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Research Article

Diffraction by a Thick Half Plane Composed of PEMC Metamaterial

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Abstract: Diffraction of a plane wave by a thick half plane composed of metamaterial was examined by employing duality transformation proposed by Lindell and Sihvola [18, 20, 21, 24]. As perfect electric conductor (PEC) and perfect magnetic conductor (PMC) media can both be derived as limiting cases of perfect electromagnetic conductor medium, so it was thought worthwhile to attempt and generalize the problem of diffraction by a thick PEC half plane to thick PEMC half plane. The boundary value problem was solved by using an integral transform, the Wiener-Hopf (WH) technique and the method of steepest descent. Infinite algebraic equations, containing infinite constants, arose in the problem under consideration which were solved numerically. The effects of arising parameters of thickness and admittance on amplitude of the diffracted field were plotted and are discussed.

Keywords: Diffraction, duality transformation, PEMC medium, Wiener-Hopf technique, thick half plane

1. INTRODUCTION

Acoustic waves diffraction by half planes satisfying various boundary conditions e.g., soft (pressure release), hard (rigid) [1], absorbing [2], modified absorbing (satisfying Myers conditions) [3], or electromagnetic waves diffraction by half planes being perfectly conducting [4], impedance [5], an imperfect half plane in bianisotropic medium [6] is a current and important topic in existing diffraction/scattering theory and has been addressed by many researchers. In most of the half plane diffraction problems thickness of half plane is assumed infinitesimal for calculational economy. However in actual practice a half plane possesses thickness which has to be taken into consideration while attempting the half plane diffraction problems. Jones [7] appears to be first to discuss the diffraction of electromagnetic plane wave by a thick half plane being perfectly conducting. Jones [7] solution contained constants satisfying infinite set of equations which were solved under the assumption that the half plane thickness is less than wavelength of incident wave. Lee and Mitra [8] considered Jones analysis [7] and solved the same problem by using WH method [1]. Crighton and Leppington [9] also analyzed and enhanced the results obtained by Jones [7] using the matched asymptotic expansions method. Volakis and Ricoy [10] also examined Jones analysis in [7] by using angular spectrum method together with generalized scattering matrix technique. Later Volakis [11] extended the problem [7] to that of thick impedance half plane. Acoustic counter parts of Jones work [7] are the plane waves diffraction by thick hard half plane with end faces being absorbent and resistive were contributed by Rawlins and McIver [12] and Buyukaksoy et al. [13]. Recently, Cinar and Buyukaksoy [14] also extended the analysis [11] to thick impendent half plane such that impedance of end face is different from impedance of lateral walls. By using the symmetry of the diffracting structure, Cinar and Buyukaksoy [14] applied the method of image bisection to divide the original problem into two problems of diffraction by a step with even and odd wave excitation modes. In even excitation case the step at $y=0$ is assumed to have Neumann boundary condition on it i.e., the normal derivative of the current become zero on it (see Fig. 2a) whereas in case of odd excitation case the step at $y=0$ is assumed to have Dirichlet boundary condition on it i.e., the total current must vanish on it (see Fig. 2b).

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More recently, Tayyar and Buyukaksoy [15] examined the plane wave diffraction by the junction of thick impedance half plane and a thick dielectric slab. Thick wall geometry is also of practical importance in the theory of diffraction of waves by parallel plate waveguides as considering plates with finite thickness give much more realistic results about the diffraction of waves in waveguides as compared to the plates of negligible thickness [16, 17]. With these motivations of considering the thickness of half plane in mind, in this paper we have examined the effect of PEMC medium on diffraction of plane waves by a thick half plane for the work reported in [14] by employing the duality transformation [18]. Plane wave diffraction from a PEMC thick half plane will help understanding the diffraction process in PEMC medium and will go a step ahead to conclude the discussion with respect to thick half plane. Since our solution methodology also depends on Fourier transform, the Wiener-Hopf (WH) technique and method of steepest descent so notation to be used principally is that of reference [14]. In order to avoid repetition we shall omit details of calculations and shall only report necessary computational steps and major calculations essential for understanding the effects of PEMC metamaterial. Some graphs showing the effects of channel width b and admittance parameter M are plotted and discussed. A good agreement with the existing literature on PEMC metamaterial is observed.

2. FORMULATION OF THE PROBLEM

The detailed problem formulation is given in [14]. For the convenience of readers here we just give a brief outline of the problem. Geometry of diffraction problem is shown in Fig. 1.

Let

$$u^i(x,y) \equiv E_z^i(x,y) = e^{-ik(x\cos\phi_0 + y\sin\phi_0)}, \tag{1}$$

be the incident plane wave where $k = \frac{2\pi}{\lambda}$ is the wave number and ϕ_0 is the angle of incidence, illuminates a semi-infinite channel of width $2b$ with side walls are represented as $S_1 = \{(x,y,z); -\infty < x < 0, y = b, z \in (-\infty,\infty)\}$ and $S_2 = \{(x,y,z); -\infty < x < 0, y = -b; z \in (-\infty,\infty)\}$ having same impedances $Z_1 = \eta_1 Z_0$ on the surfaces $S_{1,2}$ whereas surface S_3 of vertical face $S_3 = \{(x,y,z); x = 0, y = (-b,b); z \in (-\infty,\infty)\}$ has the impedance $Z_2 = \eta_2 Z_0$ with Z_0 is the specific impedance of the enclosing medium. The reflected field from half plane at $y = b$ having specific impedance η_1 is given as

$$u_1^r(x,y) = \frac{\eta_1 \sin\phi_0 - 1}{\eta_1 \sin\phi_0 + 1} e^{-ik(x\cos\phi_0 - (y-2b)\sin\phi_0)}. \tag{2}$$

To calculate the scattered field from the geometry shown in Fig. 1, we can take two equivalent configurations drawn in Fig. 2a and Fig. 2b.

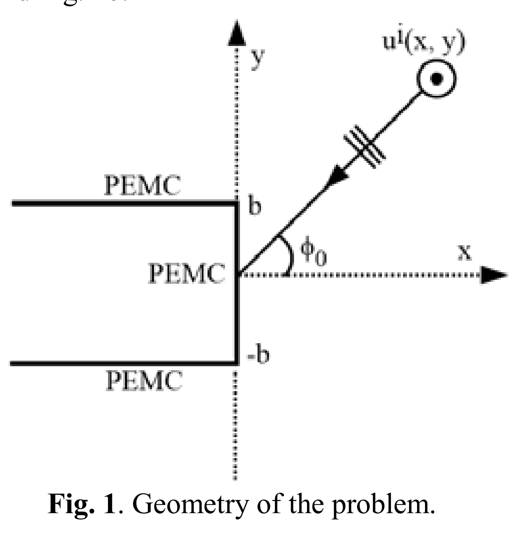

Fig. 1. Geometry of the problem.

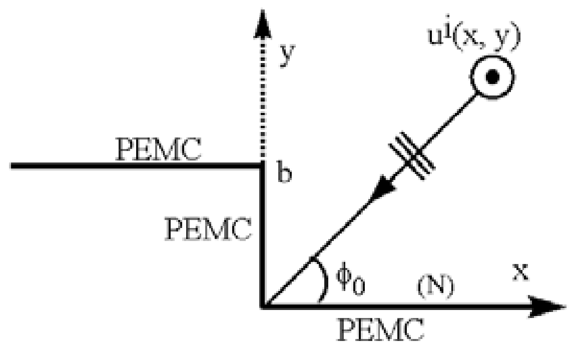

Fig. 2a. Even excitation case.

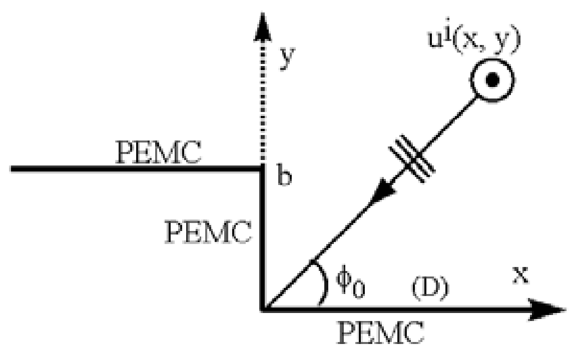

Fig. 2b. Even excitation case.

Using the method of image bisection the incident wave can be resolved in even and odd excitations. The geometry depicted in figure 1 is equivalent to a step protrusion with an impedance half plane at $y = b$ is connected through a step to a half plane at $y = 0$ satisfying Neumann boundary condition and Dirichlet boundary condition for the case of even and odd excitations respectively. The time dependence factor of the form $e^{-i\omega t}$, ω to be the angular frequency is supposed and hence not carried forward in the further analysis.

3. ANALYTIC SOLUTION OF THE PROBLEM FOR EVEN AND ODD EXCITATION CASES

Let us first consider the case of even excitation. The total field is symmetric at the plane $y = 0$ therefore the derivative of the total electric field disappear at $x \in (-\infty, \infty)$, $y = 0$. Wave equation satisfying by the total velocity potential for even excitation case u_T^e is

$$(\nabla^2 + k^2)u_T^e(x, y) = 0, \tag{3}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ where the superscripts e and o denote the odd excitation modes. The supporting boundary condition on half plane located at $-\infty < x < 0,\ y = b$ is:

$$(1 + \frac{\eta_1}{ik}\frac{\partial}{\partial y})u_T^e(x, b) = 0, \tag{4}$$

on the step located at $x = 0,\ 0 < y < b$ is:

$$(1 + \frac{\eta_2}{ik}\frac{\partial}{\partial x})u_T^e(0, y) = 0, \tag{5}$$

and the boundary condition on the step located at $0 < x < \infty,\ y = 0$ is:

$$\frac{\partial u_T^e}{\partial y}(x,0) = 0. \tag{6}$$

In addition to the related boundary conditions $(2-4)$, the continuity conditions at $0 < x < \infty,\ y = b$ should also be satisfied to achieve solution of problem under consideration.

$$u_T^e(x,b^+) = u_T^e(x,b^-), \tag{7}$$

$$\frac{\partial}{\partial y}u_T^e(x,b^+) = \frac{\partial}{\partial y}u_T^e(x,b^-). \tag{8}$$

The superscripts "+" and "−" indicate that the step is approached either from left or from right hand side along the $x-$axis. For analysis of problem, it is helpful to decompose $u_T^e(x,y)$ as follows [1, 2]:

$$u_T^e(x,y) = \begin{cases} \left(u^i(x,y) \equiv E_z^i(x,y)\right) + u_1^r(x,y) + u_1^e(x,y) & y > b \\ u_2^e(x,y)V(x) & 0 < y < b \end{cases}, \tag{9}$$

where $u_1^e(x,y)$ and $u_2^e(x,y)$ are the scattered fields, $u_1^r(x,y)$ is the reflected field from the plane at $y = b$, $V(x)$ is the unit step function. By substituting Eqs. (1), (2) and (9) into expressions $(3-8)$, we shall arrive to the situation that scattered fields $u_1^e(x,y)$ and $u_2^e(x,y)$ satisfy Helmholtz equations:

$$(\nabla^2 + k^2)u_1(x,y) = 0, \qquad y > b, \tag{10}$$

and

$$(\nabla^2 + k^2)u_2(x,y) = 0, \qquad 0 < y < b. \tag{11}$$

The boundary and continuity conditions will take the form

$$(1 + \frac{\eta_1}{ik}\frac{\partial}{\partial y})u_1^e(x,b) = 0, \text{ on } x < 0,\ y = b, \tag{12}$$

$$(1 + \frac{\eta_2}{ik}\frac{\partial}{\partial x})u_2^e(0,y) = 0, \text{ on } x = 0,\ 0 < y < b, \tag{13}$$

$$\frac{\partial u_2^e}{\partial y}(x,0) = 0, \text{ on } x > 0,\ y = 0, \tag{14}$$

$$u^i(x,b) + u_1^r(x,b) + u_1^e(x,b) = u_2^e(x,b), \text{ on } x > 0,\ y = b, \tag{15}$$

and

$$\frac{\partial\left[u^i(x,b) + u_1^r(x,b) + u_1^e(x,b)\right]}{\partial y} = \frac{\partial u_2^e(x,b)}{\partial y}, \text{ on } x > 0,\ y = b. \tag{16}$$

For analytical convergence, we assume that the wave number k has the positive imaginary part and $k = k_r + ik_i$. Since $u_1^e(x,y)$ satisfies Helmholtz equation in the range $x \in (-\infty,\infty)$, $y > b$. Let us define its Fourier transform on variable x as

$$F^e(\alpha,y) = \int_{-\infty}^{\infty} u_1^e(x,y)e^{i\alpha x}dx, \tag{17}$$

where $F^e(\alpha,y)$ is regular in the region of analyticity $\mathrm{Im}(k\cos\phi_0) < \mathrm{Im}(\alpha) < \mathrm{Im}(k)$ and inverse Fourier transform as

$$u_1^e(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F^e(\alpha,y)e^{-i\alpha x}d\alpha. \tag{18}$$

We notice that mathematical problem in the transformed complex plane α is same as that of Cinar and Buyukaksoy [14]. Therefore omitting of calculation details and employing the standard WH method [1] and the procedure used in [19] i.e., letting $\eta_1 = \eta_2 = 0$, we can obtain the results for the perfectly conducting (PEC) thick half plane in case of even excitation case as:

$$ik\frac{\xi_+(0,\alpha)}{N_+^e(\alpha)}R_+^e(\alpha) = -2k\sin\phi_0\frac{e^{-ikb\sin\phi_0}}{\alpha-\alpha_0}\frac{N_-^e(\alpha_0)}{\xi_-(0,\alpha_0)} + \sum_{m=1}^{\infty}\frac{K_m^e\sin(K_m^e b)}{\alpha+\alpha_m^e}\frac{N_+^e(\alpha_m^e)}{\xi_+(0,\alpha_m^e)}\frac{f_e^m}{2\alpha_m^e}, \tag{19}$$

and for perfectly conducting (PEC) thick half plane in case of odd excitation case as:

$$ik\frac{\xi_+(0,\alpha)}{N_+^o(\alpha)}R_+^o(\alpha) = 2k\sin\phi_0\frac{e^{-ikb\sin\phi_0}}{\alpha-\alpha_0}\frac{N_-^o(\alpha_0)}{\xi_-(0,\alpha_0)} + \sum_{m=1}^{\infty}\frac{K_m^e\cos(K_m^e b)}{\alpha+\alpha_m^o}\frac{N_+^o(\alpha_m^o)}{\xi_+(0,\alpha_m^o)}\frac{f_o^m}{2\alpha_m^o}. \tag{20}$$

where $\alpha_0 = k\cos\phi_0$, $\alpha = k\cos\phi$, $\alpha_m^e = \sqrt{k^2 - (\frac{2n\pi}{b})^2}$, $\alpha_m^o = \sqrt{k^2 - (\frac{(2n+1)\pi}{b})^2}$, $\frac{f_o^m}{2\alpha_m^o} = \frac{R_+^o(\alpha_m)}{M^o(\alpha_m^o)}$ and $\frac{f_e^m}{2\alpha_m^e} = \frac{R_+^e(\alpha_m)}{M^e(\alpha_m^e)}$.

4. DIFFRACTED FIELD ANALYSIS

Since diffracted field $u_1(x,y)$ can be evaluated as:

$$u_1(x,y) = \begin{cases} \dfrac{[u_1^e(x,y) + u_1^0(x,y)]}{2} & y > b \\ \dfrac{[u_1^e(x,y) - u_1^0(x,y)]}{2} & y < -b \end{cases}, \tag{21}$$

which can be calculated from expressions (19) and (20) as follows:

$$u_1(x,y) = \frac{1}{4\pi}\int_L \begin{cases} [\mathrm{R}_+^{\mathrm{e}}(x,y) + R_+^o(x,y)]\dfrac{e^{iK(a)(y-b)}}{[1+\frac{\eta_1}{k}K(\alpha)]}e^{-i\alpha x}d\alpha & \mathrm{y > b} \\ [\mathrm{R}_+^{\mathrm{e}}(x,y) - R_+^o(x,y)]\dfrac{e^{iK(a)(y-b)}}{[1+\frac{\eta_1}{k}K(\alpha)]}e^{-i\alpha x}d\alpha & \mathrm{y < -b} \end{cases}, \tag{22}$$

To assess far zone behavior of scattered field $u_1(x,y)$ the following transformations

$$x = \rho\cos\phi,\ y - b = \rho\sin\phi, \tag{23}$$

and

$$\alpha = -k\cos t, \tag{24}$$

are introduced in equation (22). Avoiding the details, far zone evaluation of the integral in Eq. (22) by the steepest decent method gives the diffracted field valid for $y > b$, for PEC thick half plane as [14]:

$$\begin{aligned} u_1(\rho,\phi) = &-\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{\sin\phi\sin\phi_0}{\cos\phi+\cos\phi_0}\frac{e^{-ikb\sin\phi_0}}{\sqrt{1-\cos\phi}\sqrt{1-\cos\phi_0}}\left[N_-^o(\alpha)N_-^o(\alpha_0) + iN_-^e(\alpha)N_-^e(\alpha_0)\right] + \\ &\left\{N_-^o(\alpha)\sum_{m=1}^{\infty}\frac{m\pi}{b}\frac{(-1)^m}{\alpha_m^o-\alpha}\frac{N_-^o(\alpha_m^o)}{\sqrt{k+\alpha_m^o}}\frac{f_o^m}{2\alpha_m^o} + iN_-^e(\alpha)N_-^o(\alpha)\sum_{m=1}^{\infty}\frac{(2m-1)\pi}{2b}\frac{(-1)^m}{\alpha_m^e-\alpha}\frac{N_-^e(\alpha_m^e)}{\sqrt{k+\alpha_m^e}}\frac{f_e^m}{2\alpha_m^e}\right\} \\ &\times\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{\sqrt{k}\sin\phi}{\sqrt{1-\cos\phi}}. \end{aligned} \tag{25}$$

Firstly, scattered field from PEC thick half is converted to the scattered field by PEMC thick half plane by using the concept of PEMC medium [18, 20]. PEMC medium depends upon a single scalar parameter M known as admittance of medium. Many problems involving canonical geometries in PEC medium have been transformed to PEMC medium by several investigators [21-28] through the duality transformation. In the present work, the scattering field from a PEMC thick half plane is examined. The scattered field by PEMC structures have a cross-polarized component which contributes for nonreciprocal behavior. This evidence confirms that PEC and PMC are the limiting cases of PEMC medium. Perfect electromagnetic conductor medium does not let the energy to play its role and such a barrier demonstrates an ideal boundary for the electromagnetic field. It is studied theoretically that a PEMC material acts as a best reflector of electromagnetic waves and varies from perfect electric and the perfect magnetic conductors media. Main target here is to convert diffracted field of perfectly electric conducting (PEC) thick half plane with duality transformations [18] as follows:

The dual transformed fields have been achieved from the diffracted fields via

$$\begin{pmatrix} E_d^s \\ H_d^s \end{pmatrix} = \begin{pmatrix} M\eta_0 & \eta_0 \\ \frac{-1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E^s \\ H^s \end{pmatrix}, \tag{26}$$

With

$$E_d^s = M\eta_0 E^s + \eta_0 H^s, \tag{27}$$

$$H_d^s = -\frac{1}{\eta_0} E^s + M\eta_0 H^s, \tag{28}$$

here E^s and H^s are scattered fields and E_d^s and H_d^s represent dual scattered fields from PEC half plane and obey the relation,

$$\eta_o H_d^s = -u_z \times E_d^s. \tag{29}$$

Here we set $u_1(\rho,\phi) = E^s$, so the amplitude of scattered field is given as

$$| E^s | = 10\log_{10}(u_1(\rho,\phi)\sqrt{k\rho}). \tag{30}$$

The field diffracted from PEMC thick half plane is given as

$$E = \frac{1}{(M\eta_0)^2 + 1}\left[((M\eta_0)^2 - 1)E^s - 2M\eta_0 E^s\right] \tag{31}$$

5. NUMERICAL RESULTS AND DISCUSSION

Mathematica software is used for the numerical and graphical results. The results proposed in [18, 20] are calculated to get better understanding of the behavior of the PEMC thick half plane. It is observed from [14] for the PEC case $(\eta_1 = \eta_2 = 0)$ that by increasing thickness parameter b, amplitude of the diffracted field increases. The amplitudes of scattered fields against the observation angle ϕ for PEC and PEMC thick half plane for thickness $b = \lambda/4$ can be seen in figures. 3a & 3b, for $b = \lambda/8$ can be seen in Figs. 4a & 4b and for $b = \lambda/16$ can be seen in Figs. 5a & 5b.Figures. 3b, 4b and 5b represent the amplitudes against the observation angle for the PEMC thick half plane with red dotted line for respective thickness as mentioned above. It is noted from graphs (3b, 4b & 5b) that red line for PEMC case is over lapping behavior of scattered field at $M = \pm\infty$ with green line for PEC case whereas again red line is overlapped with cyan line representing the PMC case having value $M = 0$. The crossed-polarized components are plotted for the values $M\eta_0 = 0.75$ and $M\eta_0 = -0.75$ and are represented by magenta and black lines respectively in graphs (3b, 4b & 5b). The PEMC medium exhibits maximal behavior for $M\eta_0 \to \pm 1$.

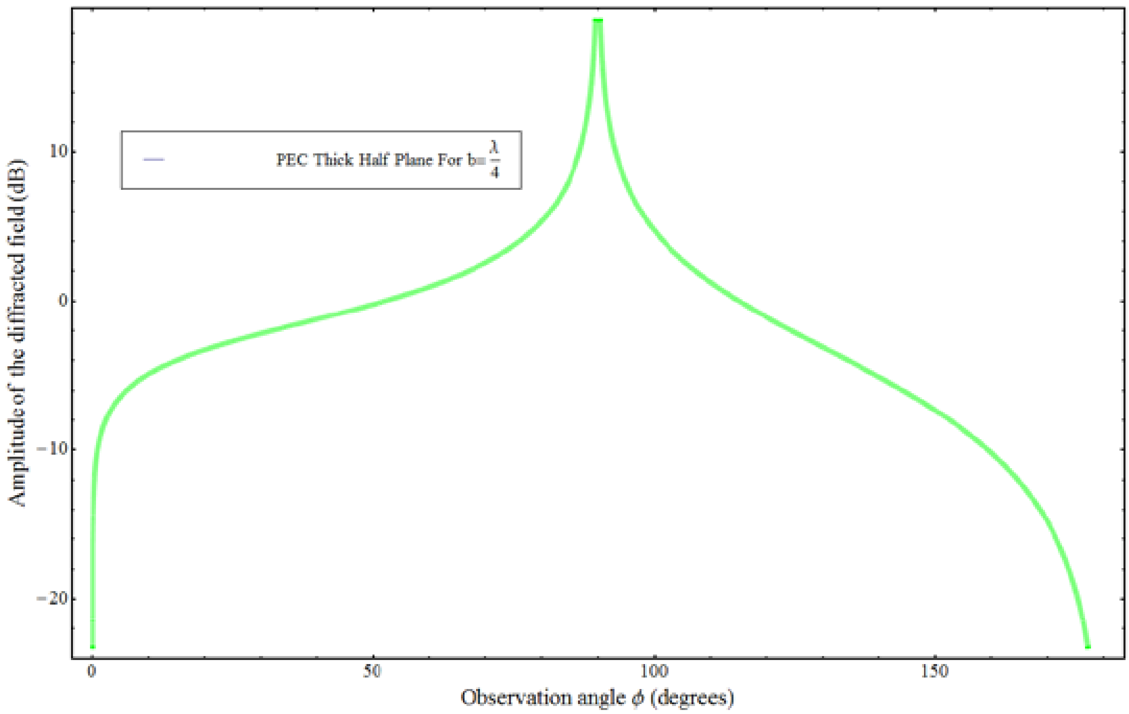

Fig. 3a. The amplitude of the diffracted field versus the observation angle for thickness parameter $b = \frac{\lambda}{4}$, for PEC case.

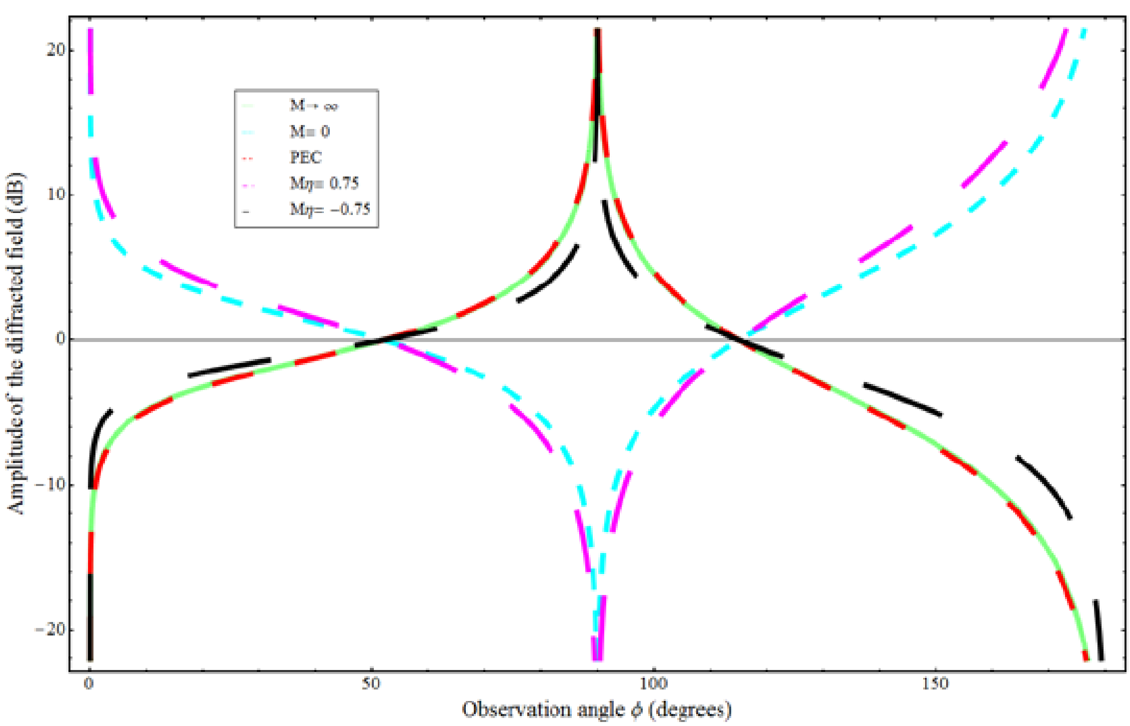

Fig. 3b. The amplitude of the diffracted field versus the observation angle for thickness parameter $b = \frac{\lambda}{4}$, for PEMC case.

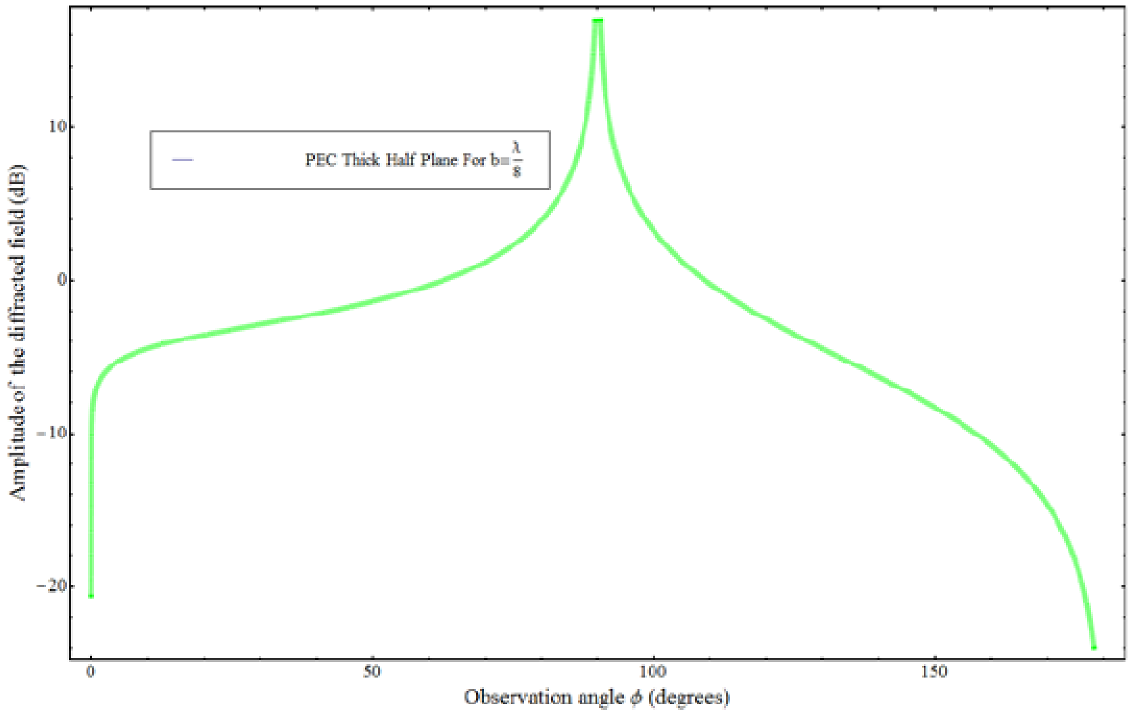

Fig. 4a. The amplitude of the diffracted field versus the observation angle for thickness parameter $b = \frac{\lambda}{8}$, for PEC case.

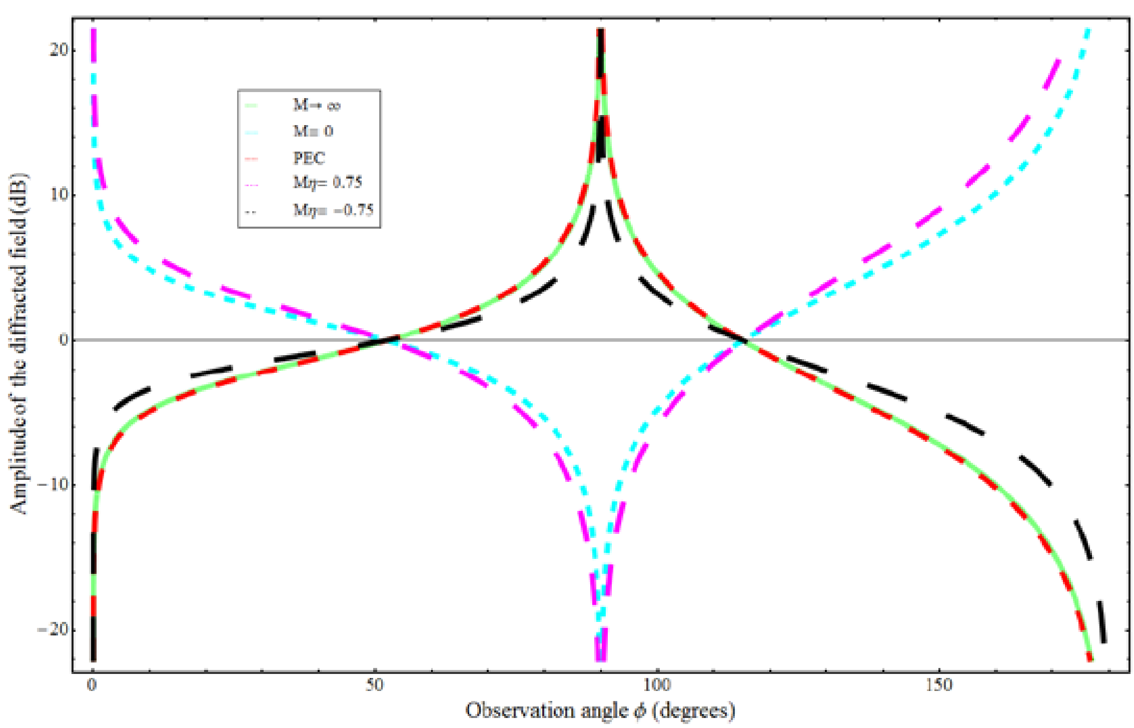

Fig. 4b. The amplitude of the diffracted field versus the observation angle for thickness parameter $b = \frac{\lambda}{8}$, for PEMC case.

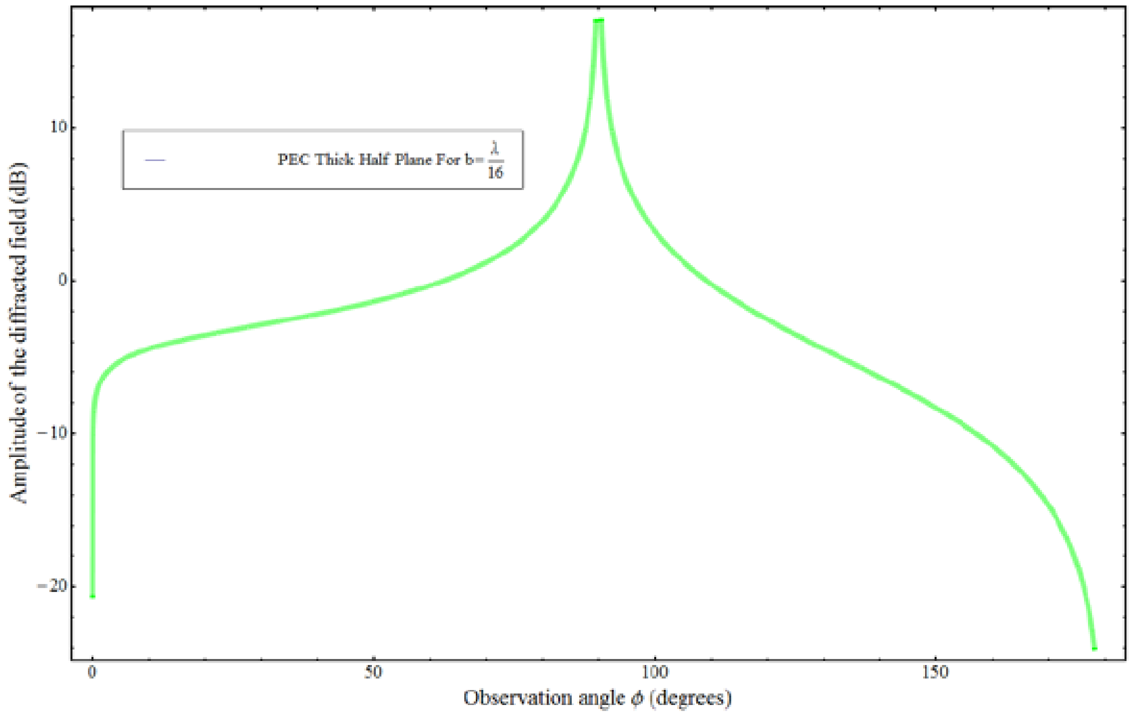

Fig. 5a. The amplitude of the diffracted field versus the observation angle for thickness parameter $b = \frac{\lambda}{16}$, for PEC case.

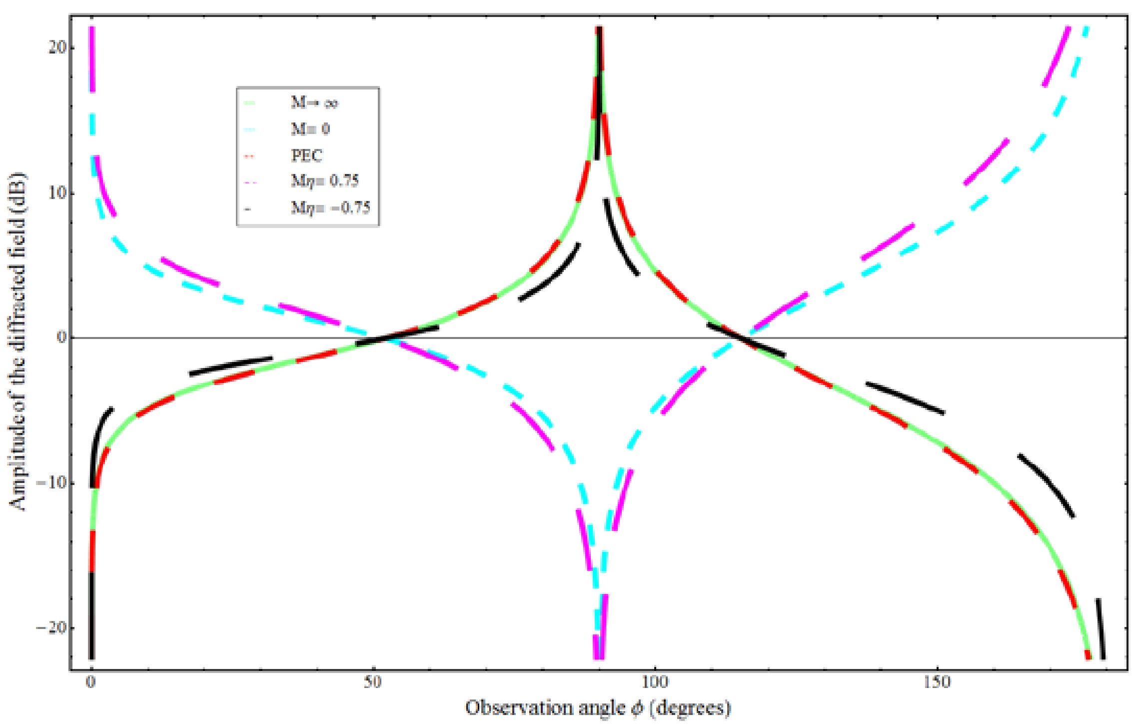

Fig. 5b. The amplitude of the diffracted field versus the observation angle for thickness parameter $b = \frac{\lambda}{16}$, for PEMC case.

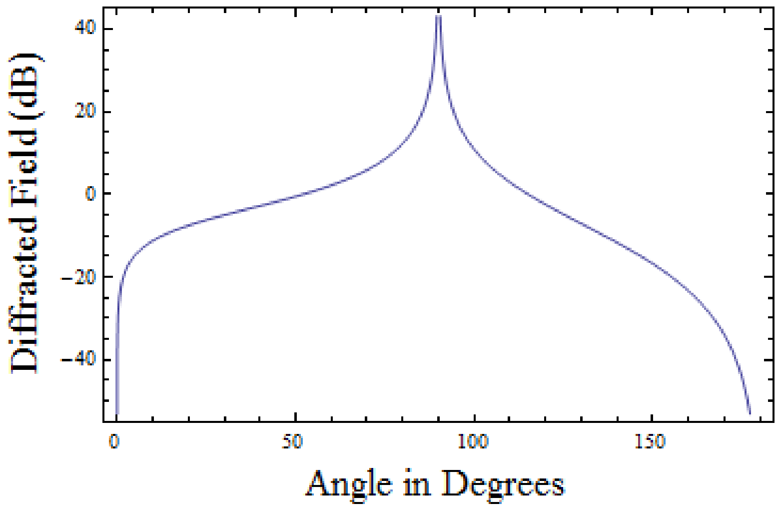

Fig. 6a. The amplitude of the diffracted field versus the observation angle for thick PEC half plane.

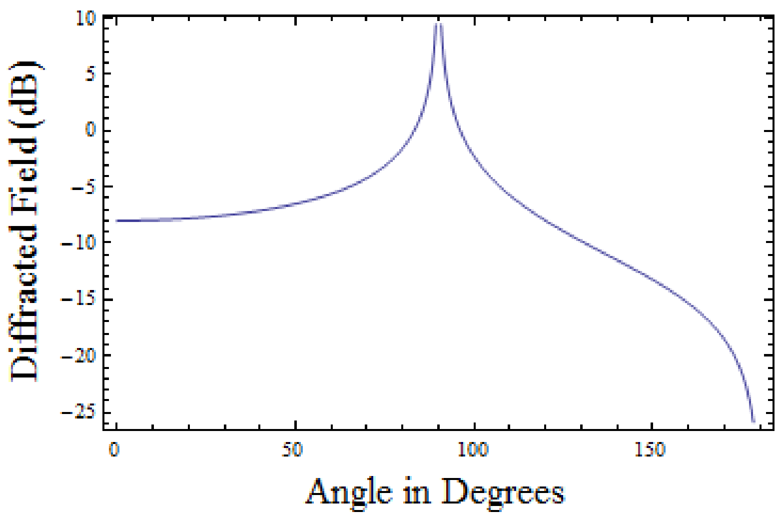

Fig. 6b. The amplitude of the diffracted field versus the observation angle for PEC half plane.

6. SPECIAL CASE

The thick PEC half plane becomes a PEC half plane when b tends to zero. The mathematical expression for scattered field from a PEC half plane becomes:

$$u_1(\rho,\phi) = -\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi}} \frac{\sin\phi \sin\phi_0}{\cos\phi + \cos\phi_0} \frac{1}{\sqrt{1-\alpha}\sqrt{1-\alpha_0}} \frac{e^{ik\rho}}{k\rho}. \tag{32}$$

It is remarked here that the graphs for the PEC thick half plane and PEC half plane $(b = 0)$ are also sown in this section in figures6a &6b respectively.

7. CONCLUDING REMARKS

Scattering by a PEMC thick half plane is studied in this paper. Mathematical expressions for scattered electric and magnetic field polarizations are obtained. It is found that the co polarized and cross polarized components depend on the parameter M only. It is noticed that analytical expressions for both electric and magnetic fields excitation can be calculated simultaneously in the PEMC medium which leads to a new aspect in scattering theory. The parameter M significantly important in perfect electromagnetic conductor theory to correlate the perfect electric and magnetic conductors' media. Cross-polarized diffracted components disappear for PEC and PMC media and are maximum when $M\eta_0 = \pm 1$. As a counter check, it is noted that $M = \pm\infty$ represents PEC case while $M = 0$ accounts for PMC case.

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