Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 54 (2): 167–178 (2017)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

Sensitivity and Generalized Sensitivity Studies of the SIR and SEIR Models of Computer Virus

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Abstract: In this article, the sensitivity and generalized sensitivity analyses of the SIR and SEIR models of the dynamics of computer virus is presented. From the sensitivity studies of the SIR model, it follows that both the parameters in the model affect the model output in the beginning. The sensitivity of the SEIR model shows that the susceptible computers in the network are affected majorly by the rate at which external computers are connected to the network and the recovery rate of the susceptible computer due to the anti-virus ability of the network. From the generalized sensitivity of the SIR model, it follows that both the infected rate and the recovery rate are sensitive in the beginning and are highly correlated. The generalized sensitivities of the SEIR model show that the recovery rate of the infected computers that are cured is insensitive with respect to the measurements from all compartments.

Keywords: Computer virus, self-replication, reproductive ratio, infectivity, sensitivity, generalized sensitivity

1. INTRODUCTION

Worldwide, there are 3,179,03,200 internet users as of August 07, 2015. They represent more than 40 percent of the population of the world. Their largest number is in China followed by the United States and India. The total number of the websites with a unique host name online increased from one website (info.cern.ch) in 1991 to one billion websites in September, 2014. This huge network aimed at sharing useful information in the form of hypertext amongst the users is open for everyone. In this network, any computer can communicate with any other computer as long as they are both connected to the Internet. However, there are many which make wrong use of this facility and share such computer codes which are self-replicating and causing severe damages to the computer community and are called the so-called computer virus.

A biological virus is an infective agent that consists of a nucleic acid molecule in a protein coat and is too small to be seen by the light microscope and has the ability to multiply only within the living cells of a host. For example, "hepatitis B virus". A computer virus is a piece of code which is capable of copying itself and typically has a detrimental effect like corrupting the system or destroying data. Computer viruses include worms, trojan horses, ransomware, spyware, adware, scareware, etc.

The first academic work on the theory of self-replicating computer programs was done in 1949 by John von Neumann. His work was published as the "Theory of self-reproducing automata", and described how a computer program could be designed to reproduce itself. Von Neumann's design for a self-reproducing computer program is considered the world's first computer virus, and he is considered to be

Received, December 2016; Accepted, April 2017
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the theoretical father of computer virology [1]. In 1980, Kraus [2] postulated that computer programs can behave in a way similar to biological viruses. Creeper virus was first detected on ARPANET, the forerunner of the internet, in the early 1970s [3]. Before the start of the computer networks, many users regularly exchanged information and programs on removable media like floppies, discs, magnetic tapes and magnetic drums, etc., so the computer virus spread from one media to another. Traditional computer viruses emerged in the 1980s, driven by the spread of personal computers. Then after the advent of the internet in 1991, their transmission became rampant from computer to computer through messaging. In the same year, the first mathematical model of the dynamics of the computer virus was proposed based on the work of Kermack and Mc Kendrick [4-7]. Several models explaining the dynamics of the computer virus were proposed which gave description of the computer virus dynamical system in the form of parameters. However, there has been no work made over the sensitivity of these parameters of the computer virus as yet.

Sensitivity analysis is a very useful tool for the analysis of mathematical models. In the analysis of such models, it is of great importance to understand how small variations in the parameters affect the model output. Precisely, the researchers need to evaluate the sensitivities of the model variables with respect to the parameters. It is desired to obtain the maximum benefit from the measurements or observations. In particular, the input to the system should be such that it maximizes the sensitivity of the state variable to parameters [8]. In the context of the measurements, Generalized Sensitivity Functions (GSFs) describe the effects of the measurements over the estimates of the parameters. The GSFs have the benefit that they show the correlation between the parameters as well.

To take initiative of this work, the author has selected two such widely-studied models by the experts in the field for their sensitivity studies. In Section 2, the two models SIR and SEIR are described briefly, and an overview of the sensitivity and the generalized sensitivity functions is given. Section 3 describes the numerical scheme of the method applied. Major results and conclusions of the study are given in Section 4 and Section 5, respectively.

2. MATERIAL AND METHODS

Two well-studied models of computer virus dynamics and the theory of the sensitivity and the generalized sensitivity functions are presented in this section.

2.1. Computer Virus Models

At any time a computer is classified as either internal or external depending on weather it is connected to internet or not respectively. The two models of dynamics of computer virus are described as under:

2.1.1. SIR Model

In the SIR model, all of the internet computers are further categorized into three classes:

i. *Susceptible computers*, i.e., uninfected computers and new computers which are connected to network,

ii. *Infectious computers*,

iii. *Removed computer*, the computers which are broken-out or were removed from the network.

Now if their corresponding number $S(t)$, $I(t)$ and $R(t)$ at any time t, be denoted simply by S, I and R, then the SIR model is given by the following system of differential equations:

$$S' = \rho SI,$$
$$I' = \rho SI - \beta I, \quad (1)$$
$$R' = \beta I,$$

With initial conditions $(S(0), I(0), R(0)) = (S_0, I_0, R_0)$. The total population $S + I + R$ is considered to be constant. The parameter ρ denotes the infected rate or the rate of interactions between susceptible and infected computers and β denotes the rate of the removing process.

The basic reproduction number (basic reproductive ratio, or incorrectly basic reproductive rate and denoted by R_0,) of the SIR model is defined by:

$$R_0 = \frac{\rho S_0}{\beta} \quad (2)$$

When $R_0 < 1$, the infection will die out in the long run and if $R_0 > 1$, the infection will spread in a population.

2.1.2. SEIR Model

In this model, all of the internet computers are further categorized into four classes at any time t, namely:

i. *Susceptible computers*, that is, uninfected computers and new computers
which connected to the network,

ii. *Exposed computers*, i.e., infected but not yet broken-out,

iii. *Infectious computers*, i.e., the ones which spread virus in the population,

iv. *Recovered computers*, i.e., the virus-free computers having immunity.

Let $S(t)$, $E(t)$, I(t), $R(t)$ denote their corresponding numbers at time t, without ambiguity; if they are abbreviated by S, E, I, R, respectively, then the SEIR model is formulated as the following system of differential equations:

$$S' = (1 - p)N - \beta_1 SI - \beta_2 SE - pS - \mu S,$$
$$E' = \beta_1 SI + \beta_2 SE - KE - \alpha E - \mu E,$$
$$I' = \alpha E - rI - \mu I, \quad (3)$$
$$R' = pS + KE + rI,$$

with initial conditions $(S(0),\ E(0).\ I(0),\ R(0)) = (S_0,\ E_0, I_0, R_0)$. The total population $S + E + I + R$ is considered to be constant. Since the first three equations in this Model (3) are independent of the fourth equation, therefore we will omit the fourth equation for our numerical calculations.

In the model (3), N represents the rate at which external computers are connected to the net-work, p is the recovery rate of susceptible computer due to the anti-virus ability of network, K is the recovery rate of exposed computer due to the anti-virus ability of the network, β_1 shows the rate at which, when having a connection to one infected computer, one susceptible computer can become exposed but has not broken-out; β_2 shows the rate of which, when having connection to one exposed computer, one susceptible computer can become exposed, α represents the rate of the exposed computers cannot be cured by anti-virus software and broken-out, r is the recovery rate of infected computers that are cured and μ represents the rate at which one computer is removed from the network.

The basic reproductive ratio of the SEIR model is:

$$R_0 = \frac{(1 - p)N(\beta_1 \alpha + \beta_2 (r + \mu))}{(p + \mu)(K + \alpha + \mu)(r + \mu)}$$

2.2 Sensitivity and Generalized Sensitivity Functions

A multiple-output system with the measurable outputs is given by:

$$f(t,\theta) = col(f_1(t,\theta), \cdots, f_M(t,\theta)), \quad 0 \le t \le T, \ \theta \in U \tag{4}$$

where $T > 0$ and the open subset $U \in R^p$ of admissible parameters are fixed. It is assumed that the output model (4) is a valid description of the system for all $t \in [0,T]$ and the open set $U \in R^p$ and the component outputs $f_i, i = 1, \dots, M$, are sufficiently smooth functions. The vector θ is the vector of parameters which is to be estimated.

The following two concepts are vital for our work in the sequel.

2.1.3. Sensitivity Functions

The sensitivity or sensitivity functions of any model output $f_i(t;\theta)$ with respect to parameter component θ_π is defined by:

$$s_{\frac{f_i}{\theta_\pi}}(t) = \lim_{\Delta\theta_\pi \to 0} \frac{\Delta f_i(t,\theta)/f_i(t,\theta)}{\Delta\theta_\pi/\theta_\pi}, \quad \pi = 1, \cdots, p, \quad i = 1, \dots, M.$$

This can be written as

$$s_{\frac{f_i}{\theta_\pi}}(t) = \frac{\partial f_i(t,\theta)}{\partial\theta_\pi} \cdot \frac{\theta_\pi}{f_i(t,\theta)}, \quad \pi = 1, \cdots, p, i = 1, \cdots, M \tag{5}$$

Here both θ_π , $\pi = 1, \cdots, p$ and $f_i(t,\theta)$, $i = 1, \cdots, M$ are assumed to be non-zero. The sensitivity functions quantify the effects of the changes in the parameters on the outputs of the model. They describe to which parameters the model output is the most or least sensitive.

The sensitivity of the system with respect to a component of the parameter vector at any time instant t in $[0, T]$ which combines the sensitivities of all model outputs $f_i, i = 1, \dots, M$, is defined by

$$S_{\theta_\pi} = \left(\sum_{i=1}^{M} \left(s_{\frac{f_i}{\theta_\pi}}(t)\right)^2\right)^{\frac{1}{2}}, \quad \pi = 1, \dots, p \tag{6}$$

2.1.4. Generalized Sensitivity Functions

In order to define the generalized sensitivity function for the multiple-output system, it is assumed that for each component f_i, of the output vector f, the measurements are taken at the sample times

$$0 \le t_1^{(i)} < \dots < t_1^{(Ni)} \le T, \quad i = 1, \dots, M.$$

The measurements are assumed to be of the form

$$y_i\left(t_j^{(i)}\right) = f_i\left(t_j^{(i)}, \theta_0\right) + \varepsilon_i\left(t_j^{(i)}\right), \quad j = 1, \dots, N_i. \tag{7}$$

where θ_0 is the `true' or nominal parameter vector and $\varepsilon_i(t_j^{(i)})$ is the measurement error which is assumed to be a representation at $t_j^{(i)}$ of some noise process $\mathcal{E}_i$(t). The measurement (7) for the whole interval [0, T] is extended by

$$y_i(t) = f_i(t,\theta_0) + +\varepsilon_i(t), \quad 0 \le t \le T, \ i = 1, \dots, M$$

$\mathcal{E}_i(t)$ are presumed to be independent and identically distributed having zero mean and constant variance $\sigma_i(t)^2$. In order to estimate the parameter vector θ i.e., to get $\hat{\theta}(\theta_0)$, the least-squares error functional $J(\theta)$ for the multiple-output problem is formulated as:

$$J(\theta) = \sum_{I=1}^{M} J_i\,(\theta), \quad \theta \in U, \tag{8}$$

where

$$J_i(\theta) = \sum_{j=1}^{N_i} \frac{1}{\sigma_i\left(t_j^{(i)}\right)^2}\left(y_i\left(t_j^{(i)}\right) - f_i\left(t_j^{(i)},\theta\right)\right)^2, \quad i = 1,\ldots,M, \quad \theta \in U \tag{9}$$

so that

$$\hat{\theta} = \underset{\theta \in U}{argmin}\, J(\theta)$$

Let $0 \le t_1 < \cdots < t_N \le T$ be the sequence of all measurement times, i.e.,

$$\{t_1,\cdots,t_N\} = \bigcup_{i=1}^{M}\left\{t_1^{(i)},\cdots,t_{N_i}^{(i)}\right\}$$

Each $t_j^{(i)}$ is contained in $\{t_1,\cdots,t_N\}$, $t_j^{(i)} = t_{k(i,j)}$, $j = 1,\ldots,N_i$, $i = 1,\ldots,M$, where $1 \le k(i,1) < \cdots < k(i,N_i) \le N$, $i = 1,\cdots,M$. Each t_k is contained in at least one sequence $\left\{t_1^{(i)},\cdots,t_{N_i}^{(i)}\right\}$. We define the $N \times N$ diagonal matrices $A^{(i)} = (\alpha_{\mu,\nu}^{(i)})_{\mu,\nu=1,\ldots,N}$ by

$$\alpha_{\mu,\nu}^{(i)} = \begin{cases} 1 \; for\ \mu = \nu = k(i,j), & j = 1,\ldots,N_i \\ 0 \quad \text{otherwise} \end{cases}$$

Let the $N \times N$-matrices $D^{(i)}$be defined by $D^{(i)}=\left(\frac{1}{\sigma_i(t_1)^2},\cdots,\frac{1}{\sigma_i(t_N)^2}\right)$, $i = 1,\ldots,M$. From [9], the generalized sensitivity matrix G for the parameter estimation problem is found as

$$G(t_k,\theta_0) = \left(F(\theta_0)\right)^{-1}F_k(\theta_0), \qquad k = 1,\ldots,N, \tag{10}$$

Where

$$F(\theta_0) = \sum_{i=1}^{M}\left(\nabla F_i(\theta_0)\right)^T D^{(i)}A^{(i)}\nabla F_i(\theta_0), \qquad \theta \in U \tag{11}$$

The matrix $F(\theta_0)$ is called the Fisher information matrix for our parameter estimation problem. If this matrix is singular then the parameter estimation problem for the linearized problem does not have a unique solution [10], and

$$F_k(\theta_0) = \sum_{i=1}^{M} F_k^i\,(\theta_0), \qquad k = 1,\cdots,N, \tag{12}$$

with

$$F_k^i(\theta_0) = \sum_{i=1}^{M}\left(\nabla F_k^i(\theta_0)\right)^T D_k^{(i)}A_k^{(i)}\nabla F_k^i(\theta_0), \quad i = 1,\cdots,M.$$

The main diagonal elements, $g_\pi(t_k,\theta_0)$ of the matrix $G(t_k\,,\theta_0)$ as a function of t_k are called the generalized sensitivity function for the parameter components θ_π, $\pi = 1,\cdots,p$.

$$g_\pi(t_k,\theta_0) = (G(t_k\,,\theta_0))_{\pi,\pi} \quad ,\pi = 1,\cdots,p, \qquad \theta \in U \tag{13}$$

The GSFs give the following information concerning the dependence of the parameter estimates on the measurement of an output variable [10].

i. Information on the correlation between parameters with respect to measurements for a specific output variable of the system. Oscillatory and monotonic behavior of the GSFs indicates a strong correlation between the parameters. A more or less monotonic increase of the GSFs from 0 to 1 indicates little correlation amongst the parameters.

ii. Information on the relative content of information carried by the measurements at different times for the parameters. If the GSF is monotonically increasing for a parameter, then the measurements taken in that time interval possess all information about the parameter in that interval, whereas the measurements taken outside that time interval are more or less irrelevant for the parameter.

The GSFs i.e., $g_\pi(t_k,\theta_0)$, $\pi = 1,\dots,p$, are defined at each time interval and $g_\pi(t_k,\theta_0) = 0$ for $t < t_1$ and $g_\pi(t_k,\theta_0) = 1$ for $t \geq t_M$. There is a transition of the GSFs from 0 to 1 for a single parameter, the GSFs are monotonic increasing [10]. For a detailed study of the GSFs, one may consult Kappel et al. [9] and Munir [10].

3. NUMERICAL IMPLEMENTATION

The *matlab* function, ode45 is used to find the solutions of the Models (1) and (3) respectively with the true value of the parameters $\theta_0 = (0.1, 10)$ as $\rho = 0.1$ and β =10 and true value of θ_0 as

N	p	K	β_1	β_2	α	r	μ
0.7	0.001	0.02	0.09	0.04	10	0.002	0.003

The initial conditions are respectively $(S_0, I_0, R_0) = (95, 2, 3)$ and $(S_0, E_0, I_0) = (95, 4, 1)$ over the time interval $[0, T]$, $T = 20$ days, for a network consisting of 100 computers each. The number of model outputs is taken as two, $M = 2$ i.e., $f_1(t,\theta) = S(t,\theta)$ and $f_2(t,\theta) = R(t,\theta)$ in the Model (1). The measurements of $S(t)$ and $R(t)$ are taken respectively as $y_1(t) = S(t,\theta_0) + \varepsilon_1(t)$ and $y_2(t) = R(t,\theta_0) + \varepsilon_2(t)$. Similarly, for the model (3), only the measurements of $S(t)$ and $I(t)$ are considered as a test case because practically taking measurements for all $S(t)$, $E(t)$, $I(t)$, $R(t)$ together is costly and makes the problem more complex. The measurement processes are assumed to have zero mean and unit variance in both the cases. The cost functional given by Equation (8) for the both the models is formed as per the theoretical framework described in sub-sub-section 2.2.2 above.

4. RESULTS

The results on the sensitivity and generalized sensitivity studies of the SIR and SEIR models are given in the paragraphs

4.1. Sensitivities of the SIR Model

The sensitivity functions and system sensitivity functions defined respectively by the Equations (5) and (6) are plotted for the SIR model. The sensitivities of the model output S and R with respect to the parameter ρ and β are drawn in the Fig. 1. It is evident that the output S and R changes with respect to these parameters up to 5 days. After this, the changes in these parameters bring no changes in the

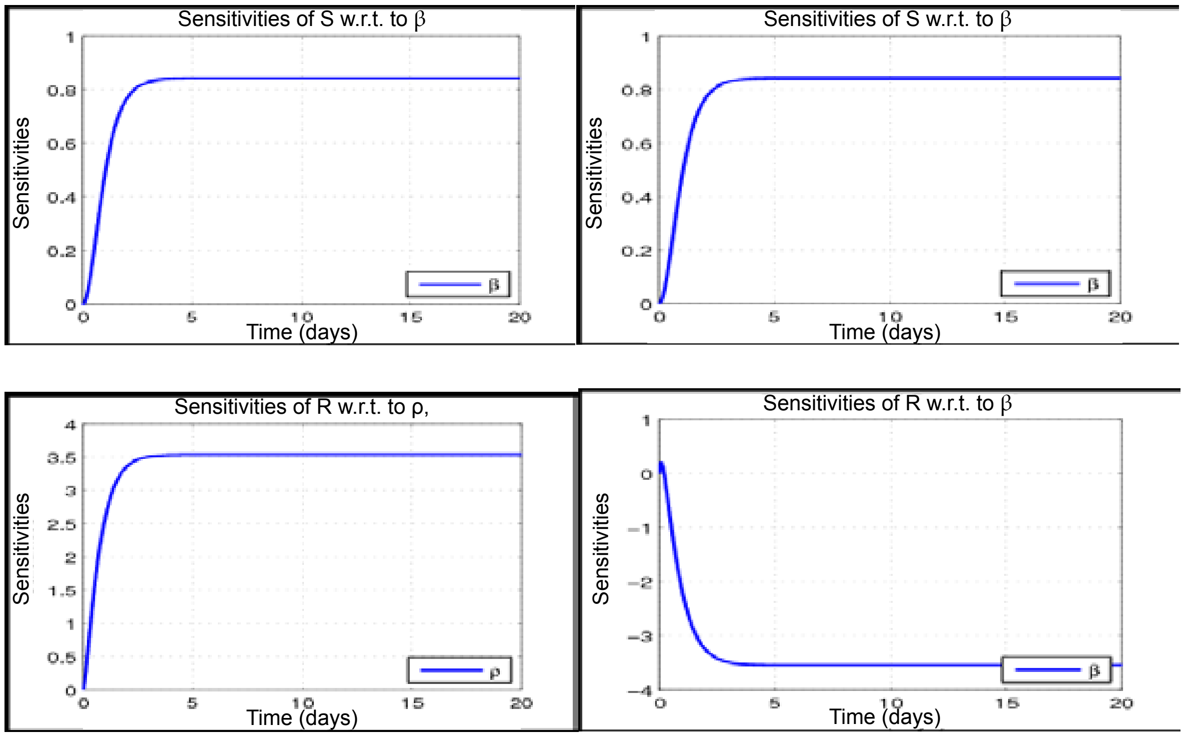

Fig. 1. Sensitivity functions of S with respect to ρ and β(upper-panel), and R with respect to ρ and β (lower-panel) for the SIR model.

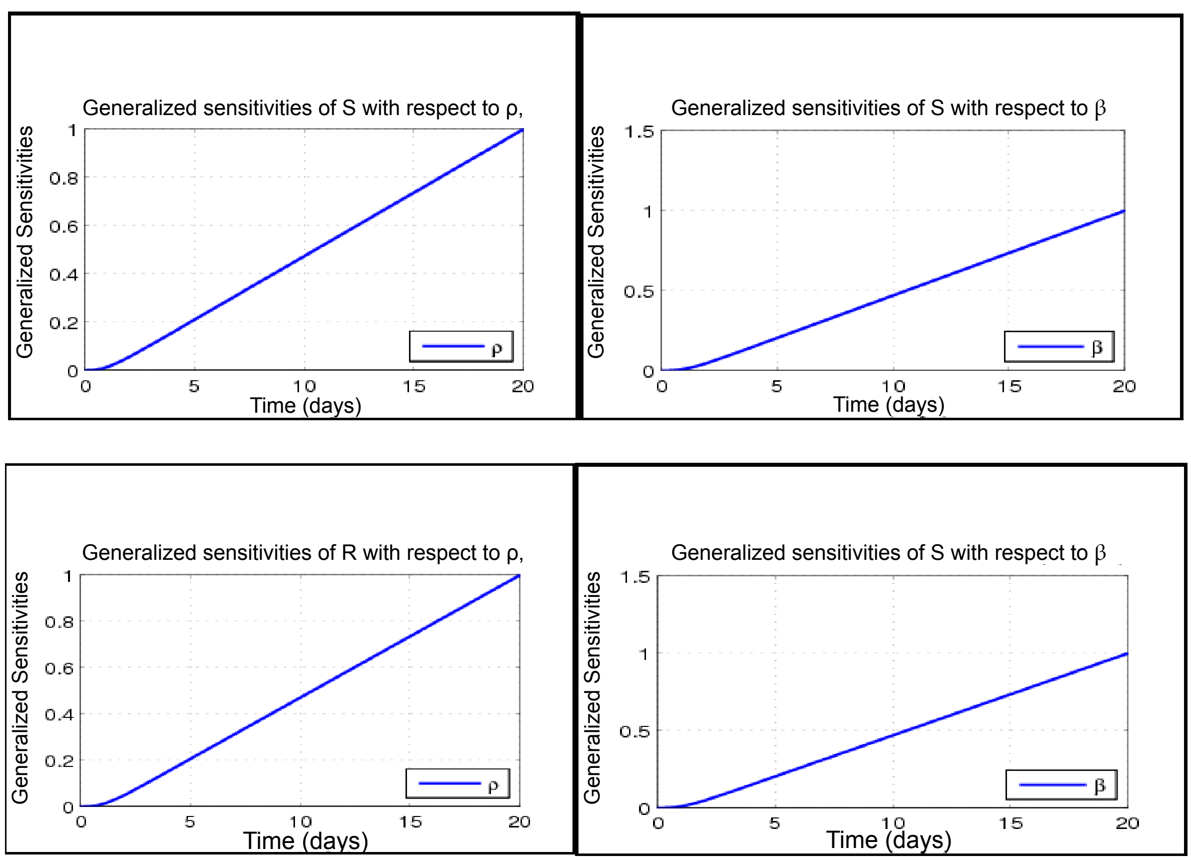

Fig. 2. GSFs of ρ respectively β when only measurements of S are taken on upper-left and upper-right panels, when only measurements of R are taken on the lower-left and lower-right panels for the SEIR model.

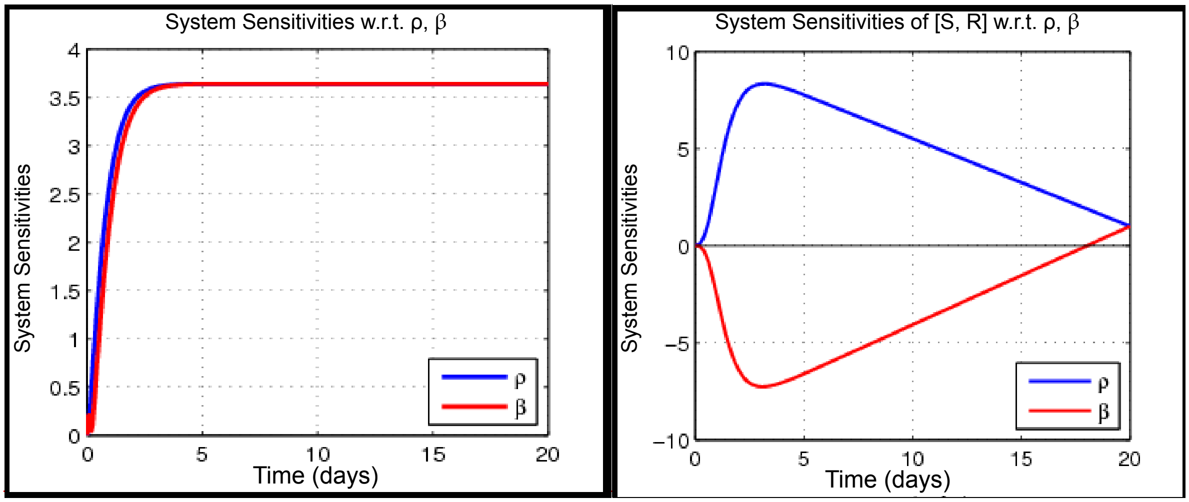

Fig. 3. System Sensitivity SIR model (left-panel) and combined GSFs (right-panel) of SIR model.

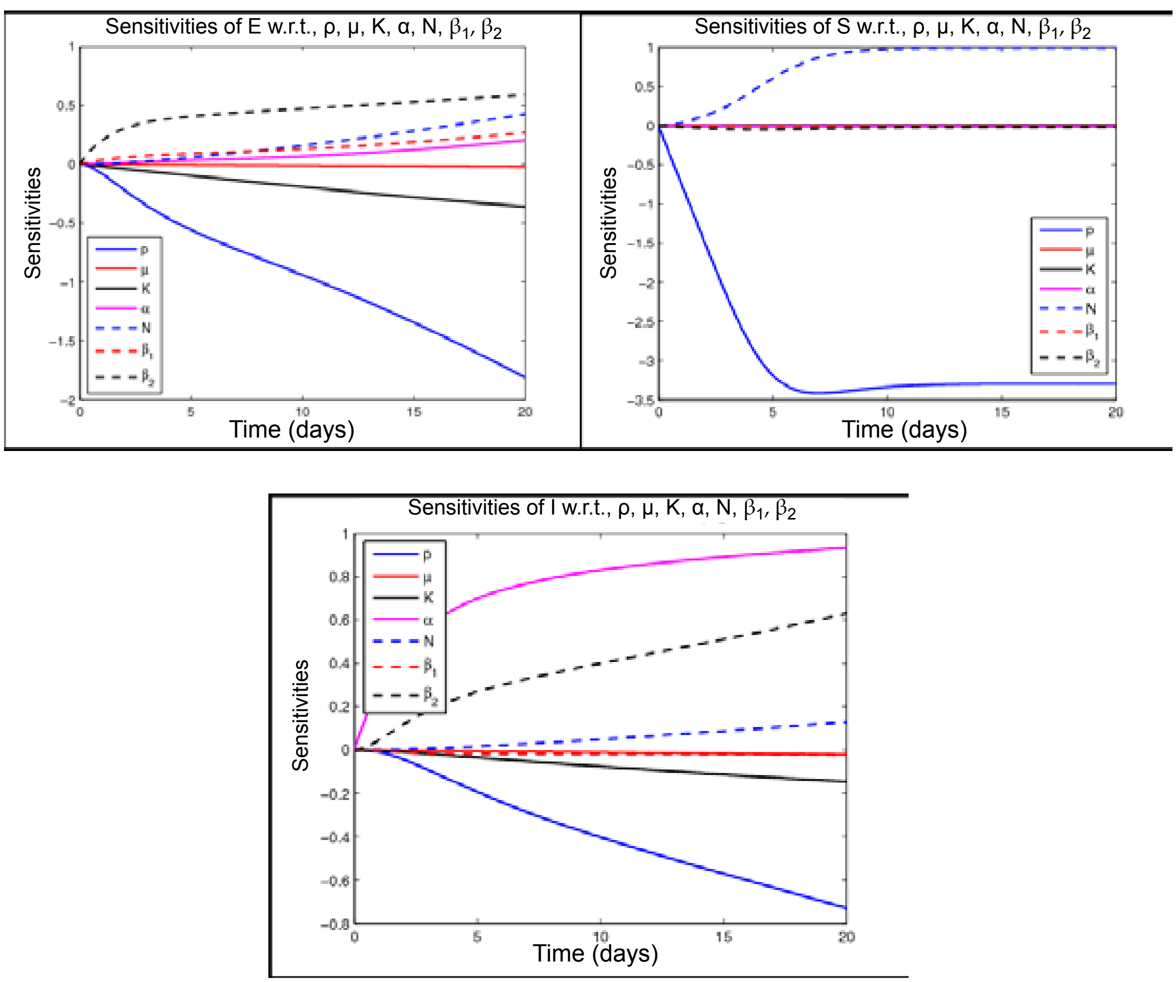

Fig. 4. Sensitivity functions of individual outputs S, E and I for selected parameters of the SEIR model.

individual outputs. This implies that changes in the infected rate ρ and the recovery rate β will change the output in the beginning and not afterwards.

System sensitivities of the parameters combine the effects of the sensitivities of both S and R. System sensitivities for the model are given in Fig. 3 (left panel). The information given by the time courses of the system sensitivities are more or less similar to the information given by their individual sensitivities. The system sensitivities indicate that the parameter ρ is a little bit more sensitive than the parameter β up to the 5 days. After 5 days, both the parameters become insensitive.

4.2. Generalized Sensitivities of the SIR Model

The individual generalized sensitivities of the model output S and R with respect to the parameter ρ and β are drawn in Fig. 2 according to Equation (13). In this case, all the GSFs are monotonic increasing reaching to 1. The parameters ρ and β give the similar results for both the outputs. These GSFs show that the changes in the values of the parameters ρ and β will have effects over their estimates. In other words, we can say that the measurements of the outputs S and R up to 5 days are enough to estimates ρ and β. Afterwords, these measurements possess no information for estimation of these two parameters.

The generalized sensitivity functions (GSFs) have the additional advantage of showing the correlation between parameters in addition to their sensitivities. By taking the number of the mesh points as the number of the measurements using as per our numerical scheme and taking only the measurements of S and R, we find the GSFs of ρ and β together as shown in Fig. 3 (right panel) which signify that the measurement of S and R possess information for the parameter estimates up to 5 days. The oscillations between GSFs show that there is a high degree of oscillations between the estimates of ρ and β.

4.3. Sensitivities of SEIR Model

First the sensitivity functions for the individual outputs and the system sensitivity defined respectively by the Equations (5) and (6) for the SEIR model are plotted. The sensitivities of the model outputs S, E and I with respect to the parameter N, p, K, β_1, β_2, α, μ are drawn in Fig. 4. The model output S is least sensitive to all the parameters except p and N. E is insensitive to the parameter μ. The compartment I is insensitive to μ and β_1. From these results one can see that the susceptible computers in the network are affected majorly by the rate at which external computers are connected to the network (N) and the recovery rate p of susceptible computer due to the anti-virus ability of network. System sensitivities for the model are given in Fig. 5. The information given by the system sensitivities are more or less similar to the information given by their sensitivities. From the system sensitivities, one can quantify the parameters in descending order of their sensitivities as p, N, α, β_2, K, β_1, μ. The parameter r is least sensitive.

4.4. Generalized Sensitivities of the SEIR Model

The GSFs are primarily drawn in two ways; one by taking a single parameter at a time and other by taking more than one parameter at a time. However, in the second case when the number of the parameters is very large, the Fisher Information Matrix becomes ill-condition. That is why one will avoid computing the GSFs for a large number of parameters. Fig. 6 describes the GSFs of different parameters. The measurements of S taken from 0 to 12 possess all information to estimate μ. The parameter r is insensitive with respect to the measurements of S, E and I up to the 18 days as is evident from the first, fourth and fifth panels of Fig. 6. That means the changes in the true value of the parameters r do not bring any changes in its estimates. In other words, r cannot be identified when we take measurements up to 18

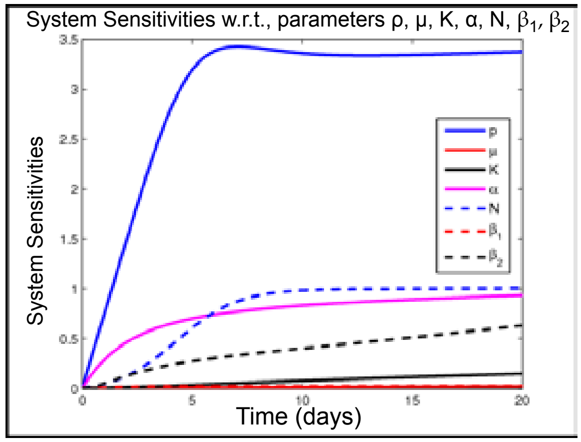

Fig. 5. System sensitivity for SEIR model.

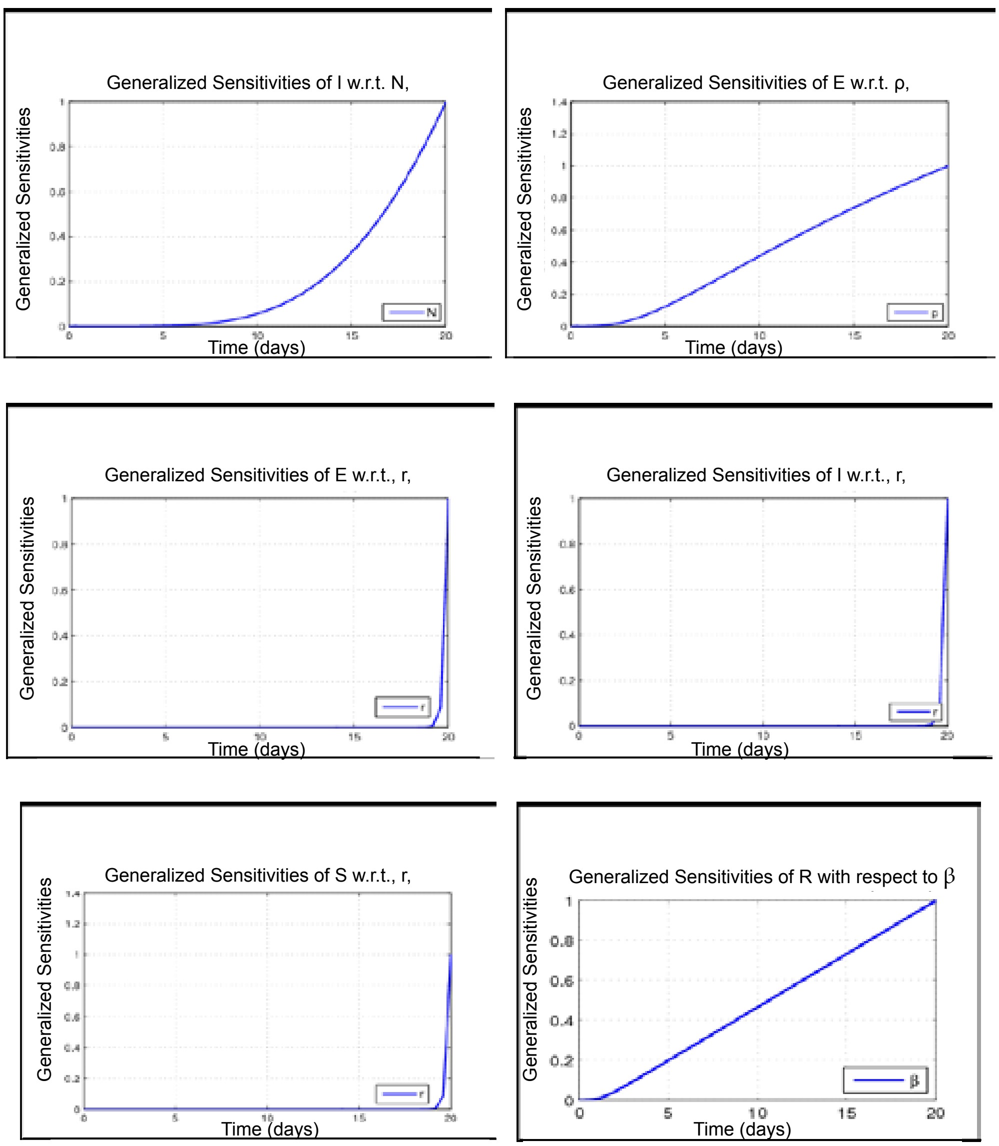

Fig. 6. Generalized sensitivity functions of individual outputs S, E and I for selected parameters of the SEIR model.

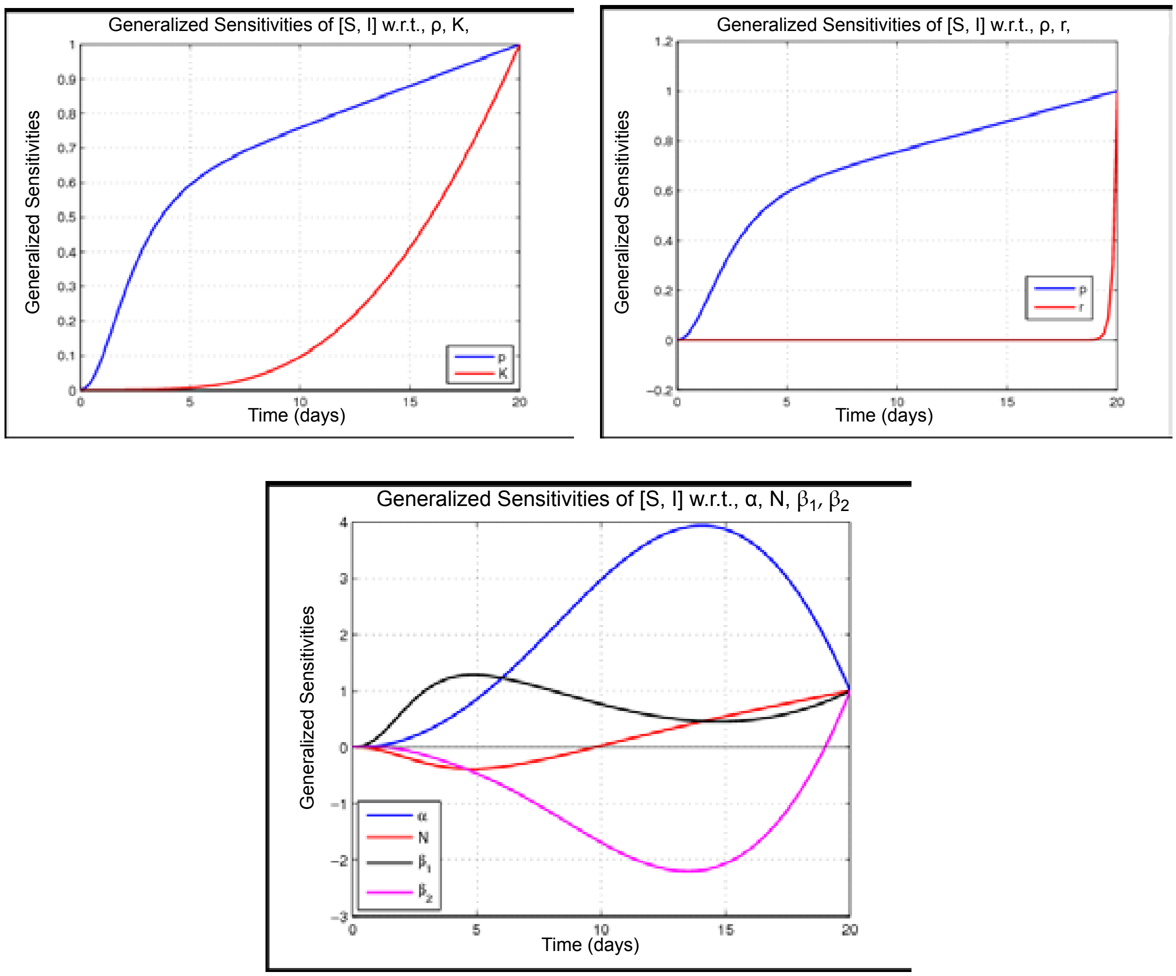

Fig. 7. Combined generalized sensitivity functions for selected parameters of the SEIR model.

days [5] and [6]. However, it becomes sensitive at the end of the time interval [0, 20]. If one restricts the measurements of I to the interval [0, 5], one cannot identify the parameter N. However, measurements taken beyond [0, 5] results in the identification of N. Similarly, the measurement of E do not possess any information about estimating the parameter p or the parameter estimate of p does not change with changes in its true value if the measurements are restricted to the beginning of [0, 20].

The GSFs have been drawn for selected parameters together by taking measurements of the outputs S and I only. This situation also shows the correlation between them.

The upper-left panel of Fig. 7 shows the GSFs of the parameters p and K. This shows that the parameter p is highly sensitive from the beginning of the interval [0, 20] whereas the parameter K is totally insensitive in the interval [0, 5]. This implies that the measurements of the S and I taken in [0, 5] possess no information about the estimate of K, whereas these do have information about the estimate of p. The oscillations between the GSFs of p and K show high degree of correlation between the estimates of these two parameters. The upper-right panel of Fig. (7) shows the GSFs of p and r. The combined measurements of S and I do possess information about the parameter p. The case of r is again different. The measurements do not identify r. Moreover, r is not correlated with p and with other parameters as was evident before. The center-down panel shows the GSFs of the parameters α, N, $\beta_1$and β_2. This shows that α and β_2 are more generalized sensitive and highly correlated whereas the parameters β_1 and N are correlated, but are not so highly generalized sensitive.

5. CONCLUSIONS

The sensitivity and generalized sensitivity analyses of the SIR and SEIR models describing the dynamics of the computer virus have been performed. The main points are as under:

i. The sensitivity functions describe the effects of changes in measurements on the model outputs whereas the generalized sensitivity functions describe the effects of changes in measurements over the parameter estimates.

ii. One can jointly use the sensitivity and generalized sensitivity studies to quantify the highly sensitive parameters.

iii. The sensitivities of ρ and β in the SIR model show that these two parameters are more or less equally sensitive for any outputs S or R or both. Their GSFs show that they are also highly correlated. So, one can suggest on this basis that this model can better describe the virus dynamics if other parameters or even a single parameter can be included in it.

iv. The sensitivity studies of the SEIR model describes that the parameter r is least sensitive for all measurements taken from the S, E or I or together. Its GSF is also negligible throughout the interval. This suggests that this parameter cannot be identified correctly through a parameter estimation process; however, it can be identified a priori.

v. The parameters p, N and α are highly sensitive whereas the parameters β_1, μ and r are the least sensitive in the SEIR model.

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