Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 53 (4): 459–467 (2016)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Matter Wave Travelling Dark Solitons in a Coupled Bose-Einstein Condensate

Muhammad Irfan Qadir* and Sumreen Naz

Department of Mathematics, University of Engineering & Technology,
Lahore 54890, Pakistan

Abstract: We investigate the existence and stability of matter wave travelling coupled dark solitons in two effectively one-dimensional parallel coupled Bose-Einstein condensates. The system can be described by linearly coupled Gross-Pitaevskii equations. In particular, we have examined the effects of changing the value of coupling strength between the condensates over the stability of travelling coupled dark solitons. It is found that the travelling coupled dark solitons are unstable but the instability of the solutions can be defeated by having a control on the coupling strength.

Keywords: Bose Einstein condensate, Bose-Josephson junction, dark solitons, Josephson effect, stability.

1. INTRODUCTION

In the last decade of the 20th century, one of the magnificent and successful achievements in the field of quantum physics was the realization of Bose-Einstein condensates (BEC) of alkali atoms [1, 2]. The first prediction about BEC was made in early 1920s by Bose and Einstein. The atoms of BEC follow Bose statistics and are linked with the essential physical phenomenon such as superconductivity in metals and superfluidity in helium [3, 4].

Dilute atomic BEC is substantially a nonlinear system that possesses the solitary wave solutions. A soliton is a localized wave which strength itself and keeps its original form unchanged when moving with fixed velocity. Solitons originate due to the balance of dispersive and nonlinear effects in the medium. They can be either bright as localized height or dark as localized depth on a continuous background. The velocity of a soliton is directly associated with its height or depth as if it is either a bright or a dark soliton respectively. Individual solitons can collide and remain unchanged in velocity, amplitude and shape but possibly not for phase shift [5].

The study of matter wave dark solitons has been a delightful area of research. The criterion for the one dimensional dynamical stability of matter wave dark soliton was presented in [6]. The snake instability was suppressed by tightly encompassing the motion in the radial direction and keeping the mean field interaction of atoms smaller than the frequency. The investigation of vortices in BEC were exhibited both theoretically and experimentally in [7].

The notion of tunneling of electrons between two superconductors linked by a very thin insulator [8] was extended to the tunneling of atoms in BEC by Smerzi et al. [9, 10, 14] and is known as the Josephson

Received, February 2016; Accepted, October 2016
*Corresponding author: Muhammad Irfan Qadir; Email: mirfan@uet.edu.pk

tunneling. The experimental realization of such tunneling for a single and a collection of small Bose-Josephson junction was presented in [12]. The concept of Bose-Josephson junction was extended to long Bose-Josephson junction [13, 14]. This idea was similar to long superconducting Josephson junction. It was suggested in [13] that atomic vortices can be viewed in weakly coupled BEC and that these atomic vortices are identical to Josephson fluxon in superconducting long Josephson junction [15]. Furthermore, it was depicted that these atomic vortices can be reversibly transformed to dark soliton and the transformation can be controlled through coupling strength.

In this study, we investigate the existence and stability of travelling dark solitons in two cigar-shaped coupled BEC. Specifically, we study the effects of variation in the value of coupling parameter on the stability of matter wave travelling dark solitons moving with a particular velocity in BEC.

The paper is formatted as follows. In section 2, we consider the coupled system of nonlinear Schrodinger equations describing BEC and find the matter wave travelling coupled dark soliton solution numerically. In section 3, we discuss the stability of the travelling soliton solution while changing the coupling strength. We conclude our results in section 4.

2. MATHEMATICAL MODEL AND DESCRIPTION

We consider a system of two parallel cigar-shaped coupled BEC with the repulsive intra atomic interactions. The system can be described by two one-dimensional coupled nonlinear Schrodinger equations which can be written as

$$i\frac{\partial Z_1}{\partial t} = -\frac{1}{2}\frac{\partial^2 Z_1}{\partial X^2} + \mu|Z_1|^2 Z_1 - \omega Z_1 - \gamma Z_2 , \tag{1}$$

$$i\frac{\partial Z_2}{\partial t} = -\frac{1}{2}\frac{\partial^2 Z_2}{\partial X^2} + \mu|Z_2|^2 Z_2 - \omega Z_2 - \gamma Z_1 , \tag{2}$$

where Z_1 and Z_2 denote the wave functions of atoms of two BEC. The variables X and t represent respectively the space and time variables. μ is the nonlinearity coefficient and γ is the coupling strength between the condensates. ω is the chemical potential which is the rate of change of energy with respect to the number of atoms. Both γ and ω can be controlled experimentally using different techniques. Typically, they can be controlled by using a combination of lasers of different intensities.

Since the soliton solutions of equations (1) and (2) are translationally invariant, this property of translational invariance motivated us to study the existence and stability of matter wave dark soliton in a moving coordinate frame of reference. So, we substitute $x = X - vt$ in equations (1) and (2) to obtain

$$i\frac{\partial Z_1}{\partial t} = -\frac{1}{2}\frac{\partial^2 Z_1}{\partial x^2} + \mu|Z_1|^2 Z_1 - \omega Z_1 - \gamma Z_2 + iv\frac{\partial Z_1}{\partial x} , \tag{3}$$

$$i\frac{\partial Z_2}{\partial t} = -\frac{1}{2}\frac{\partial^2 Z_2}{\partial x^2} + \mu|Z_2|^2 Z_2 - \omega Z_2 - \gamma Z_1 + iv\frac{\partial Z_2}{\partial x} , \tag{4}$$

where v represents the velocity of the soliton solutions.

For the steady state solutions, we substitute $\frac{\partial Z_1}{\partial t} = 0 = \frac{\partial Z_2}{\partial t}$ in equations (3) and (4) and acquire

$$-\frac{1}{2}\frac{\partial^2 Z_1}{\partial x^2} + \mu |Z_1|^2 Z_1 - \omega Z_1 - \gamma Z_2 + iv\frac{\partial Z_1}{\partial x} = 0\,, \tag{5}$$

$$-\frac{1}{2}\frac{\partial^2 Z_2}{\partial x^2} + \mu |Z_2|^2 Z_2 - \omega Z_2 - \gamma Z_1 + iv\frac{\partial Z_2}{\partial x} = 0\,. \tag{6}$$

Since Z_1 and Z_2 are complex, we substitute $Z_1 = a_1 + ib_1$ and $Z_2 = a_2 + ib_2$ in equations (5) and (6) and after equating real and imaginary parts on both sides, we get the following equations

$$-\frac{1}{2}\frac{\partial^2 a_1}{\partial x^2} + \mu\left({a_1}^3 + a_1{b_1}^2\right) - \omega a_1 - \gamma a_2 - v\frac{\partial b_1}{\partial x} = 0\,, \tag{7}$$

$$-\frac{1}{2}\frac{\partial^2 b_1}{\partial x^2} + \mu\left({b_1}^3 + {a_1}^2 b_1\right) - \omega b_1 - \gamma b_2 + v\frac{\partial a_1}{\partial x} = 0\,, \tag{8}$$

$$-\frac{1}{2}\frac{\partial^2 a_2}{\partial x^2} + \mu\left({a_2}^3 + a_2{b_2}^2\right) - \omega a_2 - \gamma a_1 - v\frac{\partial b_2}{\partial x} = 0\,, \tag{9}$$

$$-\frac{1}{2}\frac{\partial^2 b_2}{\partial x^2} + \mu\left({b_2}^3 + {a_2}^2 b_2\right) - \omega b_2 - \gamma b_1 + v\frac{\partial a_2}{\partial x} = 0\,. \tag{10}$$

We discretize equations (7), (8), (9) and (10) to obtain

$$-\frac{1}{2}\left(\frac{a_{1,j-1} - 2a_{1,j} + a_{1,j+1}}{h^2}\right) + \mu\left({a_{1,j}}^3 + a_{1,j}{b_{1,j}}^2\right) - \omega a_{1,j} - \gamma a_{2,j} - v\left(\frac{b_{1,j+1} - b_{1,j-1}}{2h}\right) = 0\,, \tag{11}$$

$$-\frac{1}{2}\left(\frac{b_{1,j-1} - 2b_{1,j} + b_{1,j+1}}{h^2}\right) + \mu\left({b_{1,j}}^3 + {a_{1,j}}^2 b_{1,j}\right) - \omega b_{1,j} - \gamma b_{2,j} + v\left(\frac{a_{1,j+1} - a_{1,j-1}}{2h}\right) = 0\,, \tag{12}$$

$$-\frac{1}{2}\left(\frac{a_{2,j-1} - 2a_{2,j} + a_{2,j+1}}{h^2}\right) + \mu\left({a_{2,j}}^3 + a_{2,j}{b_{2,j}}^2\right) - \omega a_{2,j} - \gamma a_{1,j} - v\left(\frac{b_{2,j+1} - b_{2,j-1}}{2h}\right) = 0\,, \tag{13}$$

$$-\frac{1}{2}\left(\frac{b_{2,j-1} - 2b_{2,j} + b_{2,j+1}}{h^2}\right) + \mu\left({b_{2,j}}^3 + {a_{2,j}}^2 b_{2,j}\right) - \omega b_{2,j} - \gamma b_{1,j} + v\left(\frac{a_{2,j+1} - a_{2,j-1}}{2h}\right) = 0\,, \tag{14}$$

where $j = 1, 2, \dots, N$. The equations (11), (12), (13) and (14) represent a nonlinear system of algebraic equations. We employ Newton's method with the Neumann boundary conditions $Z_{n,0} = Z_{n,1}$ and $Z_{n,N} = Z_{n,N+1}$, $n = 1, 2$, to get the travelling coupled dark soliton solutions as depicted in Fig. (1).

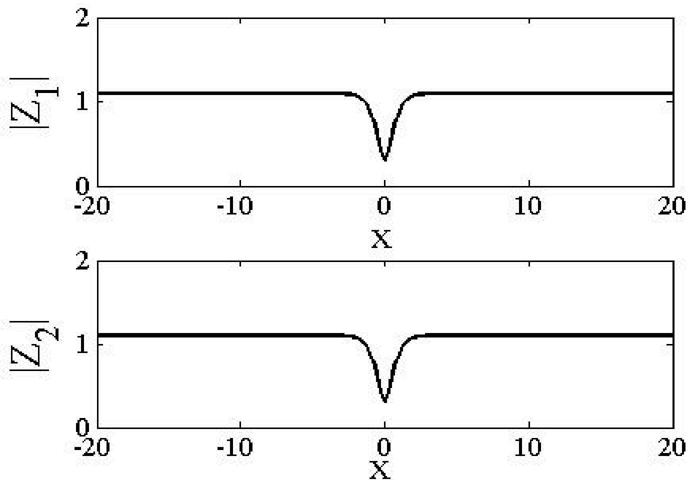

Fig. 1. Travelling Coupled dark soliton solution obtained numerically for the parameter values $\mu = 1$, $\omega = 1$, $\gamma = 0.2$ and $v = 0.3$.

3. STABILITY OF TRAVELLING COUPLED DARK SOLITONS

For investigating the stability of travelling coupled dark solitons, we first assume that $Z_1^{(0)}(x)$ and $Z_2^{(0)}(x)$ are the steady state solutions of system of equations (3) and (4). We add very small perturbations $p_1(x,t)$ and $p_2(x,t)$ in these solutions $Z_1^{(0)}$ and $Z_2^{(0)}$ respectively, i.e.

$$Z_1(x,t) = {Z_1}^{(0)}(x) + p_1(x,t), \tag{15}$$

$$Z_2(x,t) = {Z_2}^{(0)}(x) + p_2(x,t). \tag{16}$$

We substitute the values of $Z_1(x,t)$ and $Z_2(x,t)$ in equations (3) and (4) and after doing linearization, we obtain

$$i\frac{\partial p_1}{\partial t} = -\frac{1}{2}\frac{\partial^2 p_1}{\partial x^2} + \mu\left({Z_1}^{(0)}\right)^2\overline{p_1} + 2\mu\left|{Z_1}^{(0)}\right|^2 p_1 - \omega p_1 - \gamma p_2 + iv\frac{\partial p_1}{\partial x}, \tag{17}$$

$$i\frac{\partial p_2}{\partial t} = -\frac{1}{2}\frac{\partial^2 p_2}{\partial x^2} + \mu\left({Z_2}^{(0)}\right)^2\overline{p_2} + 2\mu\left|{Z_2}^{(0)}\right|^2 p_2 - \omega p_2 - \gamma p_1 + iv\frac{\partial p_2}{\partial x}. \tag{18}$$

Here bar denotes the complex conjugate. Taking complex conjugate of equations (17) and (18), we get

$$-i\frac{\partial \overline{p_1}}{\partial t} = -\frac{1}{2}\frac{\partial^2 \overline{p_1}}{\partial x^2} + \mu\overline{({z_1}^{(0)})^2}p_1 + 2\mu\left|{Z_1}^{(0)}\right|^2\overline{p_1} - \omega\overline{p_1} - \gamma\overline{p_2} - iv\frac{\partial \overline{p_1}}{\partial x}, \tag{19}$$

$$-i\frac{\partial \overline{p_2}}{\partial t} = -\frac{1}{2}\frac{\partial^2 \overline{p_2}}{\partial x^2} + \mu\overline{({z_2}^{(0)})^2}p_2 + 2\mu\left|{Z_2}^{(0)}\right|^2\overline{p_2} - \omega\overline{p_2} - \gamma\overline{p_1} - iv\frac{\partial \overline{p_2}}{\partial x}. \tag{20}$$

For the sake of simplicity, we substitute $p_1 = \delta_1$, $\overline{p_1} = \sigma_1$, $p_2 = \delta_2$, $\overline{p_2} = \sigma_2$ in equations (17), (18), (19), (20) and obtain

$$i\frac{\partial \delta_1}{\partial t} = -\frac{1}{2}\frac{\partial^2 \delta_1}{\partial x^2} + \mu\left(Z_1{}^{(0)}\right)^2\sigma_1 + 2\mu\left|Z_1{}^{(0)}\right|^2\delta_1 - \omega\delta_1 - \gamma\delta_2 + iv\frac{\partial \delta_1}{\partial x} = \lambda\delta_1\,, \tag{21}$$

$$i\frac{\partial \delta_2}{\partial t} = -\frac{1}{2}\frac{\partial^2 \delta_2}{\partial x^2} + \mu\left(Z_2{}^{(0)}\right)^2\sigma_2 + 2\mu\left|Z_2{}^{(0)}\right|^2\delta_2 - \omega\delta_2 - \gamma\delta_1 + iv\frac{\partial \delta_2}{\partial x} = \lambda\delta_2\,, \tag{22}$$

$$i\frac{\partial \sigma_1}{\partial t} = \frac{1}{2}\frac{\partial^2 \sigma_1}{\partial x^2} - \mu(\overline{Z_1{}^{(0)}})^2\delta_1 - 2\mu\left|Z_1{}^{(0)}\right|^2\sigma_1 + \omega\sigma_1 + \gamma\sigma_2 + iv\frac{\partial \sigma_1}{\partial x} = \lambda\sigma_1\,, \tag{23}$$

$$i\frac{\partial \sigma_2}{\partial t} = \frac{1}{2}\frac{\partial^2 \sigma_2}{\partial x^2} - \mu(\overline{Z_2{}^{(0)}})^2\delta_2 - 2\mu\left|Z_2{}^{(0)}\right|^2\sigma_2 + \omega\sigma_2 + \gamma\sigma_1 + iv\frac{\partial \sigma_2}{\partial x} = \lambda\sigma_2\,, \tag{24}$$

where the scalar λ represents the eigenvalues. We discretize the above four equations to get

$$-\frac{1}{2}\left(\frac{\delta_{1,j-1} - 2\delta_{1,j} + \delta_{1,j+1}}{h^2}\right) + \mu\left(Z_{1,j}{}^{(0)}\right)^2\sigma_{1,j} + 2\mu\left|Z_{1,j}{}^{(0)}\right|^2\delta_{1,j} - \omega\delta_{1,j} - \gamma\delta_{2,j} + iv\left(\frac{\delta_{1,j+1} - \delta_{1,j-1}}{2h}\right) = \lambda\delta_{1,j}, \tag{25}$$

$$-\frac{1}{2}\left(\frac{\delta_{2,j-1} - 2\delta_{2,j} + \delta_{2,j+1}}{h^2}\right) + \mu\left(\overline{Z_{2,j}{}^{(0)}}\right)^2\sigma_{2,j} + 2\mu\left|Z_{2,j}{}^{(0)}\right|^2\delta_{2,j} - \omega\delta_{2,j} - \gamma\delta_{1,j} + iv\left(\frac{\delta_{2,j+1} - \delta_{2,j-1}}{2h}\right) = \lambda\delta_{2,j}, \tag{26}$$

$$\frac{1}{2}\left(\frac{\sigma_{1,j-1} - 2\sigma_{1,j} + \sigma_{1,j+1}}{h^2}\right) - \mu(\overline{Z_{1,j}{}^{(0)}})^2\delta_{1,j} - 2\mu\left|Z_{1,j}{}^{(0)}\right|^2\sigma_{1,j} + \omega\sigma_{1,j} + \gamma\sigma_{2,j} + iv\left(\frac{\sigma_{1,j+1} - \sigma_{1,j-1}}{2h}\right) = \lambda\sigma_{1,j}, \tag{27}$$

$$\frac{1}{2}\left(\frac{\sigma_{2,j-1} - 2\sigma_{2,j} + \sigma_{2,j+1}}{h^2}\right) - \mu\left(\overline{Z_{2,j}{}^{(0)}}\right)^2\delta_{2,j} - 2\mu\left|Z_{2,j}{}^{(0)}\right|^2\sigma_{2,j} + \omega\sigma_{2,j} + \gamma\sigma_{1,j} + iv\left(\frac{\sigma_{2,j+1} - \sigma_{2,j-1}}{2h}\right) = \lambda\sigma_{2,j}, \tag{28}$$

where $j = 1, 2, \ldots, N$. Applying the Neumann boundary conditions $\delta_{n,0} = \delta_{n,1}$ and $\sigma_{n,N} = \sigma_{n,N+1}$, $n = 1, 2$, the above system of equations (25), (26), (27) and (28) can be written as an eigenvalue problem

$$CY = \lambda Y,$$

where

$$\mathrm{C}=\begin{bmatrix} C_1 & -E & D_1 & 0 \\ -E & C_2 & 0 & D_2 \\ -\overline{D_1} & 0 & C_3 & E \\ 0 & -\overline{D_2} & E & C_4 \end{bmatrix},$$

$$C_1=\begin{bmatrix} \frac{1}{2h^2}+2\mu|Z_{1,1}^{(0)}|^2-\omega-\frac{iv}{2h} & \frac{-1}{2h^2}+\frac{iv}{2h} & 0 & 0 & \dots & 0 \\ \frac{-1}{2h^2}-\frac{iv}{2h} & \frac{1}{h^2}+2\mu|Z_{1,2}^{(0)}|^2-\omega & \frac{-1}{2h^2}+\frac{iv}{2h} & 0 & \dots & 0 \\ 0 & \frac{-1}{2h^2}-\frac{iv}{2h} & \frac{1}{h^2}+2\mu|Z_{1,3}^{(0)}|^2-\omega & \frac{-1}{2h^2}+\frac{iv}{2h} & \dots & 0 \\ & \ddots & \ddots & \ddots & & \\ 0 & 0\dots & \frac{-1}{2h^2}-\frac{iv}{2h} & & & \frac{1}{2h^2}+2\mu|Z_{1,N}^{(0)}|^2-\omega+\frac{iv}{2h} \end{bmatrix},$$

$$C_2=\begin{bmatrix} \frac{1}{2h^2}+2\mu|Z_{2,1}^{(0)}|^2-\omega-\frac{iv}{2h} & \frac{-1}{2h^2}+\frac{iv}{2h} & 0 & 0 & \dots & 0 \\ \frac{-1}{2h^2}-\frac{iv}{2h} & \frac{1}{h^2}+2\mu|Z_{2,2}^{(0)}|^2-\omega & \frac{-1}{2h^2}+\frac{iv}{2h} & 0 & \dots & 0 \\ 0 & \frac{-1}{2h^2}-\frac{iv}{2h} & \frac{1}{h^2}+2\mu|Z_{2,3}^{(0)}|^2-\omega & \frac{-1}{2h^2}+\frac{iv}{2h} & \dots & 0 \\ & \ddots & \ddots & \ddots & & \\ 0 & 0\dots & \frac{-1}{2h^2}-\frac{iv}{2h} & & & \frac{1}{2h^2}+2\mu|Z_{2,N}^{(0)}|^2-\omega+\frac{iv}{2h} \end{bmatrix},$$

$$C_3=\begin{bmatrix} -\frac{1}{2h^2}-2\mu|Z_{1,1}^{(0)}|^2+\omega-\frac{iv}{2h} & \frac{1}{2h^2}+\frac{iv}{2h} & 0 & 0 & \dots & 0 \\ \frac{1}{2h^2}-\frac{iv}{2h} & -\frac{1}{h^2}-2\mu|Z_{1,2}^{(0)}|^2+\omega & \frac{1}{2h^2}+\frac{iv}{2h} & 0 & \dots & 0 \\ 0 & \frac{1}{2h^2}-\frac{iv}{2h} & -\frac{1}{h^2}-2\mu|Z_{1,3}^{(0)}|^2+\omega & \frac{1}{2h^2}+\frac{iv}{2h} & \dots & 0 \\ & \ddots & \ddots & \ddots & & \\ 0 & 0\dots & \frac{1}{2h^2}-\frac{iv}{2h} & & & -\frac{1}{2h^2}-2\mu|Z_{1,N}^{(0)}|^2+\omega+\frac{iv}{2h} \end{bmatrix},$$

$$C_4=\begin{bmatrix} \frac{-1}{2h^2}-2\mu|Z_{2,1}^{(0)}|^2+\omega-\frac{iv}{2h} & \frac{1}{2h^2}+\frac{iv}{2h} & 0 & 0 & \dots & 0 \\ \frac{1}{2h^2}-\frac{iv}{2h} & \frac{-1}{h^2}-2\mu|Z_{2,2}^{(0)}|^2+\omega & \frac{1}{2h^2}+\frac{iv}{2h} & 0 & \dots & 0 \\ 0 & \frac{1}{2h^2}-\frac{iv}{2h} & \frac{-1}{h^2}-2\mu|Z_{2,3}^{(0)}|^2+\omega & \frac{1}{2h^2}+\frac{iv}{2h} & \dots & 0 \\ & \ddots & \ddots & \ddots & & \\ 0 & 0\dots & \frac{1}{2h^2}-\frac{iv}{2h} & & & \frac{-1}{2h^2}-2\mu|Z_{2,N}^{(0)}|^2+\omega+\frac{iv}{2h} \end{bmatrix},$$

$$D_1 = \begin{bmatrix} \mu\left(Z_{1,1}^{(0)}\right)^2 & 0 & 0 & \dots & 0 \\ 0 & \mu\left(Z_{1,2}^{(0)}\right)^2 & 0 & \dots & 0 \\ 0 & 0 & \mu\left(Z_{1,3}^{(0)}\right)^2 & \dots & 0 \\ & & \ddots & & \\ & & & \ddots & \\ 0 & 0 & 0 & \dots & \mu\left(Z_{1,N}^{(0)}\right)^2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} \mu\left(Z_{2,1}^{(0)}\right)^2 & 0 & 0 & \dots & 0 \\ 0 & \mu\left(Z_{2,2}^{(0)}\right)^2 & 0 & \dots & 0 \\ 0 & 0 & \mu\left(Z_{2,3}^{(0)}\right)^2 & \dots & 0 \\ & & \ddots & & \\ & & & \ddots & \\ 0 & 0 & 0 & \dots & \mu\left(Z_{2,N}^{(0)}\right)^2 \end{bmatrix},$$

$$E = \begin{bmatrix} \gamma & 0 & 0 & \dots & 0 \\ 0 & \gamma & 0 & \dots & 0 \\ 0 & 0 & \gamma & \dots & 0 \\ & & & \ddots & \\ & & & & \ddots \\ 0 & 0 & 0 & \dots & \gamma \end{bmatrix}.$$

The solution will be stable if all the eigenvalues are real. But, if, atleast one of the eigenvalues is imaginary, the solution will be unstable.

The eigenvalues of the stability matrix C are evaluated and are depicted in Fig. 2. It is easy to see that a few of the eigenvalues are lying vertically while all the remaining eigenvalues are lying horizontally. The eigenvalues lying vertically shows that the travelling coupled dark soliton is unstable. For the verification of the results obtained, we perform the numerical integration of the system of equations (3) and (4) by perturbing the solution shown in Fig. 1. In particular, the numerical integration is done by applying the fourth order Runge-Kutta method. The contour plot of the time evolution of travelling coupled dark soliton is shown in Fig.3. The radiation are emerging at nearly $t = 35$ and reveals that the solution is unstable which justifies the results already obtained. The instability causes the solution to move away from the centre. Moreover, the density of the atoms in one of the panels go on increasing with time.

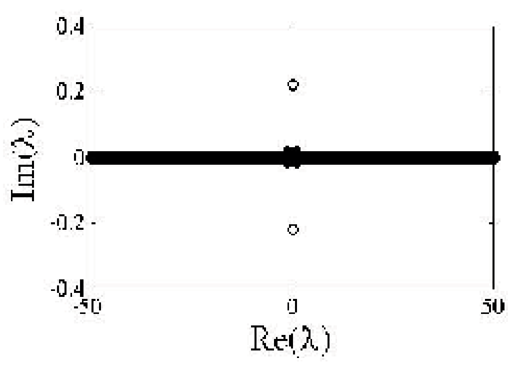

Fig. 2. The layout of eigenvalues for the solution presented in Fig. 1. Some of the eigenvalues are not on the horizontal axis and indicate the instability of the solution.

We then investigate the stability of the travelling coupled dark soliton for different values of velocity v. It is observed that the critical value γ_c of the coupling parameter γ at which the solution becomes stable varies with v. When $v = 0$, the critical value of the coupling strength is 1/3. This agrees with the result in [13] and is shown in Fig. (4) by brown dotted curve. For different nonzero values of v, we find the critical values γ_c by plotting the stability curves as displayed in Fig. (4). One can see that γ_c decreases with v and tends to zero as v goes to 1. The graph of γ_c versus v is shown in Fig. (5). The travelling coupled dark soliton exists and is unstable below the curve while it is stable above the curve in its domain of existence. This means that the instability of travelling coupled dark soliton can be managed by having a control over the coupling strength.

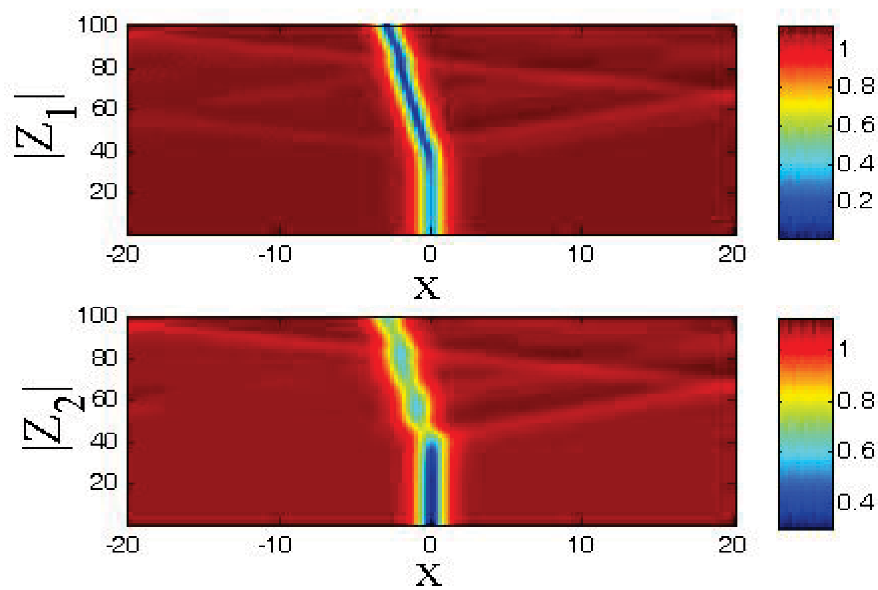

Fig. 3. The contour plot for the time evolution of the solution shown in Fig. (1). Radiation are emerging and the solution shifts away from the centre due to instability.

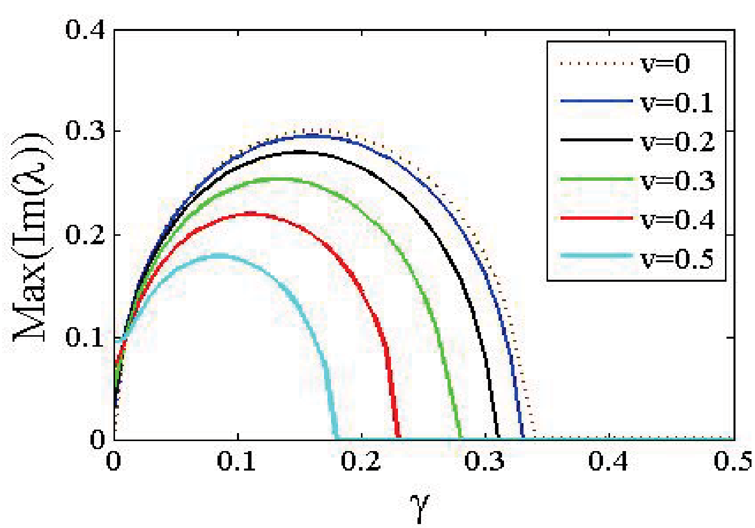

Fig. 4. The graph of coupling strength versus the maximum value of the imaginary parts of the eigenvalues corresponding to different values of velocity. The dotted curve is for zero velocity and shows that the value of critical coupling is 1/3. The other curves correspond to the non zero velocity and depict that the value of critical coupling decreases with velocity.

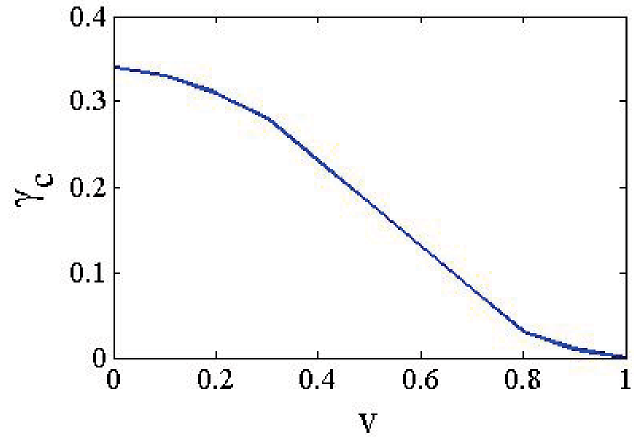

Fig. 5. The graph of velocity versus the corresponding values of critical coupling γ_c. Below the curve, the coupled dark soliton solutions exist and are unstable, while they are stable above the curve in their domain of existence.

4. CONCLUSIONS

In this paper, we have examined the existence and stability of matter wave travelling coupled dark solitons in two quasi one-dimensional parallel coupled BEC. It has been found that the travelling coupled dark solitons moving with velocity v exist for $v \leq 1$. The stability of travelling coupled dark soliton solutions has been investigated while varying the value of coupling strength. The region in the $v\gamma_c$ -plane was determined in which the travelling coupled dark solitons were found to be unstable. However, the instability of travelling dark soliton can be controlled through the coupling strength.

5. REFERENCES

1. Anderson, M.H., J.R. Ensher, M.R. Matthews, C.E. Wieman & E.A. Cornell. Observation of Bose-Einstein condensation in a dilute atomic vapor. *Science* 269: 198-201 (1995).
2. Davis, K.B., M. -O. Mewes, M. R. Andrews, N.J. van Druten, D. S. Durfee, D. M. Kurn & W. Ketterle. Bose-Einstein condensation in a gas of sodium atoms. *Physical Review Letters* 75: 3969-3973 (1999).
3. Griffin, A., D. W. Snoke & S. Stringari. *Bose- Einstein Condensation.* Cambridge University Press, UK (1995).
4. Pethick C.J., & H. Smith. *Bose- Einstein Condensation in Dilute Gases.* Cambridge University Press, UK (2001).
5. Burger, S., K. Bongs, S. Dettmer, W. Ertmer, K. Sengstock, A. Sanpera, G. V. Shlyapnikov & M. Lewenstein. Dark solitons in Bose-Einstein condensates. *Physical Review Letters* 83: 5198-5201 (1999).
6. Muryshev, A.E., H.B. Heuvell & G.V. Shlyapnikov. Stability of standing matter waves in a trap. *Physical Review A* 60: R2665-R2668 (1999).
7. Fetter, A.L., & A.A. Svidzinsky. Vortices in a trapped dilute BEC. *Journal of Physics: Condensed Matter* 13: R135-R194 (2001).
8. Josephson, B.D.. Possible new effects in superconductive tunneling. *Physics Letters* 1: 251-253 (1962).
9. Smerzi, A., S. Fantoni, S. Giovanazzi & S.R. Shenoy. Quantum coherent atomic tunneling between two trapped Bose-Einstein condensates. *Physical Review Letters* 79: 4950-4953 (1997).
10. Raghavan, S., A. Smerzi, S. Fantoni & S. R. Shenoy. Coherent oscillations between two weakly coupled Bose-Einstein condensates: Josephson effects, π oscillations, and macroscopic quantum self-trapping. *Physical Review A* 59: 620-633 (1999).
11. Giovanazzi, S., A. Smerzi & S. Fantoni. Josephson effects in dilute Bose-Einstein condensates. *Physical Review Letters* 84: 4521-4524 (2000).
12. Cataliotti, F.S., S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi & M. Inguscio. Josephson junction arrays with Bose-Einstein condensates. *Science* 293: 843-846 (2001).
13. Kaurov, V.M., & A. B. Kuklov. Josephson vortex between two atomic Bose-Einstein condensates. *Physical Review A* 71: 011601-011604 (2005).
14. Kaurov V.M., & A. B. Kuklov. Atomic Josephson vortices. *Physical Review A* 73: 013627 (2006).
15. Ustinov, A.V.. Solitons in Josephson junctions. *Physica D: Nonlinear Phenomena* 123: 315-329 (1998).