Proceedings of the Pakistan Academy of Sciences:
A. Physical and Computational Sciences 54 (4): 375–383 (2017)

ISSN: 2518-4245 (print), 2518-4253 (online)

Pakistan Academy of Sciences

Research Article

A Novel Dimension-reduced Subspace Codebook for 5G Lens-based Millimeter-Wave Massive Multiple-Input Multiple-Output (MIMO) Systems

Imran Khan*

Department of Electrical Engineering,
University of Engineering and Technology, Peshawar, Pakistan

Abstract: A novel approach is proposed to design a devoted subspace codebook for the equivalent channel feedback in lens-based millimeter-wave (mmWave) massive MIMO systems. The equivalent channel state information (CSI) between users and radio frequency RF chains required at the base station (BS) to achieve multi-user (MU) multiplexing gain. There is no such codebook designed for the channel feedback for such emerging lens-based mmWave massive multiple-input multiple-output (MIMO) system with low channel feedback overhead. For such a purpose, I propose the subspace codebook with reduced-dimension, which can substantially decrease the codebook size, RF chains and channel feedback overhead. Particularly, I first generate the high-dimensional channel vectors in the channel subspace by utilizing the limited scattering property of the mmWave channels. After this, the equivalent channel vector (ECV) is quantized by proposing the dimension-reduced subspace codebook which utilizes the function of lens and beam selector respectively. Simulation results revealed that the proposed codebook for channel feedback performs was much better than the classical channel codebook.

Keywords: 5G, mmWave, massive MIMO, equivalent channel feedback, codebook, lens

1. INTRODUCTION

The rapid increase in production of smartphones, notebooks, and data communication are posing challenges to cellular service providers. In a survey, it was estimated that by 2020 the number of new devices beyond the smartphones and tablets will be 50 billion [1]. The current cellular service providers try to deliver high quality of service (QoS), low-latency, and high data-rate to their consumers, but their bandwidth (BW) is limited and congested. The current 4G technology is unable to meet the 1000-fold increase in data-traffic which is predicted for 5G wireless systems that demand novel wider spectrum bands and devices with greater spectral efficiency. To cope with these demands, the new idea of millimeter-Wave (mmWave) massive MIMO is proposed. The MmWave massive MIMO is deemed to be a potential technology for the future 5G wireless communication systems due to its superior characteristics such as wider bandwidth availability and high spectral and energy efficiency [2].

The 5G wireless networks will have a huge available spectrum of mmWave which range from 30-300GHz [3]. The main challenging practical problem for mmWave massive MIMO systems is that each antenna requires its own dedicated RF chain, which means that the number of RF chains increases with increasing the number of antennas that consumes more energy and causes the unaffordable cost of hardware [4]. The codebook precoding is a potential technology which is deployed by the current Long-Term Evolution (LTE) system that fixes a common codebook at the transmitter and receiver [5]. The spatial multiplexing gain is improved by deploying differential codebook precoding in mmWave

---

Received, September 2017; Accepted, November 2017
Email: imran_khan@netpeshawar.edu.pk

massive MIMO systems [6]. The existing codebooks have problem of large number of RF chain requirement and complex hardware, which prompt the new idea of lens-based mmWave massive MIMO that is recently proposed which utilizes the concept of channel sparse structure [7-10]. The main function of the lens is that it focuses the energy of mmWave signals from different directions on different antennas, which in essence, perform the channel transformation from conventional spatial channel to the proposed beamspace channel which has sparsity. The number of effective channel propagation paths is limited in mmWave signals due to the considerable path loss [11]. Thus, the beamspace channel is sparse in nature which can be utilized to estimate the channel feedback with improved efficiency. Therefore, the number of RF chains can be reduced substantially by deploying beam-selector, which also reduces the equivalent channel dimensions respectively that can be estimated with less number of feedback bits.

The dimension-reduced CSI between the users and RF chains is essentially required for the BS to perform the adaptive channel operations such as power-allocation and precoding. Such CSI is termed as equivalent CSI (eCSI) in the proposed codebook. According to the literature, there is no such devoted codebook yet designed for the upcoming mmWave massive MIMO systems to perform the eCSI. The existing codebooks are based on the Rayleigh distribution CSI which cannot be used in the proposed dimension-reduced eCSI in lens-based mmWave massive MIMO systems [12] as its performance degrades significantly with increasing SNR and number feedback overhead.

I have solved this problem by proposing the subspace codebook for eCSI of dimension-reduced lens-based mmWave massive MIMO systems. The main idea is that first I utilize the mmWave channels property of limited scattering to produce the high-dimensional vectors in the channel subspace, which is dependent on the angle-of-departure (AoD) of channel path. After this, I propose the dimension-reduced subspace codebook by utilizing the function of beam-selector and lens to quantize the equivalent channel vector (ECV) using less-number of feedback bits than the conventional codebooks respectively. The performance evaluation of the proposed and the traditional codebooks [4] and [12] is illustrated in the simulation results which also provide a comparison of the proposed codebook with the ideal perfect CSI benchmark.

2. SYSTEM MODEL

The typical mmWave massive MIMO spatial channel model is first derived and explained. After this, the dimension-reduced lens-based mmWave massive MIMO equivalent channel is then proposed.

2.1 Typical Channel Model for mmWave Massive MIMO

I propose a typical mmWave massive MIMO system with M-number of BS antennas and K-number of single-antenna users, where ($M \gg K$) respectively as shown in Fig. 1. The widely-used ray-based channel model of mmWave is deployed for analytical formulations in such systems. The downlink (DL) channel-vector of $Mx1$ dimension between the k-th user and the BS can be represented by:

$$h_k = \sum_{i}^{P_k} \mathrm{g}_{k,i}\, a(\psi_{k,i}) \tag{1}$$

Where,

P_k: is the number of resolvable paths between the BS and the k-th user,

$\mathrm{g}_{k,i}$: is the k-th user complex gain of the i-th path, which is independently and identically distributed (i.i.d) with mean of zero and unit variance,

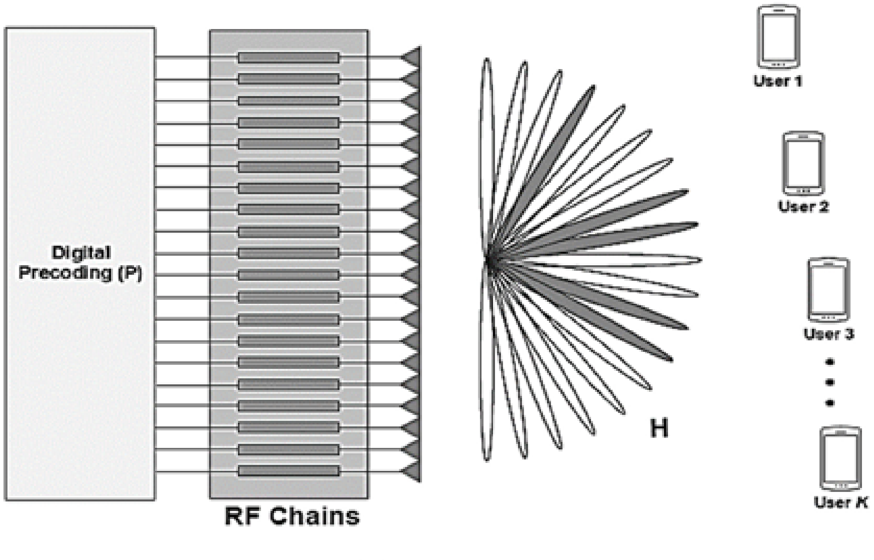
$a(\psi_{k,i})$: is the array steering-vector (ASV) of $Mx1$ dimension of the k-th user of the i-th path.

The ASV at the BS can be obtained by deploying the uniform linear array (ULA) as follows:

$$a(\psi_{k,i}) = \frac{1}{\sqrt{M}}[1, e^{-j2\pi\psi_{k,i}}, \ldots,\ e^{-j2\pi\psi_{k,i}(M-1)}]^T \tag{2}$$

Where $\psi_{k,i}$ is the spatial-direction which can be obtained as follows:

$$\psi_{k,i} = \frac{d}{\lambda}\sin(\theta_{k,i}) \tag{3}$$



(a)

Fig. 1. A typical mmWave massive MIMO system.

$\theta_{k,i}$: is the i-th path AoD of the k-th user,

λ: is the wavelength of the mmWave carrier frequency,

d: is the separation or spacing between BS antenna elements and follows the half-wave condition $\lambda/2$.

For the sake of convenience, (1) can also be represented in vector-matrix form as:

$$h_k = \mathrm{A}_k \mathrm{g}_k \tag{4}$$

Where, $\mathrm{g}_k \in \mathbb{C}^{\mathrm{P}_k x 1}$ which can be obtained from:

$$\mathrm{g}_k = [\mathrm{g}_{k,1}, \mathrm{g}_{k,2}, \mathrm{g}_{k,3}, \dots, \mathrm{g}_{k,\mathrm{P}_k}]^T \tag{5}$$

and $\mathrm{A}_k \in \mathbb{C}^{\mathrm{M}x\mathrm{P}_k}$ which can be calculated from:

$$\mathrm{A}_k = [a(\psi_{k,1}), a(\psi_{k,2}), a(\psi_{k,3}), \dots, a(\psi_{k,}\mathrm{P}_k)] \tag{6}$$

2.2 Transformation to Lens-based Beamspace Equivalent Channel Model

The proposed lens-based mmWave massive MIMO system is shown in Fig. 2. The transformation of the spatial channel of (4) into the beamspace channel is performed by the utilization of the lens which acts as a phase-shifter (PS). This lens acts as a spatial MxM discrete Fourier transform (DFT) matrix (*U*), which consists of *M* uniformly distributed orthogonal beams based array vectors, that can be represented by:

$$U = \left[1, a\left(\frac{1}{M}\right), \dots,\ a\left(\frac{1}{M}(M-1)\right)\right]^H \tag{7}$$

The received signal vector at the *k*-th user can be represented by:

$$y_k = \sqrt{\frac{\rho}{K}} h_k^H U^H S Z x_d + n_k = \sqrt{\frac{\rho}{K}} (h_k^b)^H S Z x_d + n_k \tag{8}$$

Where,

ρ: is the transmit power,

$x_d = [x_{d,1}, x_{d,2}, x_{d,3}, \dots, x_{d,K}]^T$: is the $Kx1$ signal vector,

S: is the MxN_{RF} beam-selector, which has one nonzero element "1" in each column and the location of each nonzero element in different in different columns respectively,

n_k: is the complex Gaussian noise with zero-mean and unit-variance at the *k*-th user,

$Z = [z_1, z_2, z_3, \dots, z_K]$: is the $N_{RF}xK$ zero-forcing (ZF) precoding matrix which contains K different unit-norm $N_{RF}x1$ precoding vectors,

h_k^b: is the $Mx1$ beamspace channel-vector.

It is important to note that the beamspace channel-vector has sparse characteristics, which is due to limited scattering property of the mmWave channels. Now, I define the dimension-reduced equivalent channel of $N_{RF}x1$ dimension as follows:

$$h_k^e = S^H h_k^b \tag{9}$$

Therefore,

$$y_k = \sqrt{\frac{\rho}{K}} (h_k^e)^H S Z x_d + n_k \tag{10}$$

It is clear from (9) that, the equivalent channel is not Rayleigh distributed, and thus no such

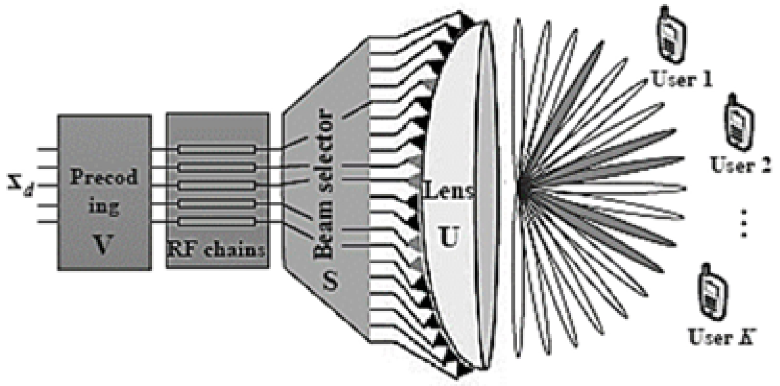


Fig. 2. Proposed lens-based mmWave massive MIMO system.

codebook is yet designed in the literature to quantize the equivalent channel. For this, purpose, I proposed the dimension-reduced subspace codebook which is discussed below.

2.3 Equivalent Channel Feedback in Lens-based mmWave Massive MIMO

The equivalent channel feedback is realized by assuming that each user knows its equivalent channel vector h_k^e via channel estimation. Then the h_k^e is quantized by the *k*-th user to feedback bit *(B)* by deploying the following codebook:

$$D_k = [d_{k,1}, d_{k,2}, d_{k,3}, \dots, x_{k,2^B}] \tag{11}$$

Now considering the classical random-vector quantization (RVQ) codebook, the $N_{RF}x1$ quantization vector $d_{k,i}$ is randomly produced by the independent vectors selection from the uniform distribution on the complex N_{RF}-dimensional unit-sphere [13]. The number of B-bits should linearly scale with N_{RF} in order to bound the loss in system capacity within an acceptable threshold level. The *k*-th user will find d_{k,F_k}, which is the nearest to the equivalent channel vector. The term nearest is expressed by the angle between two vectors, i.e., the index is calculated by:

$$F_k = \underset{i\in[1,2,3,\dots,2^B]}{arg\,min} \sin^2\left(\angle\left(h_k^e, d_{k,i}\right)\right) = \underset{i\in[1,2,3,\dots,2^B]}{arg\,max} \left|d_k^H, \check{h}_k^e\right|^2 \tag{12}$$

Where, $\check{h}_k^e$ shows the direction of equivalent channel which is given by:

$$\check{h}_k^e = \frac{h_k^e}{\|h_k^e\|} \tag{13}$$

It is clear from (13) that, the denominator is the magnitude of the equivalent channel which is just a scaler, therefore, it is assumed that channel magnitude can be perfectly fed back to the BS [13]. The F_k index can be fed back via B-bits, then the feedback equivalent channel vector can be obtained by the BS as:

$$\hat{h}_k^e = \|h_k^e\| d_{k,F_k} \tag{14}$$

2.4 Dimension-reduced Subspace Codebook

As from (4), the channel vector of the proposed lens-based mmWave channels is given by:

$$h_k = \mathrm{A}_k \mathrm{g}_k \tag{15}$$

The channel vector in (15) is specifically distributed on the column space of A_k which is of $\mathrm{M}x\mathrm{P}_k$ dimension that is generated by the linear combination of column vectors of A_k. As the number of paths P_k is much smaller than the number of BS antennas ($\mathrm{P}_k \ll M$) due to the considerable mmWave signals propagation loss [14]. It means that, the number of paths column vectors of A_k form a subspace only of the entire *M*-dimensional space, which depends on different AoDs of P_k. This subspace is also termed as channel subspace which is illustrated in Fig. 3.

As there is slower variation in AoDs than in path-gains, so it can be estimated accurately via beamspace channel estimation technique [11]. The high-dimensional vector of $Mx1$ dimension can be represented by:

$$c_{k,i} = A_k w_{k,i} \tag{16}$$

Where,

$w_{k,i}$: is of $P_k x1$ dimension and isotopically distributed on the complex unit sphere of P_k-dimension.

It is important to note that, the $c_{k,i}$ is distributed on the channel subspace, i.e., the

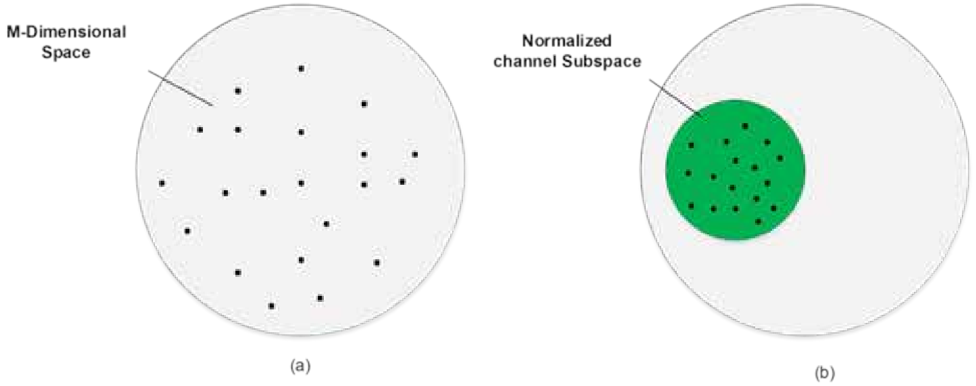


Fig. 3. Codebooks comparison: (a) Classical M-dimensional space; (b) Proposed normalized subspace.

column space of A_k. Now the dimension-reduced subspace codebook is proposed which is discussed in (11) to quantize the equivalent channel of (9) which is given by:

$$d_{k,i} = S^H U c_{k,i} \tag{17}$$

Where S and U denote the functions of beam-selector and lens respectively. The $d_{k,i}$ is normalized to fulfil the requirement of unit-norm vector which is given by:

$$d_{k,i} = \frac{d_{k,i}}{\|d_{k,i}\|} \tag{18}$$

By considering (8) and (9), it is concluded that the quantization vector of (17) and the dimension-reduced codebook of (11) has the same distribution as the equivalent channel vector in (9) respectively. Therefore, the proposed codebook has better quantization performance than the conventional RVQ-codebook. The downlink (DL) precoding is performed by the BS after channel feedback which is based on the $N_{RF}xK$ feedback equivalent channel matrix that is given by:

$$\widehat{H}^e = [\hat{h}_1^e, \hat{h}_1^e, \hat{h}_3^e, \dots, \hat{h}_K^e] \tag{19}$$

Therefore, the received signal of (10) can be rewritten as:

$$y_k = \sqrt{\frac{\rho}{K}}(h_k^e)^H z_k x_{d,k} + \sqrt{\frac{\rho}{K}} \sum_{i=1, i\neq k}^{K} (h_k^e)^H z_i x_{d,i} + n_k \tag{20}$$

Where the second term in (20) shows the multi-user (MU) interference. From the above analytical formulation, the per-user rate (R_p) which is based on the limited channel feedback and low-overhead is expressed as:

$$R_p = E\left[\log_2\left(1 + \frac{\frac{\rho}{K}\left|(h_k^e)^H z_k\right|^2}{1+\frac{\rho}{K}\sum_{i=1,i\neq k}^{K}\left|(h_k^e)^H z_i\right|^2}\right)\right] \tag{21}$$

For the ideal case of perfect CSI ($\widehat{H}^e = H^e$), the ZF precoding vector $z_{ideal,i}$ of $N_{RF}x1$ dimension can be obtained as the normalized i-th column of H^e, which is given by:

$$z_i = \frac{(H^e)^{\dagger}(:,i)}{\|(H^e)^{\dagger}(:,i)\|} \tag{22}$$

Therefore, the $z_{ideal,i}$ is orthogonal to the equivalent channel vector of k-th user for any $i \neq k$ values. From such condition, the ideal per-user rate is given by:

$$R_{ideal} = E\left[\log_2\left(1 + \frac{\rho}{K}\left|(h_k^e)^H z_{ideal,k}\right|^2\right)\right] \tag{23}$$

From (23) and (21), the data rate-gap can be obtained which is the difference between the ideal CSI and the proposed limited channel feedback per-user rate and is expressed as:

$$\Delta R(\rho) = R_{ideal} - R_p \tag{24}$$

The rate-gap of (24) can be upper-bounded by using the results of (12) index which is given by:

$$\Delta R(\rho) \leq \log_2\left(\begin{matrix} 1 + \frac{\rho(K-1)}{K} E[\|h_k^e\|^2] \\ E\left[\sin^2\left(\angle\left(\breve{h}_k^e, \hat{h}_k^e\right)\right)\right] \end{matrix}\right) \tag{25}$$

It is clear from (25) that the rate-gap depends on the equivalent channel norm $\|h_k^e\|$ and the quantization error $\sin^2\left(\angle\left(\breve{h}_k^e, \hat{h}_k^e\right)\right)$ respectively. The quantization error can be upper bounded by:

$$E\left[\sin^2\left(\angle\left(\breve{h}_k^e, \hat{h}_k^e\right)\right)\right] < 2^{-\frac{B}{P-1}} \tag{26}$$

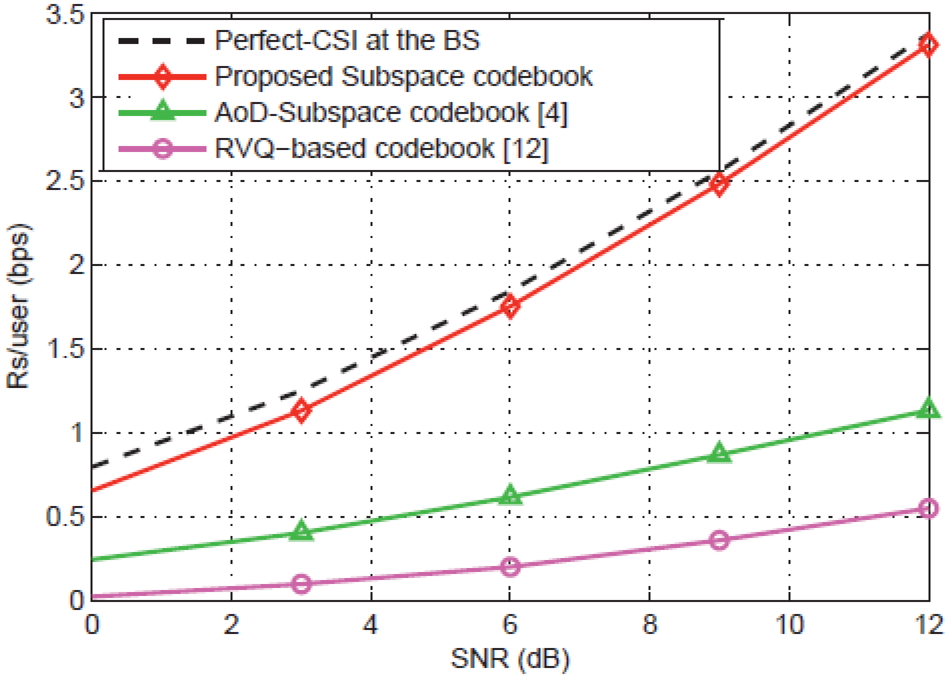


Fig. 4. Comparison of the proposed codebook with the ideal CSI and the conventional codebooks in terms of per-user rate against the SNR.

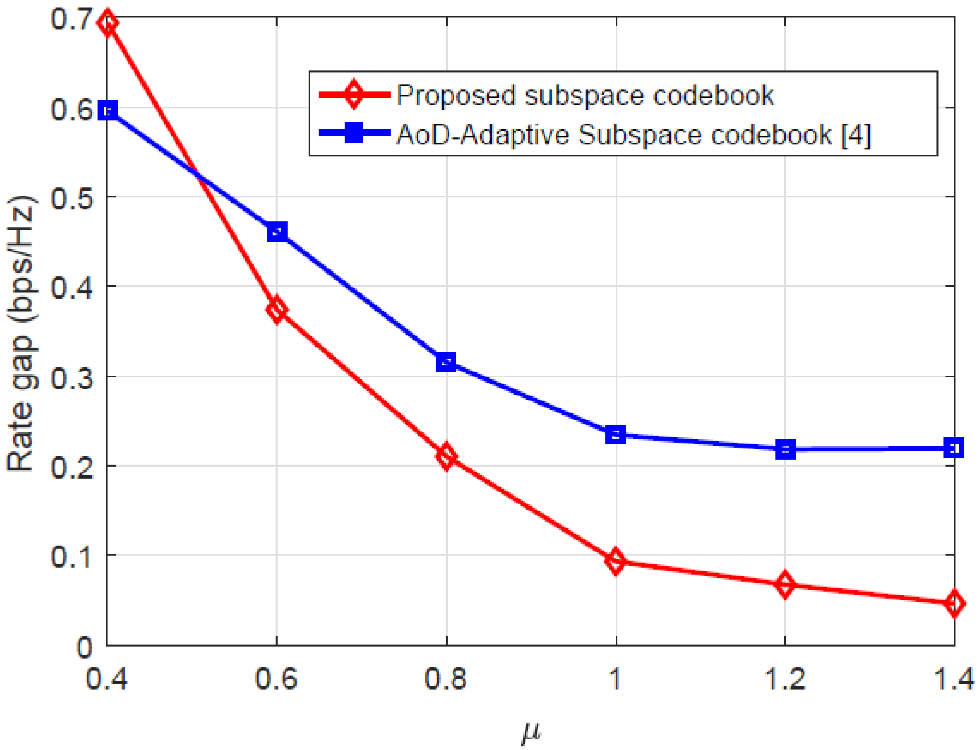


Fig. 5. Comparison of the proposed codebook with the traditional codebook of [4] in terms of rate-gap against the scaling factor μ.

Table 1. Simulation parameters.

S. No.	Parameter	Symbol	Value
1	Number of BS antennas	M	128
2	Number of receiver antennas	K	8
3	Number of RF chains	N_{RF}	24
4	Number of resolvable paths	P	3
5	Signal-to-noise ratio	SNR	15
6	BS antennas spacing	d	0.5

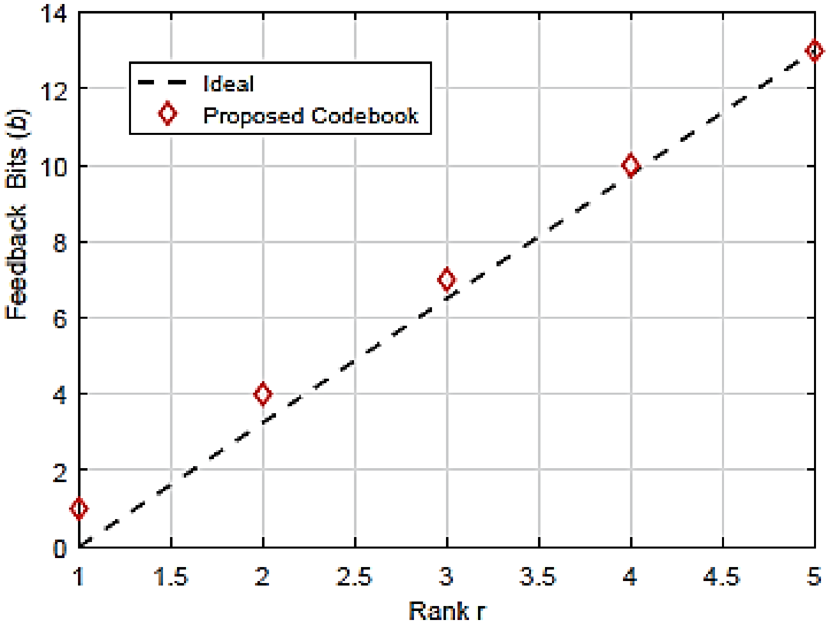


Fig. 6. Comparison of the proposed codebook with the ideal CSI condition in terms of feedback bits against the rank

Where P is the number of resolvable paths. The number of feedback bits (B) can be obtained by assuming the rate gap $\Delta R(\rho) \leq 1$bps/Hz, which can be represented by [9]:

$$B \geq \frac{P-1}{3} SNR + (P-1)\log_2(K-1) \quad (27)$$

Where the signal-to-noise ratio (SNR) at the receiver (R_x) is calculated by the following expression:

$$SNR = 10\log_{10}\frac{\rho}{K} E[\|h_k^e\|^2] \quad (28)$$

It is clear from (27) that, the number of feedback bits required (B) is directly proportional to $(P-1)$ which is the slope. This variation with the slope maintains a constant rate-gap which is desired. As the number of resolvable paths (P) is much smaller than the number of RF chains ($P \ll N_{RF}$), therefore, the required feedback overhead for the proposed dimension-reduced subspace codebook is much less than the conventional RVQ-codebook, where B linearly scales with N_{RF} respectively. Table 1 shows the proposed system simulation parameters which are used for obtaining the required results for comparison.

3. SIMULATION RESULTS

The simulation results validate the analytical formulation of the proposed dimension-reduced subspace codebook. Fig. 4 shows the comparison of the per-user rate between the ideal perfect CSI at the BS, the proposed dimension-reduced subspace codebook, the conventional AoD-adaptive subspace codebook [4] and RVQ-codebook [12] respectively. It is clear from this Figure that, the data rate-gap remains constant between the ideal perfect CSI case and the proposed dimension-reduced limited channel feedback codebook when SNR increases. It is also concluded that the proposed codebook performance is near-optimal with a reduced number of RF chains and low feedback overhead. It verifies the theoretical formulations. The proposed codebook performs much better than the conventional AoD [4] and RVQ-based [12] channel codebooks as the rate-gap between them increases with increasing SNR which is undesired from a practical perspective. The performance of the proposed subspace codebook is also superior in low SNR region which makes it suitable and attractive for lens-based mmWave massive MIMO systems. Fig. 5 compares the rate-gap between the proposed code-book and the conventional AoD-codebook against the scaling factor. It is obvious from the results that the proposed codebook rate-gap decreases exponentially and quickly than the conventional codebook [4] as the scaling factor. The high scaling factor means that the uplink channel rate is larger than the source rate. Moreover, the rate-gap between conventional codebook of [4] and the proposed codebook increases as μ increases which further verify the better performance of the proposed codebook. Fig. 6 compares the feedback bits against the correlation matrix rank for the proposed codebook and the ideal CSI case. It is clear from the results that the proposed codebook shows near-optimal performance as the rate-gap between them is limited to 0.08bps/Hz.

4. CONCLUSIONS

This study provided the basis for proposing a novel solution to the problem of high-feedback overhead and large number of RF chains requirement in mmWave Massive MIMO by designing the dimension-reduced subspace codebook for equivalent channel feedback in lens-based mmWave massive MIMO system whose channel vectors are not Rayleigh distributed. The main idea is to first utilize the mmWave channels limited scattering property to generate the high-dimensional channel vectors in the channel subspace. Then, the dimension-reduced subspace codebook is proposed to quantize the equivalent channel vector (ECV) which is according to the function of beam-selector and lens. Simulation results show that the performance of the proposed dimension-reduced subspace channel codebook is much better than the classical channel codebooks AoD [4] and RVQ [12]. This research work can also be extended by considering multiple-antennas users and analyze the system performance in terms of energy-efficiency. Moreover, the proposed codebook shows overall excellent performance which makes it suitable choice for the Lens-array based mmWave Massive MIMO systems.

5. REFERENCES

1. Ericsson. *More than 50 Billion Connected Devices*. White Paper (2011). http://www.ericsson.com/res/docs/whitepapers/wp-50 billions.pdf
2. Heath, R.W., N. Gonzalez-Prelcic, S. Rangan, W. Roh & A. Sayeed. An overview of signal processing techniques for millimeter wave MIMO systems. *IEEE Journal of Selected Topics in Signal Processing* 3:436–453 (2016).
3. Kumar, J.M. Smart cities with massive data centric living are hard to build without 5G networks. *IETE Technical Review* 4:237–239 (2015).
4. Shen W., L. Dai, G. Gui, Z. Wang, R.W. Health & F. Adachni. AoD-adaptive subspace codebook for channel feedback in FDD massive MIMO systems. In: *IEEE International Conference on Communications (ICC)* 21-25 May 2017, Paris, France, p.: 1–5 (2017). doi: 10.1109/ICC.2017.7997168
5. Wang J., M. Wu, & F. Zhen. The codebook design for MIMO precoding systems in LTE and LTE-A. In: *6th International Conference on Wireless Communications Networking and Mobile Computing (WiCOM),* 23-25 Sept. 2010, Chengdu, China, p.1–4 (2010). doi: 10.1109/WICOM.2010.5600618
6. Abe, T. & G. Bauch. Differential codebook MIMO precoding technique. In: *IEEE Global Telecommunications Conference (GLOBCOM)* 26-30 Nov., 2007, p. 3963–3968, Washington, DC, USA (2007). doi: 10.1109/GLOCOM.2007.753
7. Song, G.H, J. Brady & A.M. Sayeed. Beamspace MIMO transceivers for low-complexity and near-

optimal communication at mmwave frequencies. In: *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP),* Vancouver, BC, Canada, p. 4394–4398 (2013). https://doi.org/10.1109/ICASSP.2013.6638490

8. Zeng, Y., R. Zhang & Z.N. Chen. Electromagnetic lens-focusing antenna enabled massive MIMO: Performance improvement and cost reduction. *IEEE Journal of Selected Areas in Communication* 6:1194–1206 (2014).
9. Zeng, Y. & R. Zhang. Millimeter wave MIMO with lens antenna array: A new path division multiplexing paradigm. *IEEE Transactions on Communications* 4:1557–1571 (2016).
10. Rappaport T.S., S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi & F. Gutierrez. Millimeter wave mobile communications for 5G cellular: It will work!. *IEEE Access* 1:335–349 (2013).
11. Love, D. J., R.W. Heath & T. Strohmer. Grassmannian beamforming for multiple-input multiple-output wireless systems. *IEEE Transactions on Information Theory* 10:2735–2747 (2003).
12. Jindal, N. MIMO broadcast channels with finite-rate feedback. *IEEE Transactions on Information Theory* 11:5045–5060 (2006).
13. Alkhateeb, O. El Ayach, G. Leus & R.W. Heath. Channel estimation and hybrid precoding for millimeter wave cellular systems. *IEEE Journal of Selected Topics in Signal Processing* 5: 831–846 (2014).
14. X. Gao, L. Dai, S. Han, C.-L. I & X. Wang. Reliable beamspace channel estimation for millimeter-wave massive MIMO systems with lens antenna array. *IEEE Transactions on Wireless Communications* 9: 6010–6021 (2017).