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Research Article

Radiation and Buoyancy Effects on MHD Stagnation Point Flow of Micropolar Fluids due to a Porous Stretching Sheet

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Abstract: Computational investigation for buoyancy and radiation effects on the magneto hydrodynamics stagnation point flow of micropolar fluids has been made to study the kinematics and temperature distribution under the influence of virtual parameter of physical importance. The similarity functions have been used to obtain the ordinary differential equations for non-linear partial differential model of the problem. The results have been computed by Mathematica software and presented the graphs of microrotation, temperature and velocity.

Keywords: Micropolar fluids, radiation, Buoyancy effect, stretching sheet, similarity function

1. INTRODUCTION

Eringen [1] presented the theory of the micropolar fluids which considers the generalization of Navier-Stokes equation and it takes into account the conservation of angular momentum due to local micromotion of the fluid particles. Researchers are engaged to explore innovative results related to micropolar fluids flow problems. Sajjad et al. [2] investigated hydromagnetic micropolar fluid flow between two parallel plates, the lower plate is stretching. Barika and Dashb [3] studied the peristaltic motion of incompressible micropolar fluid through a porous medium in a two-dimensional symmetric channel. Vimala and Omega [4] analysed the steady laminar two-dimensional flow of a micropolar fluid through a porous channel with variable permeability. The magnetohyderodynamic (MHD) viscous flow of micropolar fluid over a shrinking sheet has been solved numerically by Shafique [5].Shafique et al. [6] investigated the magnetohydrodynamic viscous flow due to a shrinking sheet. Hafidz et al. [7] studied the steady magnetohydrodynamics rotating boundary layer flow and heat transfer of a viscous fluid over a permeable shrinking sheet. Adhikari et al, [8] worked on steady two-dimensional incompressible magnetohtdrodynamics micropolar fluid flow towards a stretching or shrinking vertical sheet under suction or blowing with prescribed surface heat flux. Santosh et al. [9] considered the unsteady two-dimensional, laminar flow of a viscous, incompressible, electrically conducting fluid towards a shrinking surface in the presence of a uniform transverse magnetic field. Santosh et al. [10] carried out the investigation for two-dimensional flow of aviscous incompressible electrically conducting fluid near a stagnation point of a stretching or shrinking surface in a saturated porous medium. Jena [11] considered a steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over a shrinking sheet in the presence of uniform transverse magnetic field with viscous dissipation. Sandeep et al. [12] analysed the influence of thermal radiation and chemical reaction on two dimensional steady magnetohydrodynamic flow of a nanofluid past a permeable stretching/shrinking sheet in the presence of suction/injection.

Reddy [13] analysed the effects of buoyancy and magneto hydrodynamic force on convective heat and mass transfer flow past a moving vertical porous plate in the presence of thermal radiation and chemical reaction. Babu et al. [14] obtained numerical solution to study the effects radiation and heat source/sink on the steady two dimensional magneto hydrodynamic (MHD) boundary layer flows of heat

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and mass transfer past a shrinking sheet with wall mass suction. Kishore et al. [15] described the incompressible viscous hydromagnetic flow in a porous medium in the presence of radiation, variable heat and viscous dissipation and mass diffusion. Poornima et al. [16] studied the mathematical solution of a steady convective flow of boundary-layer fluid flow of a radiating combined nanofluid to a non-linear boundary moving sheet in presence of transverse magnet field. Sharma et al. [17] investigated the heat transfer due to exponentially shrinking sheet in the existence of the thermal radiation among mass suction of the boundary layer flow of a viscous fluid.

In this article, radiation and buoyancy effects on MHD stagnation point flow of micropolar fluids due to a porous stretching sheet have been studied numerically. A comparison of the results for Newtonian and micropolar fluids has been made to extend the results of Yahaya and Simon [18]. Moreover, radiative heat effect is also observed on the thermal characteristics of the problem. The computations have been made using a very straight forward scheme in Mathematica 10.

2. MATHEMATICAL MODEL

We assume the micropolar fluid is incompressible and electrically conducting. The flow is laminar and two dimensional. The flow is due to permeable stretching surface with two equal and opposite forces along x-axis. A uniform magnetic field acts perpendicular to the surfaces. The pressure gradient, the body couple, induced magnetic field and viscous dissipation are negligible. The velocity vector is $\underline{V} = V(u,v)$ and Spin vector is $\underline{\omega} = \omega(0,0,\omega_3)$ and fluid temperature is *T*. Where $T_w(x)$ is temperature at the boundary of the surface.

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$(\mu + k)\frac{\partial^2 u}{\partial y^2} + k\frac{\partial \omega_3}{\partial y} - \sigma B_0^2(x)u + g\rho(T - T_\infty) = \rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) \tag{2}$$

$$\gamma(\frac{\partial^2 \omega_3}{\partial y^2}) - \kappa(\frac{\partial u}{\partial y} + 2\omega_3) = \rho j\left(u\frac{\partial \omega_3}{\partial x} + v\frac{\partial \omega_3}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T - T_\infty)}{\rho C_p} + \frac{\beta^* u}{\rho C_p}(T_\infty - T) + \frac{16\sigma^*}{3\rho C_p k^*}T^3{}_\infty\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where ρ is density, σ is the electrical conductivity, c_p is the specific heat capacity at constant pressure, μ is dynamic viscosity, *k* and γ are additional viscosity coefficients for micropolar fluid and *j* is micro inertia ,α is the thermal diffusivity, $\beta^* u\ (T_\infty - T)$ and $Q_0(T - T_\infty)$ are heat generated or absorbed per unit volume, B_0 is the applied magnetics induction, g is the acceleration due to gravity, *a* is the stretching rate (a constant), k^* is as mean absorption coefficient, σ^* Stefan Boltzmann constant, β is the coefficient of thermal expansion and μ_0 is magnetic permeability.

The boundary conditions are:

$$\begin{aligned} &\omega_3(x,0) = 0,\ u(x,0) = ax, \quad v(x,0) = v_w, \quad T(x,0) = T_w = T_\infty + A_0 x \\ &\omega_3(x,\infty) = 0\,, u(x,\infty) = 0, \quad T(x,\infty) = T_\infty \end{aligned} \tag{5}$$

v_w is the suction/injection.

Using similarity transformations:

The velocity components are described in terms of the stream function Ψ (x, y):

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

$$\psi(x,y)=x\sqrt{a\upsilon}\,f(\eta),\ \eta=y\sqrt{\frac{a}{\upsilon}}$$

$$u=xaf',\ v=-\sqrt{\upsilon a}\,f,\ \omega_3=\frac{a^{\frac{3}{2}}}{\upsilon^{\frac{1}{2}}}xL(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_s-T_\infty}$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4) we get

$$(1+d_1)f'''+d_1L'-H_af'+Gr\theta=f'^2-ff'' \tag{6}$$

$$d_3L''+2d_1d_2L-d_1d_2f''=f'L-fL' \tag{7}$$

$$\left(\frac{N_r+1}{P_r}\right)\theta''+\Delta\theta+f\theta'-(1+B)f'\theta=0 \tag{8}$$

and the boundary conditions are

$$\begin{aligned}&f'(0)=1,\ f(0)=R,\ L(0)=0,\ \theta(0)=1,\\&f'(\infty)=0,\ L(\infty)=0,\ \theta(\infty)=0,\end{aligned} \tag{9}$$

Whereas $H_a=\left(\frac{\sigma}{\rho a}\right)^{\frac{1}{2}}B_0$ is the Hartmann number, $G_r=g\beta\left(\frac{T_w-T_\infty}{a^2x}\right)$ is Grashof number, $P_r=\frac{\upsilon}{\alpha}$ is Prandtl number, $N_r=\frac{16\sigma^*T^3_\infty}{3k^*k}$ is thermal radiation, $B=\frac{\beta^*x}{\rho C_p}$ is heat generation coefficient and $\Delta=\frac{Q_0}{\rho C_pa}$

The dimensional less material constantsare.

$$d_1=\frac{k}{\mu},\ d_2=\frac{\mu}{\rho ja},\ d_3=\frac{\gamma}{\rho j\upsilon}$$

3. RESULTS AND DISCUSSION

The resulting highly non-linear mathematical model of the problem as given in equations (6) to (9) does not offer analytical solution. The higher order of these ordinary differential equations is reduced and then a numerical scheme has been developed in Mathematica software using classical Runge-Kutta method. The effects of the pertinent parameters have been studied on microrotation, temperature and fluid velocity. The results for skin friction coefficient $-f''(0)$ are presented in table 1. It is noticed that the magnitude of $-f''(0)$ is lesser for micropolar fluids than for Newtonian fluids. The results coincide with physical nature of the fluids. The comparison of the present results with the previous results as given by [18] is presented in table 2. The results compare very well.

Table 1. The values of skin friction coefficient -$f''(0)$ for parameters. $R=0.1, H_a=0.1, \Pr=0.71, G_r=0.1$

$f''(0)$		Parameters	$f''(0)$		Parameters
Newtonian fluid	**Micropolar fluids**		**Newtonian fluid**	**Micropolar fluid**	
0.175465	0.178963	R=0.1	0.450183	0.337531	R =0.4
0.260273	0.230792	R =0.2	0.175465	0.171963	H_a =0.1
0.346483	0.283666	R =0.3	0.275435	0.252024	H_a =0.2

Table 2. Comparison of present results with previous results of Yahya & Simon [18] for $-\theta'(0)$.

Parameter	R=0.45 B=0.5	R=0.45 B=1	R=0 B=0.5	R=0 B=1	R=-1.5 B=0.5	R=-1.5 B=1
Present results	0.824194	0.96081	0.946984	1.07789	1.57169	1.66078
Yahya & Simon [18]	0.82396	0.96190	0.94765	1.07895	1.57077	1.66182

Graphical pattern of the physical quantities of flow and heat transfer are presented. The Grashof number causes increase in the flow as indicated in fig.1, the velocity f' increases with G_r. But magnetic parameter slows down the fluid motion as shown in fig.2, the velocity f' decreases as H_a increases.Fig.3 displays the velocity f' under the effect of suction/injection. It is noticed that injection causes an increase in the fluid flow but increase in suction decreases the flow. The fluid velocity f' is lesser in magnitude for Newtonian fluids (d_1=0) than for micropolar fluids. Also the increase in micropolar parameter causes increase in f' as depicted in Fig. 4.

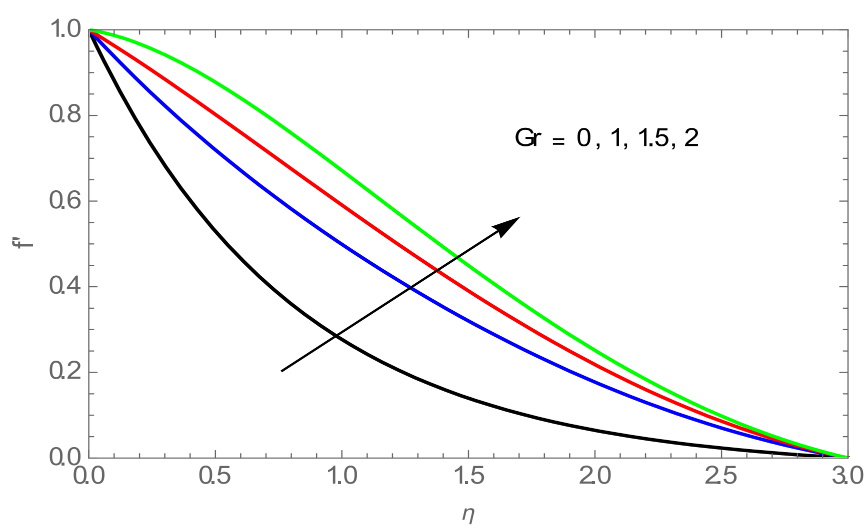

Fig. 1. The plot for curves of f' under the effect of Gr.

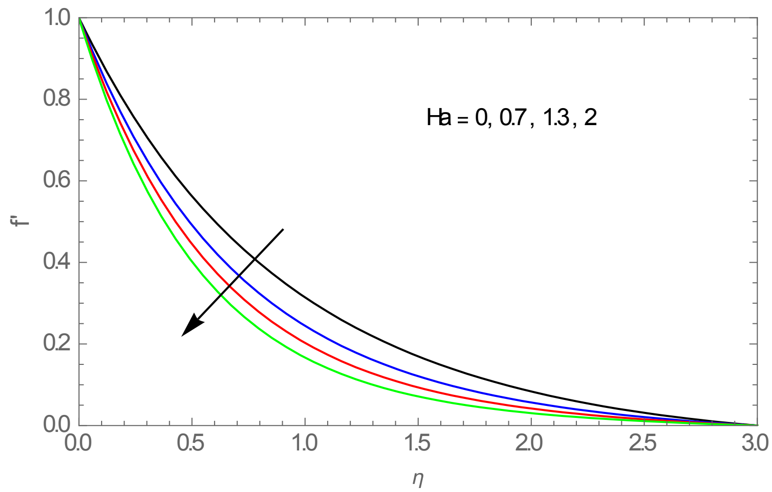

Fig. 2. The plot for curves of f' under the effect of . H_a

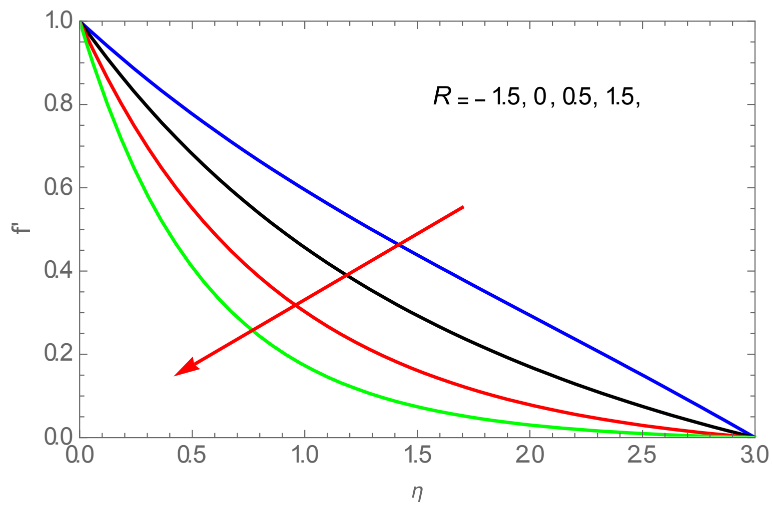

Fig. 3. The plot for curves of f' under the effect of suction/injection R.

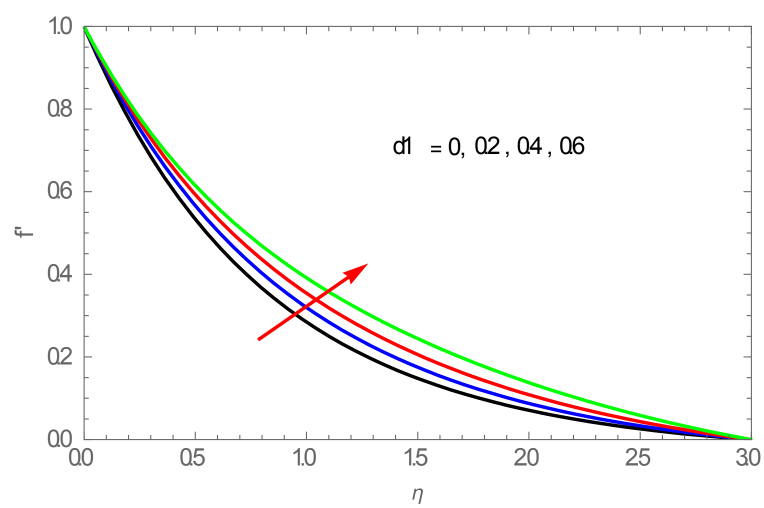

Fig. 4. The plot for curves of f' under the effect of d_1.

The microrotation *L* increases near the surface boundary and decreases with increase in the value of H_a as plotted in Fig 5. Fig. 6 shows that increase in the values of d_1 (micropolar parameter), increases the microrotation *L*.

Fig. 7 and Fig. 8, respectively, demonstrate that the temperature $\theta(\eta)$ increases with increase in radiation parameter and it decreases with increase in Prandtl number. Fig. 9 plots the temperature under the influence of suction/injection parameter R. The temperature distribution $\theta(\eta)$ decrease with injection. But it increases with increase in suction. In Fig.10, it is noticed that the temperature decreases with increase in the value of parameter *B*.

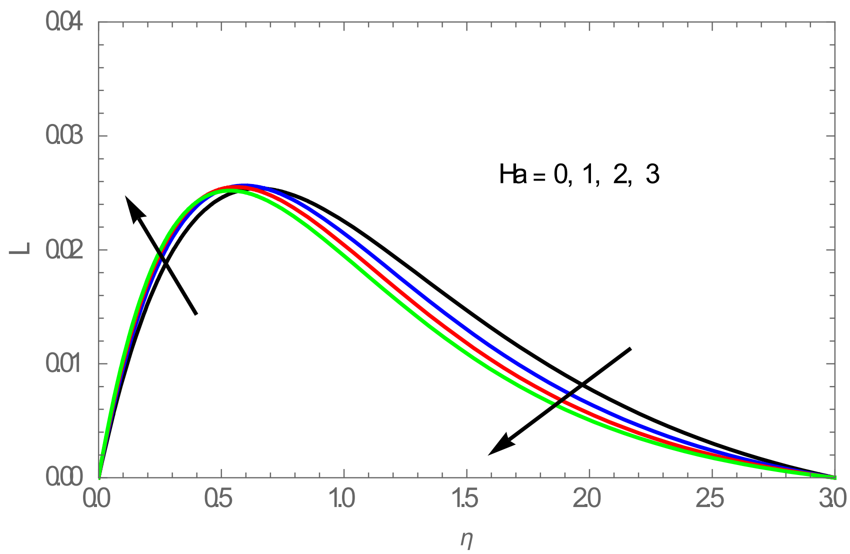

Fig. 5. The plot for curves of $L(\eta)$ under the effect of H_a.

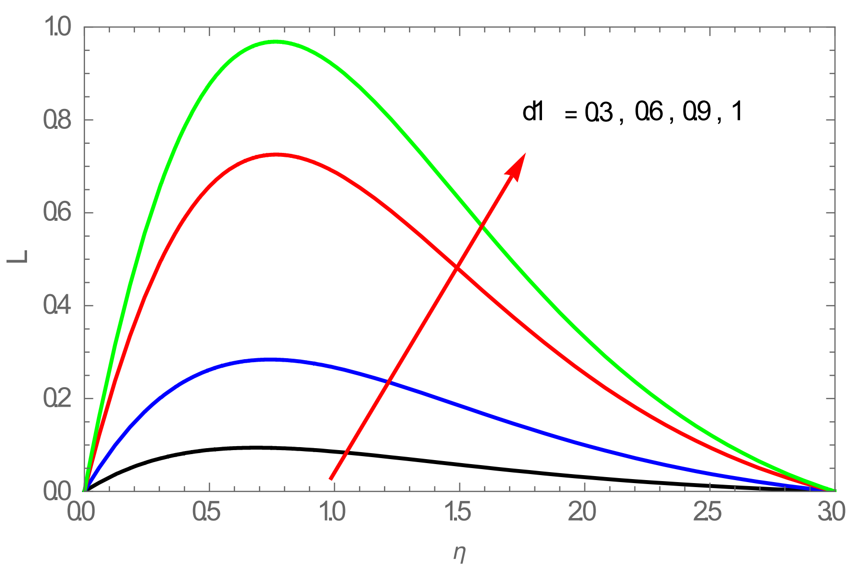

Fig. 6. The plot for curves of $L(\eta)$ under the effect of d_1.

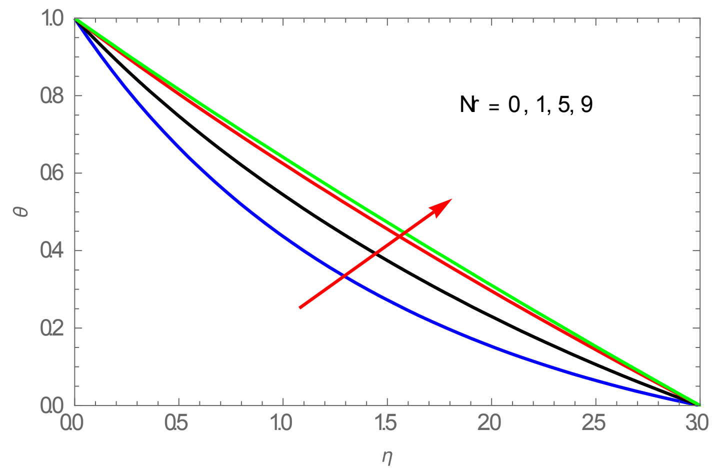

Fig. 7. The plot for curves of $\theta(\eta)$ under the effect of N_r .

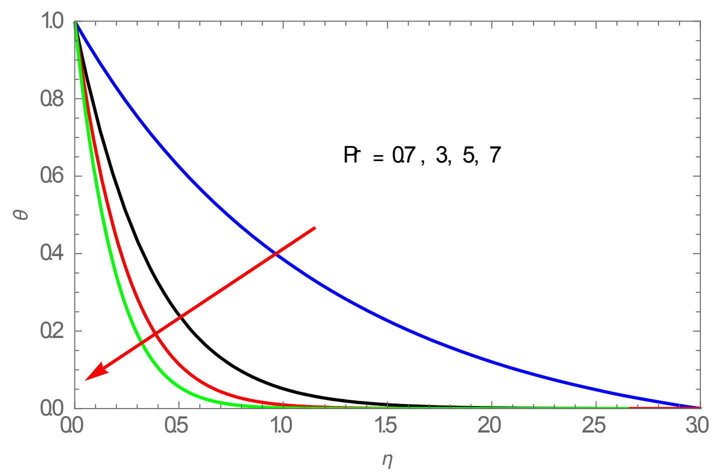

Fig. 8. The plot for curves of $\theta(\eta)$ under the effect of P_r .

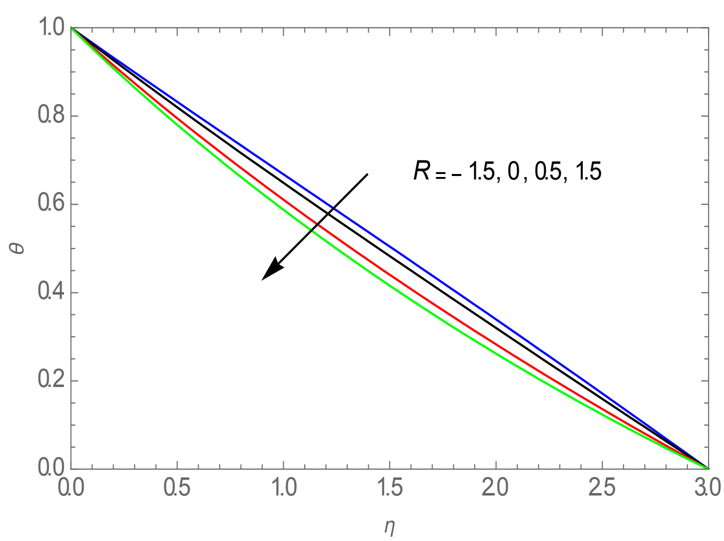

Fig. 9. The plot for curves of $\theta(\eta)$ under the effect of suction/injection *R*.

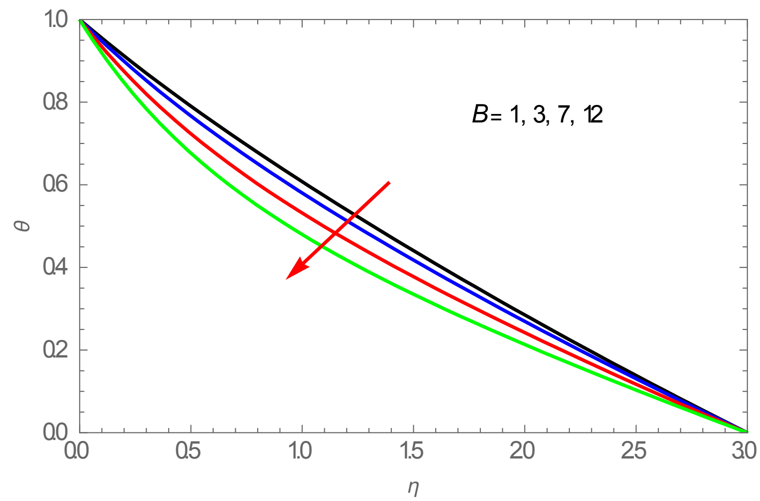

Fig. 10. The plot for curves of $\theta(\eta)$ under the effect of parameter *B*.

4. CONCLUSIONS

Main findings of the study are summarized as below:

- The Grashof number causes increase in velocity f'
- The magnetic parameter H_a slows down the fluid motion.
- The fluid velocity f' is less in magnitude for Newtonian fluids (d_1=0) than for micropolar fluids and the increase in micropolar parameter causes increase in f'.
- The microrotation *L* increases near the surface boundary and decreases with increase in the value of H_a.
- The micropolar parameter d_1, increases the microrotation *L*.
- The temperature $\theta(\eta)$ increases with increase in radiation parameter and decrease with Prandtl number.
- The temperature distribution $\theta(\eta)$ decrease with injection. But it increases with suction.
- The temperature decreases with increase in the value of parameter *B*.

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