10^3}{273+T_{H.S}}\right) p.u \quad (18)$$

$$Remaining\ Life = L * Original\ Life \quad (19)$$

ΔT_{TO} Rise of Top-Oil Temperature over Ambient temperature

T_{TO} Top oil rise in temperature under rated conditions

ΔT Under Non-linear load, rise of Hot spot temperature over Top Oil temperature

T_{HS} Hot Spot temperature

Ta Ambient temperature

L(pu) Life in per unit

5. TENTATIVE ARRANGEMENT

Data was gathered from Awais Textile Industries, situated at Abdullahpur, Faisalabad, Pakistan, which consists of printing and dyeing units. Printing section of the industry comprises of various induction motors of different rating which are used to open the fabric rolls and passing it through printing area. Most of these motors are fed through VFDs furthermore a large number of CFLs are used for lighting purpose. All of the mentioned equipment is being supplied by a 200 kVA distribution transformer. Due to non-linear load attached with the transformer, it run under the effect of harmonics. Data was collected in the form of voltage and current of the said transformer and it was processed for its analysis in MATLAB using Wavelet Transform.

A Digital Oscilloscope was used for observing the voltage profile at the terminals of transformer which has a provision of plugging in external storage device like USB for data storage. A Clamp on meter was used to measure both current and voltage to verify results, in order to, minimize the chance of error. Wave form was observed at screen of the oscilloscope and then it was stored in CSV (Comma Separated Values) format directly on USB. The procedure was repeated for several times, in order to, minimize the probability of any procedural mistake while taking data.

6. RESULTS AND DISCUSSION

Real time voltage and current profiles of 200 kVA distribution transformer working under non-linear load were taken from the above mentioned experimental set up. Real time voltage profile is shown in Fig. 2. A closer look of Fig. 2 makes it clear that real time voltage signature is distorted signal due to harmonics. Harmonics have significant impact near both the peaks of voltage signature. Mexican hat is used as Mother Wavelet because it is very symmetrical function in nature. Wavelet transform of voltage profile is depicted in Fig. 12, it shows that maximum distortion occurs up to scale of 54. Harmonics are the higher frequency components so they exist at lower scale. As the scale gets closer to zero, distortion level increases significantly. Maximum correlation between input signal and mother wavelet is shown from the brightest point at scale 250. Fig. 13 depicts the same result in 3D view. Pink and yellow humps represents the positive and negative peaks whereas at lower sale just like as depicted in Fig. 12. Translation axis shows the behavior of signal at various instants of time while scale axis illustrates various frequency components in it. Fig. 12 also makes it clear that there is resolution problem neither in time domain nor in frequency

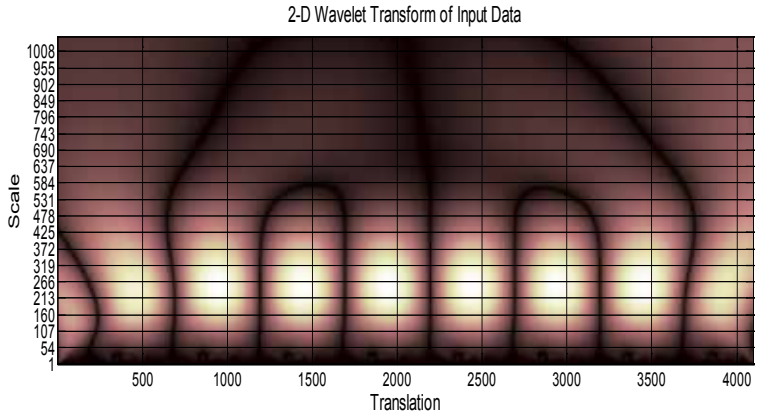

Fig. 12. Wavelet of input signal.

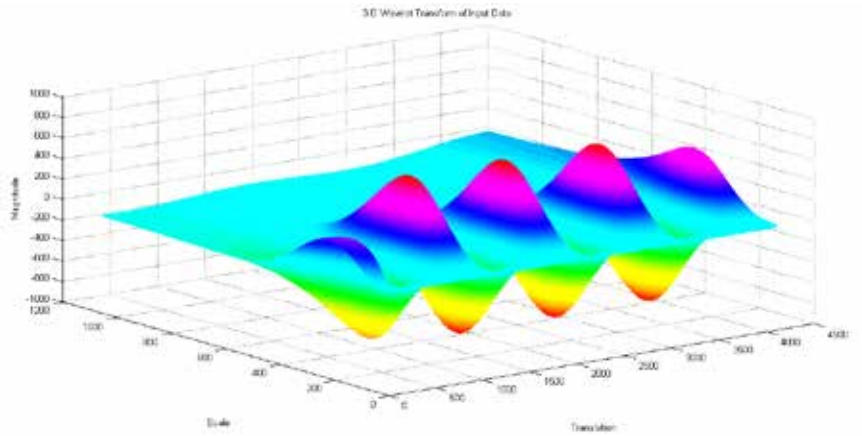

Fig. 13. 3D Wavelet transform of input data.

domain using wavelet. Harmonics which exist in the voltage signature of distribution transformer are shown in Fig. 14. It shows higher frequency component throughout the whole signal which corresponds to scale 50. In the similar fashion, Fig. 15 depicts the impact of another harmonic which exists at scale 36. Same is the case for Fig. 16 but on comparing these figures, it is obvious that frequency increases as scale becomes smaller which causes distortion level to increase. Distortion level in Fig. 16 is very high as compared to that of Fig. 14.

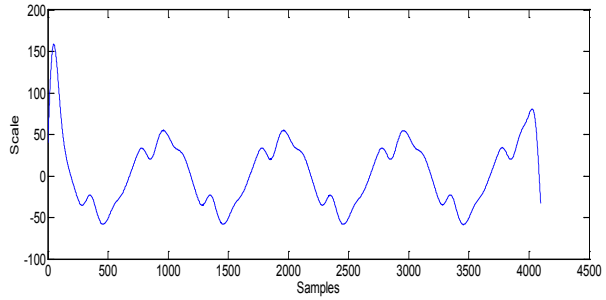

Fig. 14. Harmonics on scale 50.

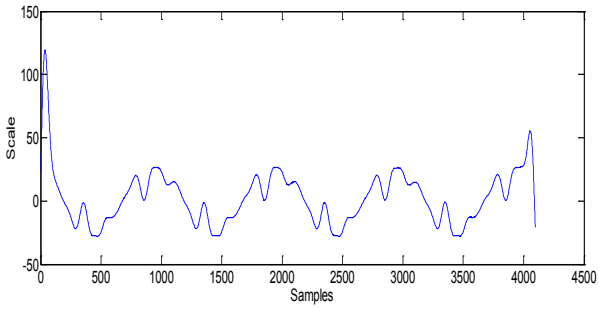

Fig. 15. Harmonics on scale 36.

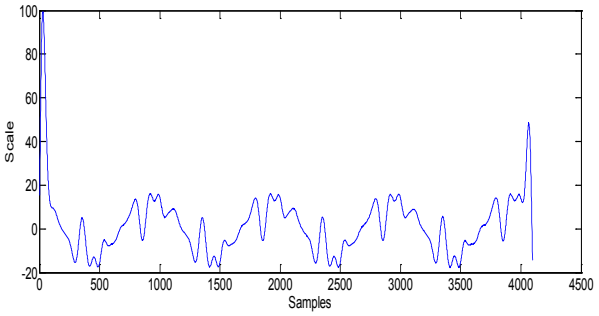

Fig. 16. Harmonics on scale 28.

By using the data of voltage and current profiles along with wavelet transform coefficients as in equations (7) and (8), power can be calculated with the help of equation (20) [13, 21].

$$P = \frac{1}{T}\int_0^T i_t v_t \cong \frac{1}{2^N}\sum_{i=0}^{2^{N-j}-1}\sum_{k=0}^{2^j-1} d_{j,k}^{i} d_{j,k}^{*i} \quad (20)$$

This is the power at i^{th} node, j level and k sample. By using equation (20), total power loss for three phase distributed transformer working under non-linear load is *3,714 W*. Rated power loss of transformer was *3000 W* and percentage rise in power loss due to non-linear load is 23.8%. Along with increase in power loss, winding temperature and hot spot temperature of transformer also increases. Power loss is directly related with temperature. Rise in hotspot temperature also causes to weaken winding insulation overall and particularly at that point where the temperature of winding is maximum. Hotspot temperature of transformer under non-linear load rises and equation (17) shows that temperature rose upto 116.47 ^{0}C. Life of transformer is also reduced due to harmonics and increased power loss, equation (18) shows useful life of distribution transformer in per unit which is 0.8 *p.u* in this case. Effective life of distribution transformer working under rated conditions is 50 years, provided by manufacturer (Transformer's data is given in appendix-I). Equation (19) shows the remaining life of transformer if it is being operated under the said conditions which is just 43 years and it is reduced to 14% compared to its normal life. With the increase in THD, life gets reduced beyond 14%.

Appendix-I. Distribution transformer's specifications.

Parameters	Ratings
Manufacturer	PEL
Distribution Transformer, kVA	200
H.T Voltage, kV	11
L.T Voltage, kV	0.4
H.T Current, A	10.5
L.T Current, A	266.7
Primary Side Resistance, Ω	14.75
Secondary Side Resistance, Ω	0.0062
Primary Side Leakage Inductance, H	0.003
Secondary Side Leakage Inductance, mH	0.067
Core Loss Resistance (Rc), kΩ	728
Magnetization Inductance, H	32105
Cooling Method	ONAN
No. of Phases	3
Insulation Class	A
Frequency, Hz	50
Ambient Temperature, ^{0}C	35
Winding Temperature, ^{0}C	65
Vector Group	Dy_n11
No-load Loss, W	500
Load Loss, W	3000
Life	50 Yrs

In order to get rid of harmonics, filters are used which filter out the unwanted signals from required fundamental component. But these filters are normally used for low voltage level and for small ratings. These filters can't be used in power system of high voltage level which requires specially designed active devices for filters for HVDC. However SHE (Specific Harmonic Elimination) method is used normally for industrial purpose which injects reverse harmonics to nullify the effect of unwanted signals. ABB, Siemens, G.E etc provides such specially designed filters for industries based upon SHE technique.

7. CONCLUSIONS

In this paper, transformer losses and reduction in its life, working in harmonics environment, are focused. Tool used for analysis purpose is Wavelet Transformation because it does not compromise between time and frequency domain. Calculations show that hotspot temperature of transformer increased to 116.47 $^\circ$C under non-linear load. Increase in transformer losses is 23.8% compared to its rated loss. Increase in power loss and hotspot temperature of transformer causes its winding insulation to deteriorate and its effective life reduces. Working of transformer under non-linear load resulted 14% reduction in its life time. Harmonics impact can be reduced by changing design considerations of winding insulation and hot spot temperature may be focused to use a transformer to its maximum life in future.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 319–324 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

An Evaluation of Approximated PWM Switching Schemes Instigating Acoustic Noise in Inverter-Fed Induction Motors

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Abstract: This research paper presents the acoustic noise produced by a three phase motor when driven by an inverter. The inverter is operated by various traditional approximated PWM switching schemes. Experiments show that the acoustic noise from the three phase motor changes with the variation of the PWM switching scheme. The PWM switching schemes included approximated sinusoidal, approximated third harmonic injected sinusoidal, approximated trapezoidal, and approximated triangular PWM switching schemes. The approximated trapezoidal PWM switching scheme yields the least acoustic noise. This paper highlights a relationship (in graphical terms) between the current in the neutral wire and the acoustic noise observed from the motor for various switching schemes. However, the relationship between the acoustic noise and switching scheme may require further research. Furthermore, results also show that the approximated trapezoidal PWM switching scheme yields the minimum current in the neutral wire.

Keywords: AC motor drive, PWM switching schemes, machinery acoustic noise control, neutral wire current

1. INTRODUCTION

AC induction motors are generally considered as the widely used electric equipment in the industry. The accumulative acoustic noise radiated from ac induction motors has a large impact on the environmental noise pollution. Nevertheless, a low-cost and efficient method for reducing the acoustic noise from ac motors still remains a quest. Remarkably, many research papers have shown various reasons originating the acoustic noise in induction motors [1-7]. In this perspective, analog electronic filters have been proposed to reduce the acoustic noise. Such analog filters tend to remove the harmonics that create the acoustic noise [8-9]. On the other hand, significant techniques considering a variety of PWM switching schemes along with vast discussion on operating frequencies have been established to reduce the acoustic noise from ac induction motors [10-15].

This research paper will present a comparison of acoustic noise as recorded when the ac induction motor is operated by four different approximated PWM switching schemes [22]. The four different approximated PWM switching schemes have been derived from traditional PWM switching scheme [16]. The advantage of using the approximated PWM switching schemes over the traditional PWM switching schemes is the reduction in switching losses. In addition, a comparison of the neutral wire current versus the acoustic noise level for the four different approximated PWM switching schemes, is also presented and a correlation is developed.

2. EXPERIMENTAL SETUP

As shown in fig. 1, the experimental setup consists of a DC power supply, V_s, two series connected electrolyte capacitors of equal value i.e., C_1 and

Received, June 2016; Accepted, August 2016
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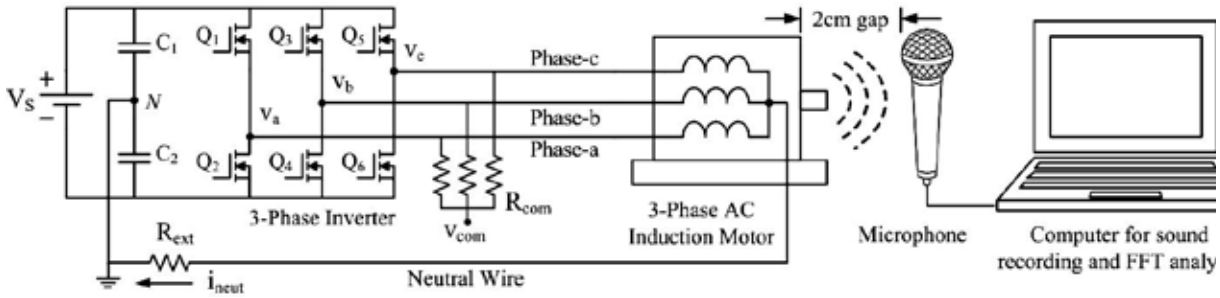


Fig. 1. The experimental setup where the DC voltage source, three-phase inverter, ac induction motor, acoustic noise recording microphone, and a computer for analysis, are shown.

C_2. Series connected electrolyte capacitors are used for reducing the voltage ripple. The common point of the two capacitors i.e., point *N* is the ground terminal. The DC voltage is fed to the three-phase inverter. The three-phase inverter consists of six MOSFETs i.e., Q_1 to Q_6. The line to ground voltages produced by the inverter, as measured from the ground terminal *N*, are represented as v_a, v_b, and v_c. Whereas the line to line voltages are represented as v_{ab}, v_{bc}, and v_{ca}. The common mode voltage found as described in [17-21], can be expressed as,

$$v_{com} = \frac{v_a + v_b + v_c}{3} \quad (1)$$

The three-phase windings of the motor are connected in Y-format. The neutral point of the motor three phase windings is connected to the ground terminal via a small resistor $R_{ext}(=1\Omega)$. The current flowing in the neutral wire is labeled as i_{neut}.

The inverter MOSFETs are controlled by a microcontroller. The microcontroller is programmed to switch the inverter for four different approximated pulse width modulation (PWM) schemes. The approximated PWM schemes include [22]:

a) 3rd Harmonic Injected Sinusoidal PWM.
b) Sinusoidal PWM
c) Trapezoidal PWM.
d) Triangular PWM.

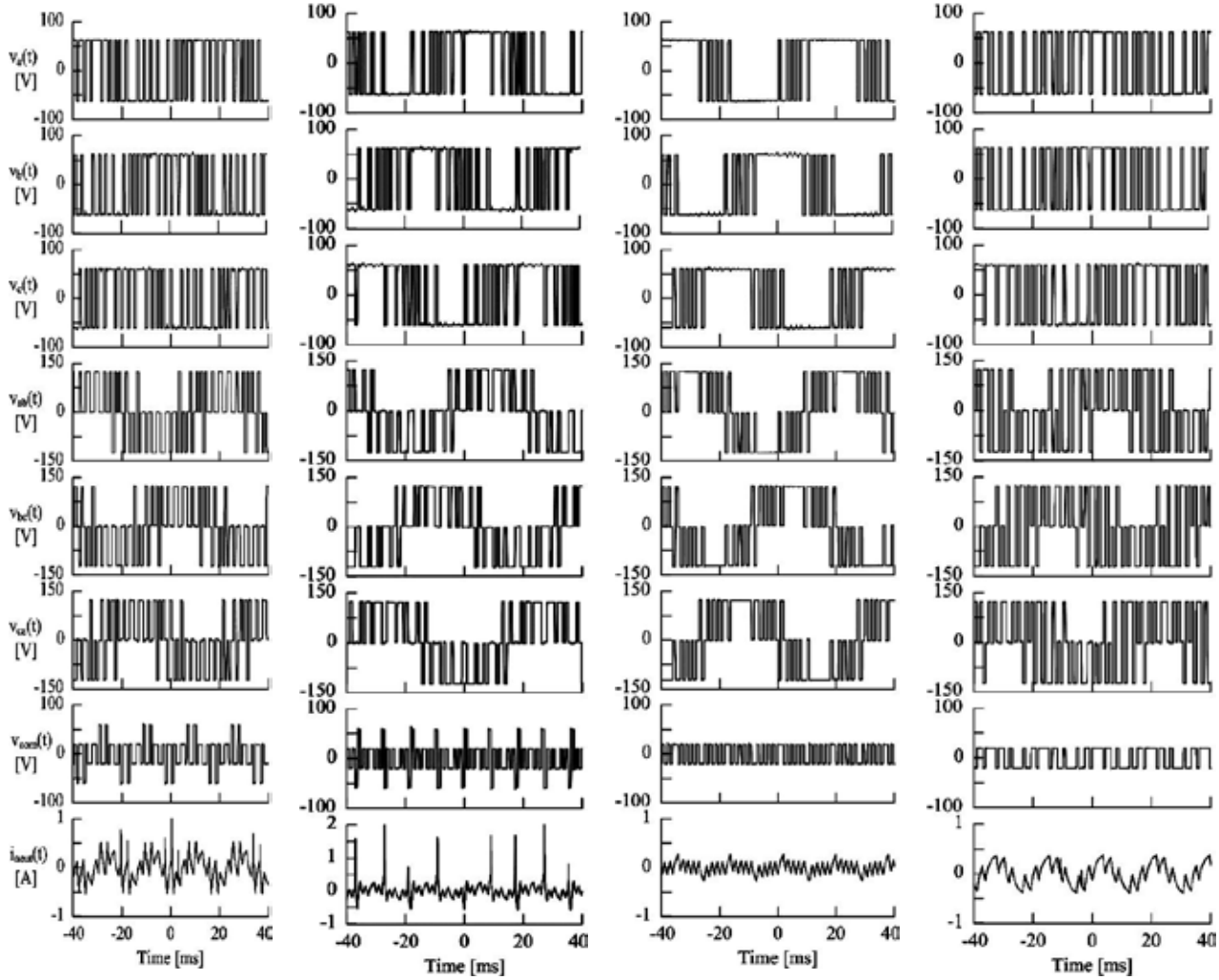
For all the above mentioned switching schemes, the microcontroller is supplied with the same carrier frequency of 1.2 kHz. A microphone, to record the acoustic noise, is placed at a distance of 2cm from the motor. The acoustic noise from the motor is recorded and stored in a computer software. The recorded acoustic noise data can be analyzed for determining the Fast Fourier Transforms (FFT). The acoustic noise is recorded while the motor is operated under no load condition.

3. EXPERIMENTAL RESULTS

The experimental results are divided into two parts. The first part presents the real-time waveforms of voltages and current for the inverter-motor systems when driven by the various approximated PWM methods. The waveforms of the voltages include line to ground voltage v_a, v_b, and v_c, the line to line voltages i.e., v_{ab}, v_{bc}, and v_{ca}, and the common mode voltage v_{com}. Whereas the waveform of the current includes the neutral wire current i_{neut}. A comparison of the common mode voltage and the neutral wire current produced from the approximated PWM switching schemes will aid in deciding the best choice for selecting the switching scheme.

The second part presents the real-time acoustic noise as recorded from the inverter-motor system while being operated under the various approximated PWM switching schemes. FFT of the acoustic noise will also be presented. A comparison of all the acoustic noise data will aid in choosing the best approximated scheme for yielding least acoustic noise.

Fig. 2(a) to (c) presents voltage waveform of the line to ground, line to line, common mode, and the waveform of the current in the neutral wire when the motor switched under the approximate 3rd harmonic injected sinusoidal PWM scheme, approximate sinusoidal PWM scheme, approximate trapezoidal PWM scheme, approximate triangular PWM scheme, respectively.



(a) (b) (c) (d)

Fig. 2 Voltage and current waveforms of the induction motor when operated under the following switching schemes, i.e., (a) 3rd Harmonic Injected Sinusoidal PWM. (b) Sinusoidal PWM. (c) Trapezoidal PWM. (d) Triangular PWM.

Figs. 3 to Fig. 6 presents experimentally obtained the acoustic noise and the FFT of the acoustic noise signals for approximate 3rd harmonic injected sinusoidal PWM scheme, approximate sinusoidal PWM scheme, approximate trapezoidal PWM scheme, and approximate triangular PWM scheme, respectively. Since the acoustic noise signal is captured by a microphone. The output of the microphone is a voltage signal that is in accordance with the acoustic noise signal. Therefore, it is to be noted the acoustic noise signals, in Figs. 3(a), 4(a), 5(a), and 6(a), are expressed in volts units. Table 1 organizes the recorded acoustic noise peak amplitude (measured in terms of volt [V]) for various approximated PWM schemes. It is seen that the Triangular PWM scheme produced the least, and 3rd Harmonic Injected Sinusoidal PWM produces the maximum acoustic noise peak amplitude.

Table 1. Peak acoustic noise for various approx. PWM schemes.

Approx. PWM Switching Scheme	**Peak Amplitude of Acoustic Noise measured in terms of volts[V]**
3rd Harmonic Inject	0.166
Sine	0.150
Trapezoidal	0.106
Triangular	0.146

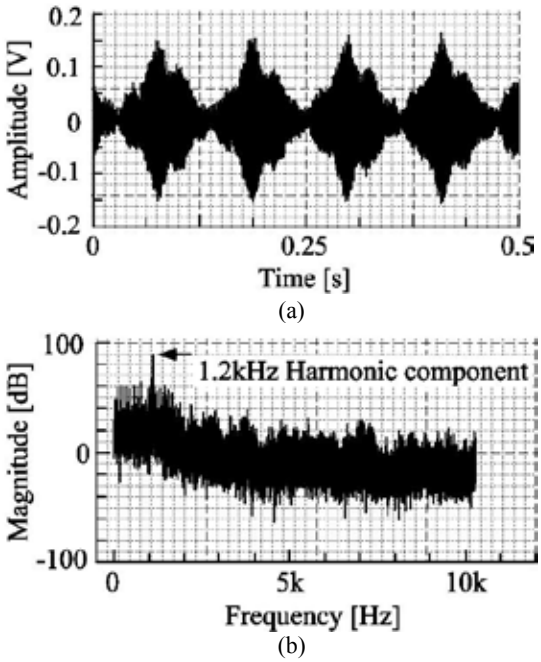


Fig. 3 Response of the induction motor switched under the 3rd Harmonic Injected Sinusoidal PWM scheme. (a) The accoustic noise signal. (b) The FFT of the accoustic noise signal.

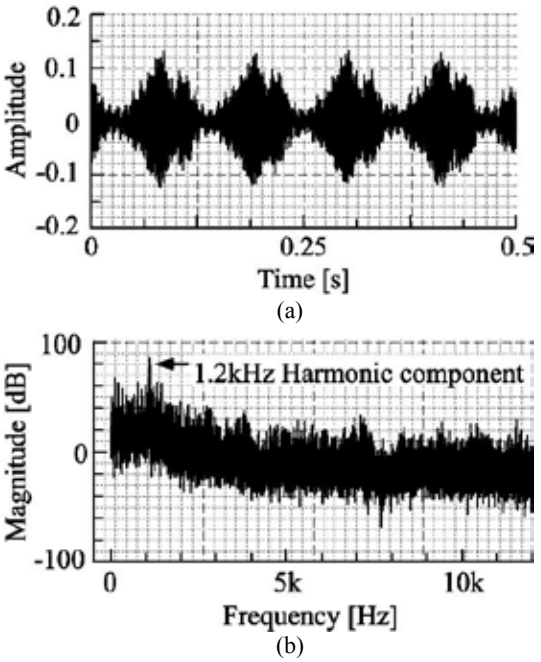


Fig. 4 Response of the induction motor switched under the Sinusoidal PWM scheme. (a) The accoustic noise signal. (b) The FFT of the accoustic noise signal.

4. ANALYSIS AND DISCUSSION

Fig. 2 shows that both the common mode voltage and the neutral wire current change with the change in the switching scheme. This is in accordance to the results demonstrated in [22]. The FFT of all the approximated switching schemes, as shown in Fig. 3, show that different acoustic noise magnitude levels (expressed in dB) exist for different switching schemes. However, a 1.2 kHz harmonic component in the acoustic noise FFT response is the most prominent harmonic component found in all four switching schemes. As discussed earlier, all the switching schemes are run with the aid of the microcontroller with a carrier signal frequency of 1.2 kHz. The existence of the harmonic component having a frequency equal to that of the carrier frequency is in accordance to [3, 5, 24]. This presence of the 1.2 kHz harmonic component indicates that a notch filter can be connected between the inverter and motor to prevent the 1.2 kHz harmonic component from entering the motor. Thereby, the acoustic noise of the motor can be reduced.

Also, a comparison of the four approximated PWM schemes as shown in Fig. 2 to the corresponding acoustic noise signals as shown in Fig. 3 to 6, illustrates that as the line to ground voltage, line to line voltage, or common mode voltage changes the acoustic noise generated from the motor. Table 2 presents the RMS common mode voltage, RMS neutral current, and RMS of Acoustic Noise, for the four approximated PWM switching schemes. Again, it is seen that the Triangular PWM scheme produces the least RMS neutral current, and RMS of Acoustic Noise. Whereas 3rd Harmonic Injected Sinusoidal PWM produced the maximum RMS neutral current and RMS of Acoustic Noise.

Table 2. RMS Values of various parameters for the four approximated switching schemes.

Approx. PWM Switching Scheme	RMS Common mode Voltage[V]	RMS Neutral Current[A]	RMS of Acoustic Noise measured in terms of volts[V]
3rd Harmonic Inject	31.77	0.238	0.053
Sine	31.59	0.214	0.045
Trapezoidal	19.63	0.103	0.031
Triangular	19.67	0.207	0.045

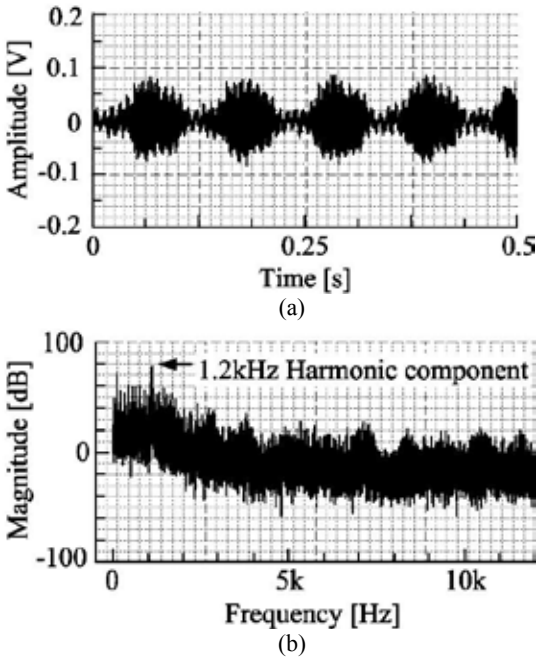


Fig. 5 Response of the induction motor switched under the Trapezoidal PWM scheme. (a) The accoustic noise signal. (b) The FFT of the accoustic noise signal.

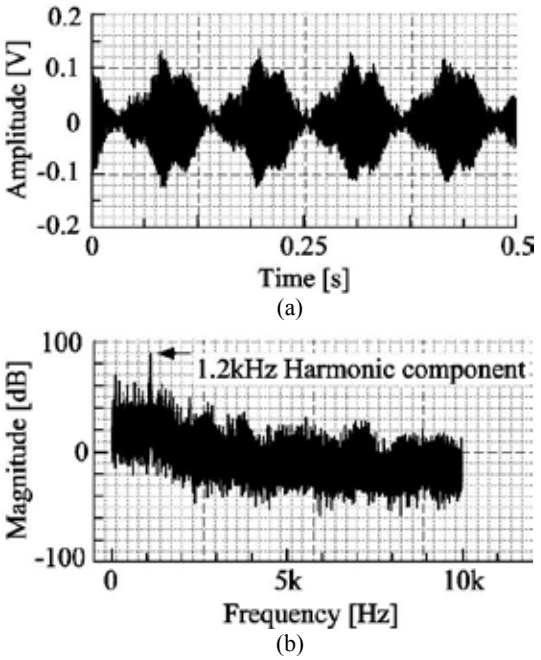


Fig. 6 Response of the induction motor switched under the Triangular PWM scheme. (a) The accoustic noise signal. (b) The FFT of the accoustic noise signal.

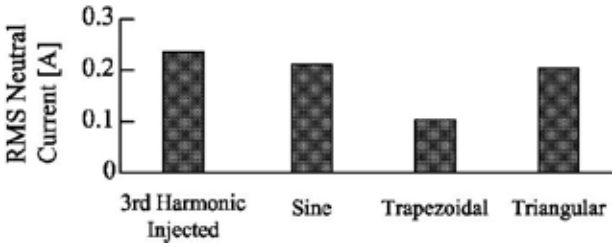


Fig. 7. Comparison of the neutral wire current as produced against the various switching schemes.

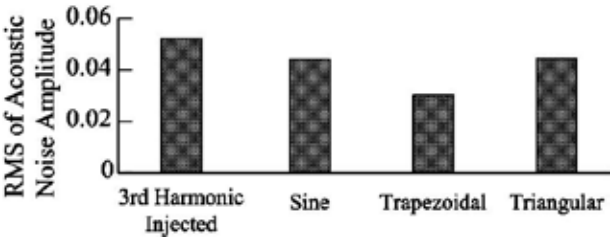


Fig. 8. Comparison of the acoustic noise produced against the various switching schemes.

Fig. 7 demonstrates in graphical arrangement, how the RMS of neutral wire current changes with the change in switching scheme. Similarly, Fig. 8 presents the change of RMS values of acoustic noise amplitude with the change in switching scheme. Comparing Fig. 7 and 8, it is interesting to note that the pattern of change of RMS neutral wire current with respect to the switching scheme is similar to the pattern of change of RMS of acoustic noise amplitude. In other words, among the four switching schemes, the 3rd harmonic injected sinusoidal PWM scheme produces the maximum neutral wire current and maximum acoustic noise, second in position is the sinusoidal PWM scheme, third is position is the triangular PWM switching scheme, whereas the least neutral wire current and the least acoustic noise is produced by the trapezoidal PWM scheme. However, the research can be extended to explain the relationship between the neutral wire current and the acoustic noise. At this stage the best choice to the least acoustic noise is to operate the motor from the trapezoidal PWM switching scheme.

5. CONCLUSIONS

This research has shown the acoustic noise characteristics for motor driven by the four approximated PWM switching schemes including 3rd harmonic injected sinusoidal PWM, sinusoidal PWM, trapezoidal PWM, and Triangular PWM. Result show that for the 3rd harmonic injected sinusoidal PWM the current in the neutral wire and the acoustic noise generated from the motor is maximum whereas for the trapezoidal PWM

switching scheme the neutral wire and the acoustic noise generated from the motor is minimum. In addition, high switching frequency (above 15 kHz) is a very effective practice but levies high stress on semiconductor switching devices and amplify switching frequency losses.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (3): 325–337 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

On Further Study of CA-AG-groupoids

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Abstract: An AG-groupoid that satisfies the identity a(bc) = c(ab) is called a CA-AG-groupoid [1]. In this article various properties of CA-AG-groupoids are explored and their relations with various other known subclasses of AG-groupoids and with some other algebraic structures are established. We proved that in CA-AG-groupoid left alternativity implies right alternativity and vice versa. We also proved that a CA-AG-groupoid having a right cancellative element is a T^1, a T^3 and an alternative AG-groupoid. We provided a partial solution to an open problem of right cancellative element of an AG-groupoid. Further, we proved that a CA-AG-groupoid having left identity is a commutative semigroup and investigated that the direct product of any two CA-AG-groupoids is again cyclic associative. Moreover, we investigated relation among CA, AG* and Stein AG-groupoids.

Keywords: AG-groupoid, CA-AG-groupoid, bi-commutative, Stein AG-groupoids, direct product

1. INTRODUCTION

A groupoid satisfying the "left invertive law" is called an Abel-Grassmann's groupoid (or simply AG-groupoid [2]). In literature different names like "left almost semigroup" (LA-semigroup) [3], left invertive groupoid [4] and right modular groupoid [5] has been used by different authors for the said structure. Many properties of AG-groupoids have been studied in [6, 7]. Various aspects of AG-groupoids have been studied in $[2, 8-14]$. In [1,15], we introduced CA-AG-groupoid as a new subclass of AG-groupoid and studies some fundamental properties of it. In the same paper we introduced CA-test for the verification of cyclic associativity for an arbitrary AG-groupoid. We also enumerated CA-AG-groupoids up to order 6 and further classified it into different subclasses.

2. PRELIMINARIES

A groupoid $(G,\cdot)$ or simply G satisfying the "left invertive law [3]: $(ab)c = (cb)a \ \forall a,b,c \in G$" is called an Abel-Grassmann's groupoid (or simply AG-groupoid [2]). Through out the article we will denote an AG-groupoid simply by S otherwise stated else. S always satisfies the "medial law: $(ab)(cd) = (ac)(bd)$ [16, Lemma $1.1(i)$], while S with left identity e is called an AG-monoid and it always satisfies the paramedial law: $(ab)(cd) = (db)(ca)$ [16, Lemma $1.2(ii)$]". A groupoid G is called right AG-groupoid or right almost semigroup (RA-semigroup) [3] if $\forall a,b,c \in G$, $a(bc) = c(ba)$. An AG-groupoid S is called:

i. cyclic associative AG-groupoid (CA-AG-groupoid) [15]; if $a(bc) = c(ab) \ \forall a,b,c \in S$.

ii. AG* [8]; if $(ab)c = b(ac)$.

iii. AG** [9]; if $a(bc) = b(ac)$.

iv. T^1-AG-groupoid [10] if $\forall a,b,c,d \in S$, $ab = cd$ implies $ba = dc$.

Received, November 2015; Accepted, July 2016
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vi. left T^3-AG-groupoid (T_l^3-AG-groupoid) if $\forall a,b,c \in S$, $ab = ac$ implies $ba = ca$.

vii. right T^3-AG-groupoid (T_r^3-AG-groupoid) if $ba = ca$ implies $ab = ac$.

viii. T^3-AG-groupoid [10] if it is T_l^3 as well as T_r^3.

ix. transitively commutative if $ab = ba$ and $bc = cb$ implies $ac = ca$ $\forall a,b,c \in S$.

x. Bol*-AG-groupoid [17] if it satisfies the identity $a(bc \cdot d) = (ab \cdot c)d$ $\forall a,b,c,d \in S$.

xi. left alternative if $\forall a,b \in S$, $(aa)b = a(ab)$ and is called right alternative if $b(aa) = (ba)a$. S is called alternative [10], if it is both left alternative and right alternative.

xii. An element $a \in S$ is left cancellative (resp. right cancellative) [17] if $\forall w,y \in S$, $aw = ax \Rightarrow w = x$ $(wa = xa \Rightarrow w = x)$.

xiii. An element $a \in S$ is cancellative if it is both left and right cancellative. S is left cancellative (resp. right cancellative, cancellative) if every element of S is left cancellative (right cancellative, cancellative).

xiv. left commutative (resp. right commutative) if $\forall a,b,c \in S$, $(ab)c = (ba)c$ $(a(bc) = a(cb))$. S is called bi-commutative AG-groupoid [18], if it is left and right commutative.

xv. Stein-AG-groupoid [18], if $a(bc) = (bc)a$ $\forall a,b,c \in S$.

xvi. An element $a \in S$ is called idempotent if $a^2 = a$ and an AG-groupoid having each element as idempotent, is called AG-2-band (or simply AG-band) [12].

xvii. A groupoid in which $(ab)c = a(bc)$, $\forall a,b,c \in S$ holds is called a semigroup. If a semigroup contains the identity element e such that $ea = a = ae$, then it is called monoid.

Due to non-associativity of AG-groupoid, left identity does not imply right identity and so the identity.

For two AG-groupoids S_1 and S_2, the set $\{(a,b)|a \in S_1, b \in S_2\}$ with the "binary operation defined by (a_1,b_1) $(a_2,b_2) = (a_1a_2, b_1b_2)$ is called the direct product of S_1 and S_2, denoted by $S_1 \times S_2$", in this case we say that S_1 and S_2 are the direct factors of $S_1 \times S_2$.

3. VARIOUS PROPERTIES OF CA-AG-GROUPOIDS

In the following, it is observed that the subclass of CA-AG-groupoid is distinct from that of T^1 and T^3-AG-groupoids. We provide a counter example to show that a CA-AG-groupoid is not a T^1-AG-groupoid, however, a CA-AG-groupoid with a right cancellative element is (i) T^1-AG-groupoid and (ii) T^3-AG-groupoid.

Example 1. Table 1 represents a CA-AG-groupoid of order 4. As $4 \cdot 3 = 2 = 3 \cdot 3$ but $3 \cdot 4 \neq 3 \cdot 3$, thus it is not a T^1-AG-groupoid.

Table 1. CA-AG-groupoid that is not T^1.

·	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	2	1
4	1	1	2	2

However, we have the following;

Theorem 1. *Every CA-AG-groupoid having a right cancellative element is a T^1-AG-groupoid.*

Proof. Let S be a CA-AG-groupoid having a right cancellative element z and $a, b, c, d \in S$. Let $ab = cd$, then by cyclic associativity, left invertive law and right cancellativity, we have;

$$\begin{aligned} z^2(ba) &= a(z^2b) = a(zz \cdot b) = a(bz \cdot z) = z(a \cdot bz) \\ &= z(z \cdot ab) = z(z \cdot cd) = z(d \cdot zc) = z(c \cdot dz) \\ &= (dz)(zc) = c(dz \cdot z) = c(z^2d) = d(cz^2) = z^2(dc) \end{aligned}$$

$$\Rightarrow z^2(ba) = z^2(dc) \Rightarrow (ba \cdot z)z = (dc \cdot z)z$$

$$\Rightarrow\ ba \cdot z = dc \cdot z \Rightarrow ba = dc.$$

Hence, S is a T^1-AG-groupoid.

Since a T^1-AG-groupoid is (i) a T^3-AG-groupoid [10], and (ii) an AG**-groupoid [11]. Thus, we immediately have the following corollary.

Corollary 1. Every cancellative CA-AG-groupoid or simply having a right cancellative element is

(i) a T^3-AG-groupoid.

(ii) an AG**-groupoid.

Lemma 1. *Every left cancellative CA-AG-groupoid S is transitively commutative.*

Proof. Let S be a left cancellative CA-AG-groupoid and $a, b, c \in S$ such that $ab = ba$ and $bc = cb$. We have to show that $ac = ca$. Using cyclic associativity and the assumption, we have $b(ac) = c(ba) = c(ab) = b(ca) \Rightarrow b(ac) = b(ca)$, which by left cancellativity imply $ac = ca$. Hence S is transitively commutative.

Now, we discuss an open problem given in [17] and provide a partial solution to that open problem. To this end, we first restate the following [17, Theorem 26].

Theorem 2. *"Every right cancellative element of an AG-groupoid S is (left) cancellative."*

The converse of the above theorem is not true in general. In 2012, M. Shah proposed an open Problem in his Ph.D thesis [17]: "Prove or disprove that in an AG-groupoid, without left identity, every left cancellative element is right cancellative". In [17], the open problem have been partially resolved by the proposer himself, that is: (a) "An AG-groupoid, a left cancellative element is right cancellative, if either S is cancellative or if S has left identity [17, Theorem 28]", (b) "In an AG-groupoid, a left cancellative element x is right cancellative if any of the following holds: (i) If x is idempotent (ii) If x^2 is left cancellative (iii) If there exists a left nuclear left cancellative element in S". The converse of the problem has also been proved for AG*-groupoid, AG**-groupoid and self-dual AG-groupoid i.e. (i) "every left cancellative element of an AG*-groupoid is right cancellative [17] (ii) every left cancellative element of an AG**-groupoid is right cancellative [17] (iii) every left cancellative element of self-dual AG-groupoid is right cancellative [19]". We claim that the converse of Theorem 2 also holds for CA-AG-groupoids and verify the claim in the following theorem.

Theorem 3. *Every left cancellative element of a CA-AG-groupoid is right cancellative.*

Proof. Let a be a left cancellative element of a CA-AG-groupoid S. To show that a is right cancellative, let $xa = ya$ for all $x, y \in S$. Then, by cyclic associativity, medial law and assumption, we have

$$\begin{aligned} a(a \cdot ax) &= (ax)(aa) = (aa)(xa) = (aa)(ya) \\ &= (ay)(aa) = a(ay \cdot a) = a(a \cdot ay) \end{aligned}$$

$$\Rightarrow a(a \cdot ax) = a(a \cdot ay).$$

This by repeated use of the left cancellativity of a implies that $x = y$. Hence a is right cancellative.

Next we prove that any cancellative element of a CA-AG-groupoid can be written as the product of its two cancellative elements.

Theorem 4. *Every cancellative element of a CA-AG-groupoid can be written as the product of its two cancellative elements.*

Proof. Let a be an arbitrary cancellative element of a CA-AG-groupoid S. Suppose $a = c_1c_2$, where c_1 and c_2 are any arbitrary elements of S. We have to show that c_1 and c_2 are cancellative. Consider $xc_1 = yc_1$, then by cyclic associativity we have

$$xa = x(c_1c_2) = c_2(xc_1) = c_2(yc_1) = c_1(c_2y) = y(c_1c_2) = ya \Rightarrow xa = ya.$$

Which by the right cancellativity of a implies $x = y$. Thus c_1 is right cancellative and hence cancellative by Theorem 2. Now let $c_2x = c_2y$. Then

$$xa = x(c_1c_2) = c_2(xc_1) = c_1(c_2x) = c_1(c_2y) = y(c_1c_2) = ya$$

this by the right cancellativity of a implies that $x = y$. Thus c_2 is left cancellative and thus cancellative by Theorem 3. Hence the result follows.

Example 2. Table 2 represent a CA-AG-groupoid having 1 and 3 as cancellative elements, while 2 as non-cancellative element. 1 and 3 are the product to two cancellative elements.

Table 2. CA-AG-groupoid with two cancellative elements.

·	1	2	3
1	1	2	3
2	2	2	2
3	3	2	1

Theorem 5. *Let k be a fixed element of a CA-AG-groupoid S such that $ak = ka$ and $bk = kb$ for some a, b in S. If k is left or right cancellative then a, b commute.*

Proof. First assume that k is left cancellative. Then, using cyclic associativity and given condition,

$$k(ab) = b(ka) = a(bk) = a(kb) = b(ak) = k(ba)$$

which by left cancellativity of k implies that $ab = ba$.

Now, let k is right cancellative, then by Theorem 2, k is left cancellative and hence the result follows.

Theorem 6. *Every CA-AG-groupoid is paramedial* [1].

Next attention is paid towards alternative AG-groupoids. The following example shows that left alternative and right alternative are distinct subclasses of AG-groupoids.

Example 3. Left alternative AG-groupoid of order 4 given in Table 3 is not right alternative because, $a(bb) \neq (ab)b$.

Table 3. Left alternative AG-groupoid that is not right alternative.

·	a	b	c	d
a	c	c	d	d
b	d	b	d	d
c	d	d	d	d
d	d	d	d	d

The right alternative AG-groupoid of order 3 represented in Table 4 is not a left alternative AG-groupoid since $(1 \cdot 1)2 \neq 1(1 \cdot 2)$.

Table 4. Right alternative AG-groupoid that is not left alternative.

·	1	2	3
1	3	2	3
2	1	3	3
3	3	3	3

However, if an AG-groupoid is cyclic associative then left alternativity implies right alternativity and vice versa, as proved in the next theorem.

Theorem 7. *Let S be a CA-AG-groupoid, then S is left alternative if and only if S is right alternative.*

Proof. Assume first that S is a left alternative CA-AG-groupoid, then for any $a, b \in S$

$$b \cdot aa = a \cdot ba = a \cdot ab = aa \cdot b = ba \cdot a$$
$$\Rightarrow b \cdot aa = ba \cdot a.$$

Conversely, assume that S is right alternative, then

$$aa \cdot b = ba \cdot a = b \cdot aa = a \cdot ba = a \cdot ab$$
$$\Rightarrow aa \cdot b = a \cdot ab.$$

Hence the theorem is proved.

In the following example it is shown that the class of CA-AG-groupoid is distinct from the class of alternative AG-groupoids.

Example 4. CA-AG-groupoid of order 4, presented in Table 5, is neither a left alternative nor a right alternative because $(4 \cdot 4)4 \neq 4(4 \cdot 4)$.

Table 5. CA-AG-groupoid that is not alternative AG-groupoid.

·	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	2	3

However, if a CA-AG-groupoid contains element either as a left or as a right cancellative then, it becomes an alternative AG-groupoid, as established in the following result.

Theorem 8. *A CA-AG-groupoid with a left cancellative element is an alternative AG-groupoid.*

Proof. Let S be a CA-AG-groupoid having a left cancellative (and hence a cancellative) element x and $a, b \in S$. Then by cyclic associativity and left invertive law:

$$x(aa \cdot b) = b(x \cdot aa) = b(a \cdot xa) = (xa)(ba)$$
$$= a(xa \cdot b) = a(ba \cdot x) = x(a \cdot ba) = x(a \cdot ab),$$

which by left cancellativity of x implies $(aa)b = a(ab)$. Thus S is left alternative AG-groupoid. By virtue of Theorem 7, S is also right alternative. Hence S is alternative.

The following example suggests that neither every cancellative AG-groupoid nor every alternative AG-groupoid is CA.

Example 5. Table 6, represents a cancellative AG-groupoid of order 3. As $3(2 \cdot 1) \neq 1(3 \cdot 2)$, hence it is not cyclic associative.

Table 6. Cancellative AG-groupoid that is not CA-AG-groupoid.

·	1	2	3
1	1	2	3
2	3	1	2
3	2	3	1

Table 7 represents an alternative AG-groupoid of order 4, which is not cyclic associative since $a(ba) \neq a(ab)$.

Table 7. Alternative AG-groupoid that is not CA-AG-groupoid.

·	a	b	c	d
a	c	c	c	b
b	d	c	c	c
c	c	c	c	c
d	c	a	c	c

Now, we demonstrate that the class of CA-AG-groupoid is distinct from the class of Stein AG-groupoid. To begin with, consider the following:

Example 6. CA-AG-groupoid of order 4, represented in Table 8, is not a Stein AG-groupoid as: $(1 \cdot 1)1 \neq 1(1 \cdot 1)$. While a Stein AG-groupoid of order 4, presented in Table 9, is not a CA-AG-groupoid since $1(1 \cdot 2) \neq 2(1 \cdot 1)$.

Table 8. CA-AG-groupoid that is not Stein AG-groupoid.

·	1	2	3	4
1	2	3	3	3
2	4	3	3	3
3	3	3	3	3
4	3	3	3	3

Table 9. Stein AG-groupoid that is not CA-AG-groupoid.

·	1	2	3	4	5
1	3	3	4	5	5
2	4	4	5	5	5
3	4	5	5	5	5
4	5	5	5	5	5
5	5	5	5	5	5

Further, the following example establish that neither every AG*-groupoid is Stein, nor every Stein AG-groupoid is AG*.

Example 7. Table 10, represents an AG*-groupoid of order 6. As $1(1 \cdot 2) \neq (1 \cdot 2)1$, hence it is not a Stein AG-groupoid. A Stein AG-groupoid of order 5 given in Table 9 of Example 6 is not an AG*-groupoid as $(1 \cdot 1)2 \neq 1(1 \cdot 2)$.

Table 10. AG*-groupoid that is not Stein AG-groupoid.

·	1	2	3	4	5	6
1	3	4	5	5	5	5
2	3	4	6	6	5	5
3	5	5	5	5	5	5
4	6	6	5	5	5	5
5	5	5	5	5	5	5
6	5	5	5	5	5	5

However, by coupling any two from CA, Stein and AG*-groupoids, we get the third one. As proved in the following;

Theorem 9. *Let S be an AG-groupoid then, any two of the following implies the third one.*

(i) *S is CA.*

(ii) *S is AG*.*

(iii) *S is Stein.*

Proof. Let S be an AG-groupoid and $a, b, c \in S$.

(i) and (ii) implies (iii): Using the properties of cyclic associativity, definition of AG* and the left invertive law, $a(bc) = c(ab) = b(ca) = (cb)a = (ab)c = b(ac) = c(ba) = (bc)a$. Hence S is a Stein AG-groupoid.

(ii) and (iii) implies (i): Using the properties of Stein AG-groupoid, the left invertive law and AG*, $a(bc) = (bc)a = (ac)b = c(ab)$. Hence S is a CA-AG-groupoid.

(iii) and (i) implies (ii): Using the definition of Stein AG-groupoid, the left invertive law and the cyclic associativity we have, $(ab)c = (cb)a = a(cb) = b(ac)$. Hence S is an AG*-groupoid.

Next we provide some counter examples to verify that (i) a Stein AG-groupoid is neither a left commutative nor a right commutative, and (ii) a bi-commutative AG-groupoid is not a Stein AG-groupoid.

Example 8. Table 9 of Example 6 represents a Stein AG-groupoid of order 5. As $(1 \cdot 2)1 \neq (2 \cdot 1)1$, hence it is not left commutative. Also, as $1(1 \cdot 2) \neq 1(2 \cdot 1)$, hence it is also not a right commutative. While Table 11, represents a bi-commutative AG-groupoid of size 3, that is not a Stein AG-groupoid as, $a(aa) \neq (aa)a$.

Table 11. Bi-commutative AG-groupoid that is not Stein AG-groupoid.

·	a	b	c
a	b	b	b
b	c	c	c
c	c	c	c

However, we have the following;

Theorem 10. *A Stein AG-groupoid S is CA, if any of the following hold.*

(i) *S is left commutative.*

(ii) *S is right commutative.*

(iii) *S is bi-commutative.*

Proof. (i) Let S be a left commutative Stein AG-groupoid and $a, b, c \in S$. Then $a(bc) = (bc)a = (cb)a = (ab)c = c(ab)$. Hence S is CA-AG-groupoid.

(ii) Let S be a right commutative Stein AG-groupoid and $a, b, c \in S$. Then $a(bc) = a(cb) = (cb)a = (ab)c = c(ab)$. Hence S is CA-AG-groupoid.

(iii) Obvious.

Lemma 2. *Every Stein CA-AG-groupoid is a semigroup.*

Proof. Let S be a Stein CA-AG-groupoid and $a, b, c \in S$, then $a(bc) = c(ab) = (ab)c$. Hence S is a semigroup.

Stein CA-AG-groupoid

Example 9. Table 12, represent a non-commutative Stein CA-AG-groupoid of order 4, where $3 \cdot 4 \neq 4 \cdot 3$.

Table 12. Non-commutative Stein CA-AG-groupod.

·	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	2	1
4	1	1	2	2

As clear from Example 8 that a Stein AG-groupoid need not to be a bi-commutative AG-groupoid. However, we have the following.

Theorem 11. *Every Stein CA-AG-groupoid is bi-commutative.*

Proof. Let S be a Stein CA-AG-groupoid and $x, y, z \in S$. Then $(xy)z = (zy)x = x(zy) = y(xz) = z(yx) = (yx)z$. Hence S is left commutative. Again, using the given properties we have, $x(yz) = (yz)x = (xz)y = y(xz) = z(yx) = x(zy)$. Thus S is also right commutative. Hence the result follows.

Here we provide a counter example to verify that a bi-commutative CA-AG-groupoid is not necessarily a Stein AG-groupoid.

Example 10. Bi-commutative CA-AG-groupoid of order 4 presented in Table 13, is not a Stein AG-groupoid as $(1 \cdot 1)1 \neq 1(1 \cdot 1)$.

Table 13. Bi-commutative CA-AG-groupoid that is not Stein.

·	1	2	3	4
1	2	3	3	3
2	4	3	3	3
3	3	3	3	3
4	3	3	3	3

Remark 1. Let S be a Stein AG-groupoid, then for all $a, b, c \in S$, $a(bc) = (bc)a = (ac)b = b(ac) \Rightarrow a(bc) = b(ac)$. Thus, every Stein AG-groupoid is an AG**. It is also proved that "every AG** is Bol* [17, Lemma 8] and that each Bol* is paramedial [17, Lemma 9]". Hence every Stein AG-groupoid is paramedial.

Example 11. In Table 9 of Example 6, represent a Stein AG-groupoid, which is not cyclic associative. Table 14 represents an AG-band of order 4, which is not CA as $1(2 \cdot 1) \neq 1(1 \cdot 2)$.

Table 14. AG-band that is not cyclic associative.

·	1	2	3	4
1	1	3	4	2
2	4	2	1	3
3	2	4	3	1
4	3	1	2	4

However, we have the following;

Theorem 12. *A Stein AG-groupoid is CA if it is an AG-band.*

Proof. Let S be a Stein AG-band and $a, b, c \in S$, then using the definition of a Stein AG-groupoid, the left invertive law, definition of AG-band, Remark 1 and the medial law we have,

$$\begin{aligned} a(bc) &= (bc)a = (ac)b = b(ac) = (bb)(ac) \\ &= (cb)(ab) = (ab \cdot b)c = c(ab \cdot b) = c(bb \cdot a) \\ &= c(ba) = (cc)(ba) = (cb)(ca) = (ab)(cc) \\ &= (ab)c = c(ab) \Rightarrow a(bc) = c(ab). \end{aligned}$$

Equivalently, S is a CA-AG-groupoid.

However, a Stein CA-AG-groupoid is not necessarily an AG-band, as clear from the following example.

Example 12. Table 15 represents a Stein CA-AG-groupoid of order 3. As $1.1 \neq 1$, hence it is not an AG-band.

Table 15. Stein AG-groupoid that is not AG-band.

·	1	2	3
1	2	1	1
2	1	2	2
3	1	2	2

As every CA-AG-band is commutative semigroup [1, Theorem 2], thus the following corollary is obvious.

Corollary 2. Every CA-AG-band is Stein AG-groupoid.

Now, we discuss role of the (left/right) identity in CA-AG-groupoids. As proved in [6, Theorem 2.3] that "in AG-groupoids the right identity element is always a left identity, while left identity does not imply right identity". Here, we prove that in CA-AG-groupoid the phenomenon is somewhat different, and prove that in CA a left identity becomes the identity, and in this case a CA-AG-groupoid becomes a commutative semigroup.

Lemma 3. *If a CA-AG-groupoid S contains the left identity, then it is also the right identity of* S.

Proof. Let S be a CA-AG-groupoid with the left identity e. Then $ae = e(ae) = e(ea) = ea = a$. Hence e is the right identity.

The following corollary is now obvious.

Corollary 3. In a CA-AG-groupoid S, the following results are equivalent.

(i) e is the left identity of S.

(ii) e is the right identity of S.

(iii) e is the identity of S.

(iv) S is a monoid.

(v) S is commutative.

We provide an example to verify that an AG-groupoid having a left identity is not necessarily a CA-AG-groupoid. In other words, any AG-monoid is not a CA-AG-groupoid.

Example 13. Table 16, represents an AG-monoid of order 3. As $a * (b * c) \neq c * (a * b)$, hence it is not a CA-AG-groupoid.

Table 16. AG-monoid that is not cyclic associative.

$*$	a	b	c
a	a	b	c
b	c	a	b
c	b	c	a

However, the following is obvious.

Corollary 4. Every monoid is CA-AG-groupoid.

It has been proved in [7] that locally associative AG-groupoids have associative powers. Here, we characterize CA-AG-groupoid by the powers of its elements.

Lemma 4. *In CA-AG-groupoid S,* $(ab)^2 = (ba)^2 \ \forall a, b \in S$.

Proof. Let S be a CA-AG-groupoid, then $\forall a, b \in S$.

$$(ab)^2 = (ab)(ab) = (aa)(bb) = b(aa \cdot b)$$
$$= b(b \cdot aa) = b(a \cdot ba) = (ba)(ba) = (ba)^2.$$

As, by medial law in AG-groupoid S, for all $a, b \in S$,

$$(ab)^2 = (ab)(ab) = (aa)(bb) = a^2b^2.$$

Thus by using this result and Lemma 4, we immediately have that squares of elements commute with each other in CA.

Corollary 5. In CA-AG-groupoid S, $a^2b^2 = b^2a^2$, $\forall a, b \in S$.

Lemma 5. *Let S be a CA-AG-groupoid. Then if for all* x *in S there exist* a *in S such that* (a) $ax = x$ *or* (b) $xa = x$, *then*

(i) $ax^2 = x^2$.

(ii) $x^2a = x^2$.

(iii) $ax^2 = x^2a$.

Proof. (a). (i) By cyclic associativity and the given condition $ax = x$,

$$ax^2 = a(xx) = x(ax) = xx = x^2 \Rightarrow ax^2 = x^2.$$

(ii) By left invertive law and by the given condition

$$x^2a = (xx)a = (ax)x = xx = x^2 \Rightarrow x^2a = x^2.$$

(iii) By (i) and (ii).

(b). (i) By cyclic associativity and given condition $xa = x$

$$ax^2 = a(xx) = x(ax) = x(xa) = xx = x^2 \Rightarrow ax^2 = x^2.$$

(ii) By left invertive law, given condition and cyclic associativity

$$\begin{aligned} x^2a = (xx)a = (ax)x = (ax)(xa) &= a(ax \cdot x) \\ &= x(a \cdot ax) = x(x \cdot aa) = x(a \cdot xa) = x(ax) \\ &= x(xa) = xx = x^2 \Rightarrow x^2a = x^2. \end{aligned}$$

(iii) By (i) and (ii).

Now, we prove that the direct product of two CA-AG-groupoids with same binary operation is cyclic associative and will generalize this idea to two CA-AG-groupoids having arbitrary binary operations.

Theorem 13. *The direct product $S_1 \times S_2$ of two CA-AG-groupoids with same binary operation $(S_1,\cdot)$ and $(S_2,\cdot)$ is a CA-AG-groupoid.*

Proof. Let S_1 and S_2 be two CA-AG-groupoids with same binary operation "$\cdot$", then $S_1 \times S_2$ is also an AG-groupoid by [13]. To prove that $S_1 \times S_2$ is CA, let (a_1, b_1), (a_2, b_2), $(a_3, b_3) \in S_1 \times S_2$, where $a_1, a_2, a_3 \in S_1$ and $b_1, b_2, b_3 \in S_2$. Then

$$\begin{aligned} (a_1, b_1)\big((a_2, b_2)\,(a_3, b_3)\big) &= (a_1, b_1)\,(a_2a_3,\ b_2b_3) \\ &= (a_1 \cdot a_2a_3,\ b_1 \cdot b_2b_3) = (a_3 \cdot a_1a_2,\ b_3 \cdot b_1b_2) \\ &= (a_3, b_3)(a_1a_2, b_1b_2) = (a_3, b_3)((a_1, b_1)\,(a_2, b_2)) \\ \Rightarrow (a_1, b_1)((a_2, b_2)\,(a_3, b_3)) &= (a_3, b_3)\,((a_1, b_1)\,(a_2, b_2)). \end{aligned}$$

Hence the direct product of two CA-AG-groupoids is cyclic associative.

As proved in [17, Theorem 32] that the direct product of two cancellative AG-groupoids is cancellative. Hence we have the following.

Corollary 6. The direct product $S_1 \times S_2$ of two cancellative CA-AG-groupoids S_1 and S_2 is cancellative CA-AG-groupoid.

Next, we generalize the idea of direct product of CA-AG-groupoids with same binary operation to two arbitrary binary operations and prove that the direct product of any two CA-AG-groupoids is again a CA-AG-groupoid.

Theorem 14. *Let (S_1, α_1) and (S_2, α_2) be two CA-AG-groupoids with α_i binary operations defined on each S_i for $i = 1, 2$. The direct product of S_1 and S_2 denoted by $S = S_1 \times S_2 = \{(a, b) | \ a \in S_1, b \in S_2\}$ by component wise multiplication on S, then S becomes a CA-AG-groupoid.*

Proof. As (S_1, α_1) and (S_2, α_2) are CA-AG-groupoids with binary operations α_1 and α_2. If $a = (a_1, b_1)$, $b = (a_2, b_2) \in S_1 \times S_2$, where $a_1, a_2 \in S_1$ and $b_1, b_2 \in S_2$, define * on S as follows; $a * b = \{(a_1\alpha_1a_2,\ b_1\alpha_2b_2)\}$. Clearly, $a * b \in S$. Hence S is a groupoid.

To prove that $S = S_1 \times S_2$ is an AG-groupoid, let $c = (a_3, b_3) \in S_1 \times S_2$, where $a_3 \in S_1$ and $b_3 \in S_2$. Then

$$\begin{aligned} (a * b) * c &= \big((a_1,\ b_1)\,(a_2, b_2)\big)\,(a_3, b_3) \\ &= (a_1\alpha_1a_2,\ b_1\,\alpha_2b_2)\,(a_3,\ b_3) \end{aligned}$$

$$= ((a_1\alpha_1 a_2)\alpha_1 a_3, (b_1\alpha_2 b_2)\alpha_2 b_3)$$
$$= ((a_3\alpha_1 a_2)\alpha_1 a_1,\ (b_3\alpha_2 b_2)\alpha_2 b_1)$$
$$= (a_3\alpha_1 a_2,\ b_3\alpha_2 b_2)\ (a_1, b_1)$$
$$= ((a_3, b_3)\ (a_2, b_2))\ (a_1, b_1)$$
$$\Rightarrow (a * b) * c = (c * b) * a.$$

Hence S is an AG-groupoid. Now to prove that $S = S_1 \times S_2$ is CA, consider

$$a * (b * c) = (a_1, b_1)\ ((a_2, b_2)\ (a_3, b_3))$$
$$= (a_1, b_1)\ (a_2\alpha_1 a_3,\ b_2\alpha_2 b_3)$$
$$= (a_1\alpha_1(a_2\alpha_1 a_3),\ b_1\alpha_2(b_2\alpha_2 b_3))$$
$$= (a_3\alpha_1(a_1\alpha_1 a_2),\ b_3\alpha_2(b_1\alpha_2 b_2))$$
$$= (a_3, b_3)\ (a_1\alpha_1 a_2,\ b_1\alpha_2 b_2)$$
$$= (a_3, b_3)\ ((a_1, b_1)\ (a_2, b_2))$$
$$\Rightarrow a * (b * c) = c * (a * b).$$

Hence S is a CA-AG-groupoid.

4. CONCLUSIONS

We precisely discussed some fundamental characteristics of CA-AG-groupoids and established their relations with some other subclasses of AG-groupoids and with semigroup, monoid etc. We used the modern techniques of GAP, Prover-9 and Mace-4 to produce counterexamples and provide several other examples to improve the standard of this research work.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Compzutational Sciences 53 (3): 339–348 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Integral Form of Popoviciu Inequality for Convex Function

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Abstract: In this paper, the new integral form of Popoviciu inequality for convex functions is constructed and also the new refinement of integral form of Jensen's inequality is given. For the purpose of application some new quasi arithmetic means are defined along with their monotonicity property.

Keywords: Convex function, Popoviciu inequality, Jensen's inequality, quasi arithmetic means

1. INTRODUCTION AND PRELIMINARY RESULTS

A function $g : C \to \mathbb{R}$, where C is a convex subset of real vector space, is said to be convex if

$$g(ax+by) \le ag(x)+bg(y), \tag{1}$$

for all $x, y \in C$ and $a, b \ge 0$, such that $a+b=1$ (see [10, page ~1]).

In [10, page ~43] the Jensen's inequality in discrete version is given as follows:

Theorem 1.1 *Let C be a convex subset of real vector spaceX, $g : C \to \mathbb{R}$ be convex function, $p_1, \ldots, p_n \in (0,1]$ such that $\sum_{i=1}^{n} p_i = 1$, and $c_1, \ldots, c_n \in C$, then*

$$g\left(\sum_{i=1}^{n} p_i c_i\right) \le \sum_{i=1}^{n} p_i g(c_i). \tag{2}$$

In [10, page ~63] the integral form of Jensen's inequality is defined as follows.

Theorem 1.2 *Let h be an integrable function on a probability space (X, A, μ) taking values in an interval $I \subset \mathbb{R}$. If g is a convex function on I such that the composition function $g \circ h$ is integrable, then*

$$g\left(\int_X h d\mu\right) \le \int_X g \circ h d\mu. \tag{3}$$

In [2], Brneti c′, Pearce and Pe c̆aric give the refinement of integral form of Jensen's inequality (3) by using refinement of discrete Jensen's inequality. Moreover in [7], László Horváth and Pečarić give

Received, May 2016; Accepted, August 2016
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the improvement of integral form of Jensen's inequality (3) by using some refinement of discrete Jensen's inequality which is generalization of result given in [2], they also give new quasi arithmetic means and prove their monotonicity.

The Popoviciu inequality is given by (see [10, page 173]).

Theorem 1.3 *Let* $m,n\in\mathbb{N}$, *such that* $n\geq 3$, $2\leq m\leq n-1$, $[a,b]\subset\mathbb{R}$ *be an interval,* $\mathbf{x}=(x_1,\ldots,x_n)\in[a,b]^n$, $\mathbf{p}=(p_1,\ldots,p_n)$ *be a* n*-tuple such that* $p_i\geq 0, i=1,2,\ldots,n$ *with* $\sum_{i=1}^{n}p_i=1$. *Also let* $g:[a,b]\to\mathbb{R}$ *be a convex function. Then*

$$g_{m,n}(\mathbf{x},\mathbf{p})\leq\frac{n-m}{n-1}g_{1,n}(\mathbf{x},\mathbf{p})+\frac{m-1}{n-1}g_{n,n}(\mathbf{x},\mathbf{p}), \tag{4}$$

where

$$g_{m,n}:=\frac{1}{C_{m-1}^{n-1}}\sum_{1\leq i_1<\ldots<i_m\leq n}\left(\sum_{j=1}^{m}p_{i_j}\right)g\left(\frac{\sum_{j=1}^{m}p_{i_j}x_{i_j}}{\sum_{j=1}^{m}p_{i_j}}\right), \tag{5}$$

and

$$C_m^n=\frac{n!}{m!(n-m)!}.$$

In the current century, the Popoviciu inequality (4) is studied by many authors. In the monograph [6], the generalization of (4) for real weights, mixed symmetric means, exponential convexity, mean value theorems and Cauchy means are studied. In [8, 9], the integral version and refinement of a special case of (4) is proved respectively. In [1], the higher dimension analogue of a special case of (4) is given. Moreover, in [3, 4, 5] (4) is generalized for higher order convex functions via different interpolating polynomials. We use the idea of Brneti$\acute{c}$, Pearce and Pe$\breve{c}$aric given (for Jensen's inequality) in [2] to construct the integral form of Popoviciu inequality (4). Also following the way of László Horváth and J. Pe$\breve{c}$ari$\acute{c}$ given (for refinements of Jensen's inequality) in [7] we give application to the quasi arithmetic means.

2. MAIN RESULTS

We now consider some hypotheses which are used in our work.

(H_1) Let (X,E,μ) be a probability space, and let $p_1,\ldots,p_n$ be positive numbers with $\sum_{i=1}^{n}p_i=1$.

(H_2) Let $h:X\to I\subset\mathbb{R}$ be an integrable function.

(H_3) Let g be a convex function on interval I such that the composition $g\circ h$ is integrable.

Let $m \geq 2$ be a fixed integer. The σ-algebra in X^k generated by the projection mapping $pr_l : X^k \to X \ (l = 1, \ldots, m)$

$$pr_l(x_1, \ldots, x_m) := x_l \tag{6}$$

is denoted by E^k. And μ^m is defined as the product measure on E, this measure is uniquely (μ is σ-finite) specified by

$$\mu^m(B_1 \times \ldots \times B_m) := \mu(B_1) \ldots \mu(B_m), \quad B_l \in \mathsf{E}, \quad l = 1, \ldots, m. \tag{7}$$

Theorem 2.1 *Assume* (H_1)-(H_3), *then the following inequalities hold.*

a.

$$\frac{1}{C_{m-1}^{n-1}} \sum_{1 \leq i_1 < \ldots < i_m \leq n} \left(\sum_{j=1}^{m} p_{i_j} \right) \int_{X^m} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} \right) d\mu^m(x_{i_1}, \ldots, x_{i_m})$$

$$\leq \frac{n-m}{n-1} \sum_{i=1}^{n} p_i \int_X g(h(x_i)) d\mu(x_i)$$

$$+ \frac{m-1}{n-1} \int_{X^n} g\left(\sum_{i=1}^{n} p_i h(x_i) \right) d\mu^n(x_{i_1}, \ldots, x_{i_n}).$$

b.

$$\frac{1}{C_{m-1}^{n-1}} \sum_{1 \leq i_1 < \ldots < i_m \leq n} \left(\sum_{j=1}^{m} p_{i_j} \right) \int_{X^m} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} \right) d\mu^m(x_{i_1}, \ldots, x_{i_m})$$

$$\leq \sum_{i=1}^{n} p_i \int_X g(h(x_i)) d\mu(x_i).$$

Proof. (a) On integrating the inequality (4) over X^n and replacing x_{i_j} by $h(x_{i_j})$, we have

$$\frac{1}{C_{m-1}^{n-1}} \sum_{1 \leq i_1 < \ldots < i_m \leq n} \left(\sum_{j=1}^{m} p_{i_j} \right) \int_{X^n} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} \right) d\mu^n(x_{i_1}, \ldots, x_{i_n})$$

$$\leq \frac{n-m}{n-1} \sum_{i=1}^{n} p_i \int_{X^n} g(h(x_i)) d\mu^n(x_{i_1}, \ldots, x_{i_n})$$

$$+\frac{m-1}{n-1}\int_{X^n} g\left(\sum_{i=1}^{n} p_i h(x_i)\right) d\mu^n(x_{i_1},\ldots,x_{i_n}).$$

On simplification we have

$$\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)\int_{X^m} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}}\right) d\mu^m(x_{i_1},\ldots,x_{i_m})$$

$$\times\int_X d\mu(x_{i_{m+1}})\ldots\int_X d\mu(x_{i_n})$$

$$\le\frac{n-m}{n-1}\sum_{i=1}^{n} p_i\int_X g\circ h(x_i)d\mu(x_i)\times\int_X d\mu(x_{i_1})\ldots\int_X d\mu(x_{i_m})\int_X d\mu(x_{i_{m+1}})\ldots\int_X d\mu(x_{i_n})$$

$$+\frac{m-1}{n-1}\int_{X^n} g\left(\sum_{i=1}^{n} p_i h(x_i)\right) d\mu^n(x_{i_1},\ldots,x_{i_n}).$$

This gives

$$\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)\int_{X^m} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}}\right) d\mu^m(x_{i_1},\ldots,x_{i_m})$$

$$\le\frac{n-m}{n-1}\sum_{i=1}^{n} p_i\int_X g(h(x_i))d\mu(x_i)$$

$$+\frac{m-1}{n-1}\int_{X^n} g\left(\sum_{i=1}^{n} p_i h(x_i)\right) d\mu^n(x_{i_1},\ldots,x_{i_n}).$$

(b) Using the discrete Jensen's inequality in the last term of inequality given in (a) and on solving, we have

$$\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)\int_{X^m} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}}\right) d\mu^m(x_{i_1},\ldots,x_{i_m})$$

$$\le\frac{n-m}{n-1}\sum_{i=1}^{n} p_i\int_X g(h(x_i))d\mu(x_i)$$

$$+\frac{m-1}{n-1}(p_1\int_{X^n} g(h(x_1))d\mu^n(x_{i_1},\ldots,x_{i_n})+\ldots+p_n\int_{X^n} g(h(x_n))d\mu^n(x_{i_1},\ldots,x_{i_n})),$$

this gives

$$\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)\int_{X^m}g\left(\frac{\sum_{j=1}^{m}p_{i_j}h(x_{i_j})}{\sum_{j=1}^{m}p_{i_j}}\right)d\mu^m(x_{i_1},\ldots,x_{i_m})$$

$$\le\sum_{i=1}^{n}p_i\int_X g(h(x_i))d\mu(x_i).$$

Under the hypothesis (H_1), (H_2) and (H_3), define the function $H_m(t)$ on $[0,1]$ given by

$$H_m(t)=\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)$$

$$\times\int_{X^m}g\left(t\frac{\sum_{j=1}^{m}p_{i_j}h(x_{i_j})}{\sum_{j=1}^{m}p_{i_j}}+(1-t)\int_X hd\mu\right)d\mu^m(x_{i_1},\ldots,x_{i_m}). \tag{8}$$

Theorem 2.2 *For $m\ge 2$ be an integer, we assume (H_1)-(H_3) and consider H_m :$[0,1]\to R$* as defined in (8) then the following statements are valid.

a. H_m is convex.

b. $\min_{t\in[0,1]}H_m(t)=H_m(0)=g\left(\int_X hd\mu\right)$

c. $\max_{t\in[0,1]}H_m(t)=H_m(1)$

d. H_m is increasing.

Proof. (a) Suppose $\alpha,\beta\in[0,1]$ with $\alpha+\beta=1$ and $u,v\in[0,1]$, then from (8) we have

$$H_m(\alpha u+\beta v)=\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)$$

$$\times\int_{X^m}g\left((\alpha u+\beta v)\frac{\sum_{j=1}^{m}p_{i_j}h(x_{i_j})}{\sum_{j=1}^{m}p_{i_j}}+(\alpha+\beta-\alpha u-\beta v)\int_X hd\mu\right)d\mu^m(x_{i_1},\ldots,x_{i_m}).$$

On simplification we have

$$H_m(\alpha u+\beta v)=\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)$$

$$\times \int_{X^m} g(\alpha\left(u\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} + (1-u)\int_X h d\mu \right) +$$

$$\beta\left(v\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} + (1-v)\int_X h d\mu \right))d\mu^m(x_{i_1},\ldots,x_{i_m}).$$

By convexity of g, we have

$$H_m(\alpha u+\beta v) \le \alpha \frac{1}{C_{m-1}^{n-1}} \sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)$$

$$\times \int_{X^m} g\left(u\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} + (1-u)\int_X g d\mu \right) d\mu^m(x_{i_1},\ldots,x_{i_m}) +$$

$$\beta \frac{1}{C_{m-1}^{n-1}} \sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)\int_{X^m} g\left(v\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} + (1-v)\int_X h d\mu \right) d\mu^m(x_{i_1},\ldots,x_{i_m}),$$

that is

$$H_m(\alpha u+\beta v) \le \alpha H_m(u) + \beta H_m(v).$$

Therefore H_m is convex function.

(b) By the integral from Jensen's inequality (8) yields

$$H_m(t) \ge \frac{1}{C_{m-1}^{n-1}} \sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)$$

$$\times g\left(\int_{X^m}\left(t\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} + (1-t)\int_X g d\mu \right) d\mu^m(x_{i_1},\ldots,x_{i_m}) \right)$$

or

$$H_m(t) \ge \frac{1}{C_{m-1}^{n-1}} \sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right) g(I), \tag{9}$$

where

$$I = \int_{X^m} \left(t \frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} + (1-t) \int_X g d\mu \right) d\mu^m (x_{i_1}, \ldots, x_{i_m})$$

$$= t \int_{X^m} \frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}} d\mu^m (x_{i_1}, \ldots, x_{i_m}) + (1-t) \int_{X^m} \left(\int_X g d\mu \right) d\mu^m (x_{i_1}, \ldots, x_{i_m})$$

$$= \frac{t}{\sum_{j=1}^{m} p_{i_j}} \sum_{i=1}^{m} p_i \int_X h d\mu + (1-t) \int_X g d\mu$$

$$= \int_X g d\mu$$

so from (9), we have

$$H_m(t) \geq \frac{1}{C_{m-1}^{n-1}} \sum_{1 \leq i_1 < \ldots < i_m \leq n} \left(\sum_{j=1}^{m} p_{i_j} \right) g \left(\int_X g d\mu \right)$$

$$= H_m(0), \qquad \forall\ t \in [0,1].$$

(c)

$$H_m(t) = H_m(1.t + (1-t)0) \leq t H_m(1) + (1-t) H_m(0)$$

$$\leq t H_m(1) + (1-t) H_m(1)$$

$$= H_m(1), \quad \forall\ t \in [0,1].$$

(d) Since $H_m(t)$ is convex and $H_m(t) \geq H_m(0) (t \in [0,1])$, therefore for $0 \leq t_1 < t_2 \leq 1$, we have

$$\frac{H_m(t_2) - H_m(t_1)}{t_2 - t_1} \geq \frac{H_m(t_2) - H_m(0)}{t_2} \geq 0,$$

so

$$H_m(t_2) \geq H_m(t_1).$$

Theorem 2.3 *Assume* (H_1), (H_2) *and* (H_3), *then*

$$g \left(\int_X h d\mu \right) \leq H_m(t) \leq H_m(1) \leq \int_X g \circ h d\mu. \tag{10}$$

Proof. Using (b) and (c) of Theorem 2.2 we get first two inequalities, and for the last inequality

$$H_m(1) = \frac{1}{C_{m-1}^{n-1}} \sum_{1 \leq i_1 < \ldots < i_m \leq n} \left(\sum_{j=1}^{m} p_{i_j} \right)$$

$$\times \int_{X^m} g\left(\frac{\sum_{j=1}^{m} p_{i_j} h(x_{i_j})}{\sum_{j=1}^{m} p_{i_j}}\right) d\mu^m(x_{i_1},\ldots,x_{i_m}).$$

Using the discrete Jensen's inequality, we have

$$H_m(1) \le \frac{1}{C_{m-1}^{n-1}} \sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)$$

$$\times \frac{\sum_{j=1}^{m} p_{i_j}}{\sum_{j=1}^{m} p_{i_j}} \int_{X^m} g\left(h(x_{i_j})\right) d\mu^m(x_{i_1},\ldots,x_{i_m}),$$

this gives

$$H_m(1) \le \int_X g\circ h d\mu.$$

Remark 2.4 *A refinement similar to (10) of integral form of Jensen's inequality is proved in Proposition 7 of [7].*

3. NEW QUASI-ARITHMETIC MEANS

Now we introduced some new quasi arithmetic means. For this first assume some conditions:

(H_4) Let $h: X \to I$, where $I \subset \mathsf{R}$ be an interval, is measurable.

(H_5) Let α, $\beta: I \to \mathsf{R}$ are continuous and strictly monotone functions.

Definition 1 *Assume* (H_1), (H_4) *and* (H_5).

For $t \in [0,1]$ we define the class of quasi-arithmetic mean given by

$$M_{\lambda,\chi}(t,g,\mu) := \chi^{-1}\left(\frac{1}{C_{m-1}^{n-1}} \sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m} p_{i_j}\right)\right.$$

$$\left.\times \int_{X^m} \chi\circ\lambda^{-1}\left(t\frac{\sum_{j=1}^{m} p_{i_j}\lambda(g(x_{i_j}))}{\sum_{j=1}^{m} p_{i_j}} + (1-t)\int_X \lambda(g)d\mu\right) d\mu^m(x_{i_1},\ldots,x_{i_m})\right), \tag{11}$$

where the integrals are supposed to be exist.

Assume (H_6), let $\eta: I \to \mathsf{R}$ be a continuous and strictly monotone function such that the composition $\eta\circ h$ is integrable on X. Define the mean

$$M_\eta(h,\mu) = \eta^{-1}\left(\int_X \eta\circ h d\mu\right). \tag{12}$$

Theorem 3.1 *Assume* $(H_1),(H_4),\ (H_5)$ *and assume that* $\lambda\circ h$ *and* $\chi\circ h$ *an integrable on* X.

(a) If $\chi\circ\lambda^{-1}$ is convex with χ is increasing or $\chi\circ\lambda^{-1}$ is concave with χ is decreasing, then

$$M_{\lambda}(h,\mu)\le M_{\chi,\lambda}(t,h,\mu)\le M_{\chi}(h,\mu), \tag{13}$$

holds for all $t\in[0,1]$.

(b) If $\chi\circ\lambda^{-1}$ is convex with χ is decreasing or $\chi\circ\lambda^{-1}$ is concave with χ is increasing, then

$$M_{\lambda}(h,\mu)\ge M_{\chi,\lambda}(t,h,\mu)\ge M_{\chi}(h,\mu), \tag{14}$$

holds for all $t\in[0,1]$.

Proof. (a) Using pair of functions $\chi\circ\lambda^{-1}$ and $\lambda(h)$ ($\lambda(I)$ is an interval) in Theorem 2.3, we have

$$\chi\circ\lambda^{-1}\left(\int_X\lambda(h)d\mu\right)\le\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)$$

$$\times\int_{X^m}\chi\circ\lambda^{-1}\left(t\frac{\sum_{j=1}^{m}p_{i_j}\lambda(h(x_{i_j}))}{\sum_{j=1}^{m}p_{i_j}}+(1-t)\int_X\lambda(h)d\mu\right)d\mu^m(x_{i_1},\ldots,x_{i_m})$$

$$\le\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)\int_{X^m}\chi\circ\lambda^{-1}\left(\frac{\sum_{j=1}^{m}p_{i_j}\lambda(h)}{\sum_{j=1}^{m}p_{i_j}}\right)d\mu^m(x_{i_1},\ldots,x_{i_m}).$$

Using the discrete Jensen inequality on the right side of last inequality we get

$$\chi\circ\lambda^{-1}\left(\int_X\lambda(h)d\mu\right)\le\frac{1}{C_{m-1}^{n-1}}\sum_{1\le i_1<\ldots<i_m\le n}\left(\sum_{j=1}^{m}p_{i_j}\right)$$

$$\times\int_{X^m}\chi\circ\lambda^{-1}\left(t\frac{\sum_{j=1}^{m}p_{i_j}\lambda(h(x_{i_j}))}{\sum_{j=1}^{m}p_{i_j}}+(1-t)\int_X\lambda(h)d\mu\right)d\mu^m(x_{i_1},\ldots,x_{i_m})$$

$$\le\int_X\chi(h)d\mu.$$

On taking χ^{-1} on both sides we have (13).

(b) Similarly using the pair of functions $-\chi\circ\lambda^{-1}$ and $\lambda(h)$ in Theorem 2.3, where $\chi\circ\lambda^{-1}$ is concave. On taking χ^{-1} the we have (14).

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Compzutational Sciences 53 (3): 349–357 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Single-Tile Semigroups of Shift Operators

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Abstract: We introduce some new subsemigroups of the finite full transformation semigroups T_n. We consider various irregular boards of different shapes and sizes to generate subsemigroups of T_n using the four idempotent operators *L, R, U, D*. These operators shift tiles on a board in four different directions (left, right, up and down) in their respective rows and columns. In this way each operator $O \in G = \{L, R, U, D\}$ defines a member of T_n on the base set $X = \{1, 2, \ldots, n\}$ and the semigroups of various properties are given as $S = \langle G \rangle \leq T_X$.

Keywords and phrases: Finite semigroups, shift operators, transformation semigroups, $\mathcal{D}$-classes

1. INTRODUCTION AND PRELIMINARIES

We will consider different shaped boards consisting of tiles. We assume that the tiles of the board can be dragged in four different directions (left, right, up and down) by introducing the four idempotent operators *L, R, U, D* on the board. We denote the operator *L(T) = L*, as that which moves the tile T from co-ordinates (i, j) to $(i^{'}, j)$ that can be reached by sliding T to the left until it encounters a barrier. (It is assumed that each row and column has barriers at the edges of the boards and perhaps elsewhere as well.) In a similar way we define the right operator *R*, while the up operator U and the down operator *D* acts on the second co-ordinate in an entirely analogous fashion.

In this way each operator $O \in G = \{L, R, U, D\}$ defines a member of the full transformation Semigroup "Ganyushkin and Mazorchuk[5]" on the base set $X = \{1, 2, \ldots, n\}$ and put $S = \langle G \rangle \leq T_X$. We wish to study this finite semigroup, which is generatedby a pair of disjoint two-element left-zero subsemigroups $R_h = \{L, R\}$ and $R_v = \{U, D\}$. (The subscript h, v stands respectively for horizontal and vertical.) A similar finite subsemigroup of T_n on rectangular $m \times n$ bi-coloured board B with $2 \leq m \leq n$ with m rows and n columns has been discussed recently in "Ahmad [1, 6]". We will apply the operators from right to left so the operator *UD = UD(B)* will mean, first operate *D* followed by *U*.Before going into details of the semigroups, we will define some basic notions. For undefined semigroup terminology we will refer to "Peter [2], Howie [3], Ganyushkin and Mazorchuk [5]".

An element a of a semigroup S will be called idempotent if $a^2 = a$. A non-emptysubset A of S is called a left ideal if $SA \subseteq A$, a right ideal if $AS \subseteq A$ and an (two-sided) ideal if it is both a left and right ideal. If a is an element of a semigroup S, the smallest left ideal of S containing a is $aSa \cup \{a\}$, denoted by S^1a and will be called the principal left ideal generated by a. For any set X, the full transformation Semigroup $(T_X, \circ)$ is the semigroup of all mappings from X into X under the operation of composition of mappings. An element$a$of a semigroup S is called regular if there exists an element x

Received, May 2016; Accepted, August 2016
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in S such that $a = axa$; S is called regular if all its elements are regular. The set of all regular elements of S is denoted by Reg(S). A semigroup S is calleda rectangular band if $aba = a$ for all a, b in S. The term band is used in general for a semigroup consisting of idempotents. A commutative band is called a semilattice. Any band is a semilattice of rectangular bands "Howie[3,Theorem 4.4.1]". For $a \in S$, we say that a is an inverse of a if $aa\,a = a$ and $a\,aa = a$. The set of all inverses of an element a in S is denoted by $V(a)$. Obviously every idempotent e is regular $(e = eee)$ and every regular element a has an inverse.

The equivalence $\mathcal{L}$ on S is defined by the rule that $a \mathcal{L} b$ if and only if a and b generate the same principal left ideal, that is, if and only if $S^1a = S^1b$. Similarly the equivalence $\mathcal{R}$ is defined by the rule that $a \mathcal{R} b$ if and only if $aS^1 = bS^1$. The $\mathcal{H}$ relation is defined to be the intersection of the $\mathcal{L}$ and $\mathcal{R}$ relations.The join $\mathcal{L} \vee \mathcal{R}$ of $\mathcal{L}$ and $\mathcal{R}$ will be called the $\mathcal{D}$-relation. Similarly, since the principal two-sided ideal of S generated by a is S^1aS^1, we can definethe equivalence $\mathcal{J}$ by the rule that $a \mathcal{J} b$ if and only if $S^1aS^1 = S^1bS^1$. It is immediate that $\mathcal{L} \subseteq \mathcal{J}$ and $R \subseteq J$. Hence, since $\mathcal{D}$ is the smallest equivalence containing $\mathcal{L}$ and $\mathcal{R}$, we must have $\mathcal{D} \subseteq \mathcal{J}$. For finite semigroups and for any T_X, we have the equality $\mathcal{D} = \mathcal{L}$ as appears in "Howie [3, Proposition 2.1.4]".

2. MATERIALS AND METHODS

We will use different types of boards, like L-shape, T and Y- shape boards consisting of n tiles to introduce various finite subsemigroups of the finite semigroups T_n.

3. RESULTS AND DISCUSSION

One-Tile Semigroup that is Regular but not a Band

Here, we will consider an L-shaped one-tile board of four cells that will be labelled {1, 2, 3, 4}. We will assume that this board has one moving cell that will be called its tile.

Definition 1. A board B will be called convex if we can travel between any two cells by a horizontal and then a vertical movement (in some order).

Let B be the L-shaped board as in the following Fig. 1.

The board B in the Fig. 1 has four cells with base set {1, 2, 3, 4}. The *L, R, U, D* operators, as defined in the previous section act on the moveabletile $T \in \{1, 2, 3, 4\}$ as follows:

3.1. Calculations of Presentation for the L-semigroup.

For this type of calculations, we follow the method as outlined by "Pin [4, Section 3.1]" whichguarantees to produce a presentation of a semigroup S (but the presentation maycontain redundant relations).

The Actions of Operators of Length 1

The actions of operators *L, R, U,* and *D* on the L-shaped board are given as follows:

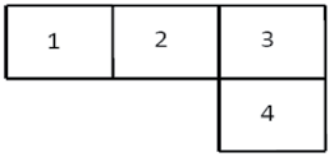

Fig. 1. L-shaped board with a single movable tile.

Operators	Tiles			
	1	**2**	**3**	**4**
L	1	1	1	4
R	3	3	3	4
U	1	2	3	3
D	1	2	4	4

Note that the operator *L* moves the tiles 1, 2, 3, 4 of the board B respectivelytowards 1, 1, 1, 4.

The Actions of Operators of Length 2

We calculate successively (1). *LL* to (16).*DD* and list the relatedrelations as in the following table.

Operatros	Tiles				Relations	
	1	**2**	**3**	**4**		
L	1	1	1	4	*(1) LL = L*	*(15) DU = D*
R	3	3	3	4	*(2) LR = L*	*(16) DD = D*
U	1	2	3	3	*(5) RL = R*	*(21) LUL = LU*
D	1	2	4	4	*(6) RR = R*	*(22) LDR = DR*
(3) LU	1	1	1	1	*(10) UR = RU*	*(28) RDR = DR*
(4) LD	1	1	4	4	*(11) UU = U*	*(29) ULU = LU*
(7) RU	3	3	3	3	*(12) UD = U*	*(38) DRD = DR*
(8) RD	3	3	4	4	*(13) DL = L*	
(9) UL	1	1	1	3		
(14) DR	4	4	4	4		
(30)ULD	1	1	3	3		

The Actions of Operators of Length 3

We calculate successively the actions of operators of length 3from (17) to (40) as (17) *LLU*, (18) *LLD*, (19*) LRU,* ... (39) *DU L*, (40) *DDR*.

The relations already known enable us to avoid the calculation of (17), . . . , (20),(22), . . . , (28), (32), . . . , (37), (39), (40), (42), (43), (44) since $LLU = LU, LLD = LD, LRU = LU$ and so on.

We now have the following representation for the semigroup generated by the idempotents *L, R, U, D.*

$$S_1 = \langle L, U, R, D : L^2 = L, R^2 = R, U^2 = U, D^2 = D, LR = L, RL = R, UR = RU, DL = L,$$

$$DU = D, U D = U, U LU = LU, LU L = LU, LDR = DR, DRD = DR, RD R = DR\rangle$$

Removing the redundancies we have,

$$S_1 = \langle L, U, R, D : LR = L, RL = R, UR = RU, DL = L, DU = D,$$

$$U D = U, U LU = LU, LU L = LU, LDR = DR, DRD = DR, R DR = DR\rangle .$$

As in the case of previous boards, we can show that $|S_1| = |S| = 11$ and so $S_1 = S$. Now we have Table 1.

Table 1.

	L	*R*	*U*	*D*	*UL*	*RU*	*DR*	*LU*	*LD*	*RD*	*ULD*
L	*L*	*L*	*LU*	*LD*	*LU*	*LU*	*DR*	*LU*	*LD*	*LD*	*LU*
R	*R*	*R*	*RU*	*RD*	*RU*	*RU*	*DR*	*RU*	*RD*	*RD*	*RU*
U	*UL*	*RU*	*U*	*U*	*UL*	*RU*	*RU*	*LU*	*ULD*	*RU*	*ULD*
D	*L*	*DR*	*D*	*D*	*L*	*DR*	*DR*	*LU*	*LD*	*DR*	*LD*
UL	*UL*	*UL*	*LU*	*ULD*	*LU*	*LU*	*RU*	*LU*	*ULD*	*ULD*	*LU*
RU	*RU*	*RU*	*RU*	*RU*	*RU*	*RU*	*RU*	*RU*	*RU*	*RU*	*RU*
DR	*DR*	*DR*	*DR*	*DR*	*DR*	*DR*	*DR*	*DR*	*DR*	*DR*	*DR*
LU	*LU*	*LU*	*LU*	*LU*	*LU*	*LU*	*LU*	*LU*	*LU*	*LU*	*LU*
LD	*L*	*DR*	*LD*	*LD*	*L*	*DR*	*DR*	*LU*	*LD*	*DR*	*LD*
RD	*R*	*DR*	*RD*	*RD*	*R*	*DR*	*DR*	*RU*	*RD*	*DR*	*DR*
ULD	*UL*	*RU*	*ULD*	*ULD*	*UL*	*RU*	*RU*	*LU*	*ULD*	*RU*	*ULD*

Note that each element is idempotent except *UL&RD*, where $(U\,L)^2 = LU$ and $(RD)^2 = DR$, hence *S* is not an orthodox. However, it can be checked that *UL* and *RD* are still regular, since *(U L)D(U L) = UL* and *(RD) L (RD) = RD.*

3.2. D-classes for the L-semigroup

The D-classes are given as follows:

$\mathcal{D}_1$: | *U* | / | *D* |

$\mathcal{D}_2$: | *L* | *LD* | / | *R* | *RD* | / | *UL* | *ULD* |

$\mathcal{D}_3$: | *RU* | / | *DR* | / | *LU* |

4. OPERATIONS ON A T-SHAPED BOARD

Now we consider a T-shaped 4-cell board with one moveable cell with the base set {1, 2, 3, 4} as in the following Fig. 2.

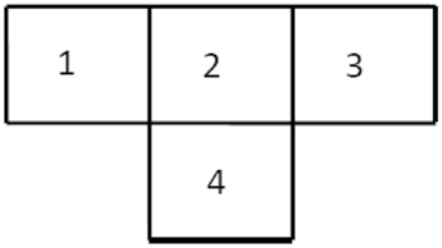

Fig. 2. T-shaped board with a single movable tile.

The *L, R, U, D* operators then act as follows and generate an idempotent-generated semigroup that is not a band but still is regular and furthermore it is conventional. The operators are given as follows:

4.1. Calculations of Presentation for the T-semigroup

The Actions of Operators of Length 1: The actions of the operators *L, R, U*, and *D* are given as

OPERATORS	TILES			
	1	**2**	**3**	**4**
L	1	1	1	4
R	3	3	3	4
U	1	2	3	2
D	1	4	3	4

The Actions of Operators of Length 2: We calculate successively *(1). LL, (2). LR, up to (16).DD* as follows.

Operator	Tiles				Relations
	1	**2**	**3**	**4**	
L	1	1	1	4	***(1) LL = L***
R	3	3	3	4	*(2) LR = L*
U	1	2	3	2	*(5) RL = R*
D	1	4	3	4	*(6) RR = R*
(3) LU	1	1	1	1	*(11) U U = U*
(4) LD	1	4	1	4	*(12) U D = U*
(7) RU	3	3	3	3	*(13) DL = L*
(8) RD	3	4	3	4	*(14) DR=R*
(9) UL	1	1	1	2	*(15) DU = D*
(10) UR	3	3	3	2	*(16) DD = D*

The Actions of Operators of Length 3: We continue to calculate successively from (17) to (40) as (17) *LLU*, (18) *LLD*, (19) *LRU*, ..., (39*) DU L*, (40) *DU R*.

The known relations enable us to avoid the calculations of (17), . . . , (20),(23), . . . (26), (33), . . . , (40) since *U LL = U L, DLL = DL, URL = U R* andso on. The continuation of the calculations, then gives

Calculaitons of Length 3

OPERATORS	TILES				RELATIONS	
	1	2	3	4		
L	1	1	1	4	*(1) LL = L*	*(15) DU = D*
R	3	3	3	4	*(2) LR = L*	*(16) DD = D*
U	1	2	3	2	*(5) RL = R*	*(21) LUL = LU*
D	1	4	3	4	*(6) RR = R*	*(22) LUR = LU*
(3) LU	1	1	1	1	*(11) UU = U*	*(27) RUL = RU*
(4) LD	1	4	1	4	*(12) UD = U*	*(28) RU R = RU*
(7) RU	3	3	3	3	*(13) DL = L*	*(29) ULU = LU*
(8) RD	3	4	3	4	*(14) DR = R*	*(31) URU = RU*
(9) UL	1	1	1	2		
(10) UR	3	3	3	2		
(30) ULD	1	2	1	2		
(32) URD	3	2	3	2		

The Actions of Operators of Length 4: No more new calculations for length 4 are possible as $LU\,L = LU = LU\,R$ and $RU\,L = RU = RU\,R$, etc.is avoiding the new entries.

We now have the following representation for the semigroup of the **T**-shaped board.

$$S_1 = \langle L, U, R, D : L^2 = L, R^2 = R, U^2 = U, D^2 = D, LR = L, RL = R, DR = R, DU = D,$$
$$DL = L, UD = U, RU\,L = RU, ULU = LU, LU\,L = LU, LU\,R = LU, RU\,R = RU, U\,RU = RU\rangle$$

Again, on removing the redundancies we have

$$S_1 = \langle L, U, R, D : LR = L, RL = R, DR = R, DU = D, U\,D = U, DL = L,$$
$$RU\,L = RU, ULU = LU, LU\,L = LU, LU\,R = LU, RU\,R = RU, URU = RU\rangle$$

Using these relations and generators we have Table 2 for the semigroup S of the **T**-shaped board having one moveable tile under the shift operators

Table 2.

	L	R	U	D	UL	UR	LU	RU	LD	RD	ULD	URD
L	L	L	LU	LD	LU	LU	LU	LU	LD	LD	LU	LU
R	R	R	RU	RD	RU	RU	RU	RU	RD	RD	RU	RU
U	UL	UR	U	U	UL	UR	LU	RU	ULD	URD	ULD	URD
D	L	R	D	D	L	R	LU	RU	LD	RD	LD	RD
UL	UL	UL	LU	ULD	LU	LU	LU	LU	ULD	ULD	LU	LU
UR	UR	UR	RU	URD	RU	RU	RU	RU	URD	URD	RU	RU
LU	LU	LU	LU	LU	LU	LU	LU	LU	LU	LU	LU	LU
RU	RU	RU	RU	RU	RU	RU	RU	RU	RU	RU	RU	RU
LD	L	L	LD	LD	L	L	LU	LU	LD	LD	LD	LD
RD	R	R	RD	RD	R	R	RU	RU	RD	RD	RD	RD
ULD	UL	UL	ULD	ULD	UL	UL	LU	LU	ULD	ULD	ULD	ULD
URD	UR	UR	URD	URD	UR	UR	RU	UR	URD	URD	URD	URD

Clearly the table shows that the elements UL and UR are not idempotents but are regular as

$U\,L = U\,L(LD)U\,L$

$U\,R = U\,R(LD)U\,R$

4.2. $\mathcal{D}$-classes for the T-semigroup.

The $\mathcal{D}$-classes for the T-shaped board semigroup are given as follows:

$\mathcal{D}_1$: | U |
|---|
| D |

$\mathcal{D}_2$: | L | LD |
|---|---|
| R | RD |
| UL | ULD |
| UR | URD |

$\mathcal{D}_3$: | LU |
|---|
| RU |

The **L** and **T**-shaped boards semigroups are not orthodox. However, the **T**-semigroup is an example of a finite conventional semigroup that is notorthodox as follows by the following result.

Proposition 2. The **T**-semigroup associated with the **T**-shaped board is conventional.

Proof. To show the single tile semigroup T is conventional, we need to show that *aea* is always idempotent (as we know that T is regular but not orthodox). However, there are only two non-idempotent elements, $U\,L$ and $U\,R$, and since the tile has left-right symmetry, we only need to check that there is no factorization of *UL* of the form *aea* .

Suppose that $UL = aea$. The Table 2 shows this is possible only if a is a member of the set $X = \{U,\ U\,L,\ U\,LD\}$ and a is a member of the set $Y = \{L,\ R,\ U\,L,\ U\,R\}$. On the other hand, since $V(U) = U$, $V(U\,L) = \{LD,\ RD\}$ and $V(U\,LD) = U\,LD$. Clearly the inverses of each element of the set *X* has an empty intersection with *Y*. This shows that no factorization of the from, $UL = ata$ (for any $t \in S_1$) is possible. Hence T is conventional Semigroup that is not orthodox.

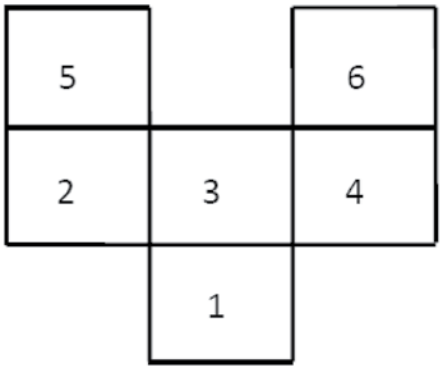

Fig. 3. Y-shaped board with a single movable tile.

Definition 3. A semigroup is called E-solid if the idempotents *e, f, g* satisfy, $e\mathcal{R}f$ and $f\mathcal{L}g$, then there is an idempotent *h* such that $e\mathcal{L}h$ and $h\mathcal{R}g$.

T. E. Hall, recently asked (personal communication) if conventional semigroups are E-solid. The answer is indeed 'no' as T is not E-solid: $R,\ RD,\ ULD \in E(S)$ but $UL \notin E(S)$.

5. IRREGULAR ONE-TILE SEMIGROUP

We will include here another special kind of one-tile board of the shape **Y** consisting of six cells on the base set {1, 2, 3, 4, 5, 6}. We will assume again that the board has a single movable tile.

The *L, R, U, D* operators on the **Y**-shaped board are defined as follows:

$$L(B) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 2 & 2 & 5 & 6 \end{pmatrix} \qquad R(B) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 4 & 4 & 5 & 6 \end{pmatrix}$$

$$U(B) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 3 & 6 & 5 & 6 \end{pmatrix} \qquad D(B) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 1 & 4 & 2 & 4 \end{pmatrix}$$

$$LU(B) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 2 & 6 & 5 & 6 \end{pmatrix}$$

Proposition 4. The semigroup S generated by the operators on the **Y-** shaped board is irregular.

Proof. $LU(1) = L(2) = 2$. Let $W \in S^1$. Then $W(2) \in \{2, 4, 5, 6\}$. Hence

$(LU)W(LU)(1) = LUW(2) \subseteq LU\{2, 4, 5, 6\} = L\{5, 6\} = \{5, 6\}$. Since $2 \notin \{5, 6\}$ it follows that $(LU)W(LU) = LU, \forall W \in S^1$. Hence $LU \notin Reg(S)$.

Remark 5. We observe that the semigroups associated with the convex boards, for example, any **T, L-**shaped boards are all regular while the semigroup of **Y-**shaped board is irregular. Hence we conjecture that if the board is convex then the associated semigroup is regular.

Definition 6. A semigroup S is called aperiodic semigroup if all subgroups of S are trivial.

Conjecture. The one-tile semigroup on a board with no internal barrier is aperiodic.

We include an example of a non-aperiodic semigroup when internal barriers are allowed. In the Fig. 4 hard lines represent the barriers.

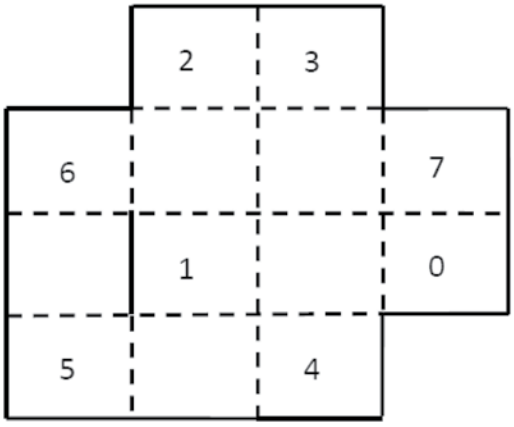

Fig. 4. A board with an internal barrier.

Since an internal barrier to the left of square 1 exists. We have

$$DRUL(0) = DRU(1) = DR(2) = D(3) = 4;$$
$$DRUL(4) = DRU(5) = DR(6) = D(7) = 0.$$

Hence, $DRUL(0) = 4$, $DRUL(4) = 0$ and so, $S = DRUL$ contains a nontrivial subgroup. Whence, the semigroup of this board is not aperiodic.

6. ONE-TILE SEMIGROUPS CORRESPONDING TO RECTANGLE OF ANY SIZE ARE ISOMORPHIC

Theorem 7. One-tile semigroups corresponding to any $m \times n$, $(2 \le m, n)$ rectangle are isomorphic

Proof. Let $2 \le m, n$. Consider a one-tile $m \times n$ rectangular board B as follows;

$B = \{(i, j) : 1 \le i \le m, 1 \le j \le n\}$.

We list the action of the four operators *L, R, U, D* on the board B as under;

$L(i, j) = (i, 1) = LL(i, j),\ i \leq m; \quad R(i, j) = (i, n) = RR(i, j),\ i \leq m;$

$U\ (i, j) = (1, j) = U\ U\ (i, j), j \leq n; \quad D(i, j) = (m, j) = DD(i, j), j \leq n,$

Similarly,

$LU\ (i, j) = (1, 1) = U\ L(i, j); \quad RU\ (i, j) = (1, n) = U\ R(i, j);$

$LD(i, j) = (m, 1) = DL(i, j); \quad RD(i, j) = (m, n) = DR(i, j).$

Since *L, R* commute with *U, D* we can write any word in the form $w = uv$ where $u \in \{L, R\}$ and $v \in \{U, D\}$. Further, since $\{L, R\}$ and $\{U, D\}$ are left-zero semigroups w equals one of the 8 listed elements in the one-tile semigroup *S*. Hence the semigroup *S* on a one-tile rectangular board has at most 8 elements *L, R, U, D, LU, RU, LD* and *RD*. Hence one-tile semigroups corresponding to any $m \times n,\ (2 \leq m, n)$ rectangle are isomorphic.

7. CONCLUSIONS

Finite semigroups are very rare in the literature of Semigroup theory. This work provides a variety of subsemigroups of the finite transformation semigorups in an amazing way while playing with tiles of irregular boards. We provide various several finite subsemigroups of different types.

8. ACKNOWLEDGEMENT

The author is very thankful to Prof. Dr. Peter M. Higgins for his helpful suggestions to improve this work.

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Proceedings of the Pakistan Academy of Sciences:
A. Physical and Compzutational Sciences 53 (3): 359 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Obituary

Prof. Dr. Nasir-ud-Din (1937–2016)

We are grieved on the sad demise of an eminent scientist and a senior Fellow of the Pakistan Academy of Sciences, Prof. Dr. Nasir-ud-Din, who breathed his last in Lahore on Thursday, 18th February, 2016. He was elected Fellow of the Pakistan Academy of Sciences in 1996.

Prof. Dr. Nasir-ud-Din was born in Amritsar, British India on 15th August, 1937. He obtained his BSc in 1955 and MSc in 1957 from University of the Punjab, Lahore. Later, he earned PhD from Edinburgh University, Scotland in 1963. In 1996, Prof. Dr. Nasir-ud-Din was conferred DSc by his alma mater, i.e., Edinburgh University, Scotland.

Prof. Dr. Nasir-ud-Din was Chairman, Institute of Molecular Sciences & Bioinformatics, Lahore since 2000, till his demise. Earlier, he had served as Professor of Biochemistry and Dean, Postgraduate Studies and Research, The Lahore University; Executive Director, Institute of Biomedical Sciences, Lahore; Adjunct Professor, H.E.J. Research Institute of Chemistry, University of Karachi, Karachi; Visiting Professor, Georgetown University, Washington, DC, USA; University of Geneva, Switzerland and Harvard University, Cambridge, USA. In University of Balochistan, Quetta, Prof. Nasir-ud-Din had served as Director, Institute of Biochemistry; Professor and Chairman, Department of Chemistry/ Biochemistry. He was Visiting Scientist, Institute National de la Sante et de la Recherche Medicale, Unite de Recherches sur la Biochimie des Proteines, Lille, France; Assistant Biochemist, Department of Medicine, Massachusetts General Hospital, Boston, USA; Research Fellow in Biochemistry, Massachusetts General Hospital, Boston, USA; Technical Director (Advisor), Johnson & Johnson Pakistan, Karachi; Chief Chemist and Manager Quality Control, Johnson & Johnson Pakistan, Karachi; Lecturer, University of the Punjab, Lahore; Research Fellow, Biological Chemistry, Harvard Medical School, Harvard University, Cambridge, USA; Demonstrator, Chemistry Department, Edinburgh University, Scotland; and Demonstrator, Institute of Chemistry, University of the Punjab, Lahore.

Prof. Dr. Nasir-ud-Din was Fellow of Pakistan Institute of Chemists; The Academy of Sciences for the Developing World (TWAS); Pakistan Society of Biochemistry and Molecular Biology; and Chemical Society of Pakistan. Also, he was Member of Pakistan Biological Sciences Society; Pakistan Institute of Chemists; and Chemical Society of Pakistan.

In recognition of his outstanding contributions in the field of chemistry/biochemisty, Prof. Dr. Nasir-ud-Din was conferred Edinburgh University, Studentship, 1961; Fulbright-Hays Award, 1963; Senior Fulbright-Hays Award, 1969; *Sitara-i-Imtiaz,* Govt. of Pakistan, 1991; Gold Medal (B. Khan), University of Balochistan, Quetta, 1992; Visiting Scientist Award, World Health Organization, 1989; and Visiting Scientist Award, National Medical Research Institute, France, 1982.

Prof. Dr. Nasir-ud-Din's areas of research were Biochemistry, Chemistry, Immunology of Complex Carbohydrates; Structure-Function Relationship of Glycoproteins; Secretory Proteins, Glycoproteins of Epithelial Cells and Cell Surface Carbohydrates; and Malaria Glycoproteins.

Prof. Dr. Nasir-ud-Din was a kind and humble person, and, despite health problems, remained committed to his scientific pursuits till his demise. In the death of Prof. Dr. Nasir-ud-Din, Pakistan has lost a committed and accomplished scientist. May Allah Almighty rest his soul in eternal peace and give fortitude to his family to bear this irreparable loss! Aameen.

Dr. Abdul Rashid, *FPAS*

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3. Kay, R.R. & C.R.L. Thompson. Forming patterns in development without morphogen gradients: differentiation and sorting. *Cold Spring Harbor Perspectives in Biology* 1: doi: 10.1101/cshperspect.a001503 (2009).

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6. Sarnthein, M.S. & J.D. Stanford. Basal sauropodomorpha: historical and recent phylogenetic developments. In: *The Northern North Atlantic: A Changing Environment.* Schafer, P.R. & W. Schluter (Ed.), Springer, Berlin, Germany, p. 365–410 (2000).
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