



## $\rho\pi$ Puzzle: Its Perspectives and Possibilities

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**Abstract:** In order to understand Charmonium Physics there are much theoretical explanations on “ $\rho\pi$  puzzle” of Charmonium decays with a comparative study of theoretical concepts and experimental data. “ $\rho\pi$  puzzle” is the violation of the 12% rule satisfied by most of the hadronic decay modes. In this paper, experimental and theoretical perspectives and possibilities are reviewed. Charmonium decay into light hadrons is a rich source of information about strong interactions between quarks and gluons. Flavor change in  $c\bar{c}$  is an ideal case to study light hadron spectroscopy to probe their gluon contents and flavor. In order to understand Chardynamics and the mysterious “ $\rho\pi$  puzzle” systematic, theoretical, experimental as well as concrete phenomenological knowledge is presented. Final state interaction, suppressed and enhanced hypothesis always play leading role to answer this question. Exploration of “ $\rho\pi$  puzzle” will be an expedition for opening new worlds of hadron phenomenology and a great scientific triumph for the pedestrians and scientists of high energy physics. For future developments, it is much urgent to construct new phenomenology, systematic measurements in charmonium decays are needed to answer “ $\rho\pi$  puzzle”. In this context, a large expected data of CLEO-c and BESIII will open a new era of chardynamics will give a profound understanding of “ $\rho\pi$  puzzle”.

**Keywords:**  $\rho\pi$  puzzle, charmonium annihilations, hadron spectroscopy, charmonia phenomenology, two body decay of charmonia

### 1. INTRODUCTION

In high energy physics, experimental puzzle always led to new prospects of scientific discoveries. In 1956,  $\theta - \tau$  puzzle led to the prediction of parity revolution. Such puzzles in high energy physics draw much attention of theorists and experimentalists. The Ratio of decay mode of charmed mesons  $J/\psi(3096)$  and  $\psi'(3686)$  to leptons and hadrons is also a type of puzzle since 1983.  $\psi'(3686)$  to  $J/\psi(3096)$  decay to hadrons is an OZI suppressed process, may decay via a photon or three gluons, for each case OZI rule defines suppressed decays of  $\psi'$  and  $J/\psi$  into photon or three gluons [1], anyhow perturbative QCD defines a relation,

$$Q_h = \frac{B(\psi' \rightarrow h)}{B(J/\psi \rightarrow h)} = \frac{B(\psi' \rightarrow e^+e^-)}{B(J/\psi \rightarrow e^+e^-)} \approx 12.7\% \quad (1)$$

This  $Q_h$  value is known as “12 % rule” holds good for both cases inclusive and exclusive decay

channels, but violates in some of the decay modes e.g.  $\rho\pi$  and  $K^*K$ . Where  $h$  is representation of any hadron and  $e^+e^-$  is leptonic representation. Also the  $\rho\pi$  and  $K^*K$  puzzle dramatically conflicts with perturbative QCD Hadronic Helicity Conservation. If HHC is considered as conserved factor for vector/pseudo scalar two body final states for each vertex as these are forbidden for leading twist [2]. This equation is known as “ $\rho\pi$  puzzle” which predicts that Eq. (1) is badly violated especially in “ $\rho\pi$  puzzle” and many other decay modes [3].

Following contents comprises of review of theoretical efforts on “ $\rho\pi$  puzzle” and the forecasts are compared with available experimental details, different methods are discussed to estimate the ratio of  $\psi'$  to  $J/\psi$ . Debates on “ $\rho\pi$  puzzle”, including theoretical aspects, phenomenological trails and experimental perspectives are given at the end following a brief summary.

## 2. EXPERIMENTAL EVIDENCE

In 1983 Mark-II found this effect for the first time. Since that time many theoretical efforts have done to solve this dramatic puzzle. Recent experimental results from CLEO-c and BESII about  $J/\psi$  and  $\psi'$  to two body decay modes, like vector tensor (VT), pseudo scalar vector (PV), and pseudoscalar-pseudoscalar (PP) states are clear evidence for existence of this puzzle. Where  $J/\psi$  attracted most attention in both theory and experiment for multibody decay channels of  $J/\psi$ ,  $\psi'$  and  $\psi''$  [3]. Various solutions have been proposed which can be verified at higher level of accuracy. Theory says that  $Q_h$  value is less than 12 % for this astonishing puzzle. Suppressed and enhanced  $J/\psi$  decay may be the cause of this longstanding puzzle. Both factors may be reason of this mystery. So, the relevant facts may be classified into three steps:

1.  $J/\psi$ - enhancement theory, which offers small  $Q_h$  value with enhanced branching ratio of  $J/\psi$  decays.
2.  $\psi'$  suppression theory, [4] which proves small  $Q_h$  value with suppressed branching ratios of  $\psi'$  decays.
3. Other concepts, which are rather than above two thoughts.

In this paper, firstly, there is a review of theoretical works on “ $\rho\pi$  puzzle” and theoretical predictions are compared with experimental facts and figures. Then some methods are discussed to estimate  $\psi'$  to  $J/\psi$  ratio for its decay into final states. Some comments are also discussed for the implication of “ $\rho\pi$  puzzle”, at the end a brief summary is given. BES group investigating [5] the largest sample of events considering the systematic investigation of  $\psi'$  decay mode to explain lowest lying final states. This process should be dealt in a systematic way by both experimental and theoretical efforts to solve the  $\rho\pi$  puzzle of charmed mesons like  $J/\psi$  and  $\psi'$ . In this context pedestrain tests are feasible for BES sample with proper phenomenology. It leads to the measurements of  $\psi' \rightarrow \omega\eta$ ,  $\rho\eta'$ ,  $\rho\eta$  to know about VP decay modes. It definitely requires a large amount of data and possibly there is need of the construction of new MDCHI for the future tau-charm physics at IHEP [5].

## 3. THEORETICAL EFFORTS ON “ $\rho\pi$ PUZZLE”

Some important theoretical features are discussed below.

### 3.1 $J/\psi$ Enhancement Hypothesis

$J/\psi \rightarrow \rho\pi$  is a process in which HHC (Hadronic Helicity Conservation) violates and it is a suppressed decay channel. So, there were efforts to find mechanism for the enhancement of this  $1^{--} \rightarrow \rho\pi$  decay channel. Two mechanisms are discussed below for this puzzle [3].

### 3.2 Glueball Admixture/ $\mathcal{O}$ meson

Freund and Nambu (FN hereafter) [6] proposed the idea of  $J/\psi \rightarrow \rho\pi$  via a glueball just after the discovery to understand the violation of OZI (Okubo-Zweig-Iizuka) rule [7]. In such schemes, violation is the result of  $\omega$ ,  $J/\psi$ ,  $\phi$  mesons mixing with SU (4) singlet vector state  $\mathcal{O}$ . It was found that such  $\mathcal{O}$  meson lie in the mass range 1.4-1.8 GeV/ $c^2$  with decay width more than 40 MeV/ $c^2$  and there should be its copious decay to  $\rho\pi$ ,  $K^*K$ . Quantitative predictions for experimental research is given for two channels.

$$\frac{\Gamma_{J/\psi \rightarrow \rho\pi}}{\Gamma_{\phi \rightarrow \rho\pi}} = 0.0115 - 0.087 \quad (2)$$

$$\frac{\Gamma_{J/\psi \rightarrow K\bar{K}}}{\Gamma_{J/\psi \rightarrow \rho\pi}} < 8 \times 10^{-5} \quad (3)$$

New available results by considering three pions as substitute of  $\rho\pi$  in  $J/\psi$  decays [8] first ratio  $R_1$  is obtained as,  $R_1 \approx 0.003$ . This is one order less than the predicted value. For the second ratio  $R_2$ , PDG [9] results for  $K^+K^-$ , and new experimental value for  $K_S^0 K_L^0$  [10], estimated results for  $B(J/\psi \rightarrow K\bar{K}) \sim 10^{-4}$  and for  $\rho\pi$  [3]  $\sim 10^{-2}$ , which is greater than the predicted value [3].

Hou and Soni [HS] proposed an earlier conception of [FN] [11] that the channel  $J/\psi \rightarrow \rho\pi$  is an enhancement by the mixation of glueball  $\mathcal{O}$  with  $J/\psi$  which further decays to  $\rho\pi$ . The main difference between HS and FN'S work is as follows:

1. According to the result of potential model, gluon mass is around  $2.4 \text{ GeV}/c^2$  [12] not  $1.4\text{-}1.8 \text{ GeV}/c^2$  [6];
2.  $\mathcal{O}$  Meson mixing was considered, as it was ignored in FN's work; and
3. In QCD gauge coupling constant is momentum dependent and mixing parameter is function of  $q^2$ , invariant mass, and mixing parameter decreases sharply with increasing  $q^2$ .

This propagation effect enhances more suppression of  $\psi'$  to  $J/\psi$  decay to  $\rho\pi$  and  $K^*\bar{K}$  modes. According to this assumption, HS proposed more search of vector gluonium state in hadronic decay channels of  $\psi' \rightarrow \pi\pi + X, \eta + X, \eta' + X$ , where X may be any VP final state [13]. By virtue of HS's idea, Brodsky, Tuan and Lepage [BTL] researched for the glueball theory of  $\rho\pi$  [3]. [BTL] confessed that  $J/\psi \rightarrow \rho\pi$  is the violation of helicity conservation selection rule of pQCD and emphasized that glueball  $\mathcal{O}$  should be fairly degenerate and narrow resonance with  $J/\psi$ . Although BTL model was recognized long for as "fly in the ointment" in the isospin violating decay mode  $J/\psi \rightarrow \omega\pi^0$ . Chen and Braten [CB] suggested this mass or even lightest glueball mass  $\mathcal{O}$  with  $J^{PC} = 1^{--}$  appeared in simulation of lattice QCD without active quarks also known as "quenched" approximation [14].

In  $Q_h$  factor, the final state  $h$  proceeds through gluonium satisfying the ratio,

$$Q_h = \frac{B_{\psi' \rightarrow e^+e^-} (M_{J/\psi} - M_{\mathcal{O}})^2 + \frac{\Gamma_{\mathcal{O}}^2}{4}}{B_{J/\psi \rightarrow e^+e^-} (M_{\psi'} - M_{\mathcal{O}})^2 + \frac{\Gamma_{\mathcal{O}}^2}{4}} \quad (4)$$

$Q_h$  factor will be small, if mass of  $\mathcal{O}$  state is near to the mass of  $J/\psi$ . Experiment states that mass of  $\mathcal{O}$  state [15] lies near  $80 \text{ MeV}/c^2$  of the  $J/\psi$  and total width lies less than  $160 \text{ MeV}/c^2$ . Brodsky et. al., recommended a simple way for the search of  $\mathcal{O}$  mass by scanning of  $e^+e^- \rightarrow V P$  cross section of  $J/\psi$  resonances. BES has searched this hypothetical particle experimentally in  $\rho\pi$  scanning across  $J/\psi$  region in  $e^+e^-$  decay as well as in  $\psi' \rightarrow \pi\pi\mathcal{O}$ , where  $\mathcal{O} \rightarrow \rho\pi$ , but no evidence found for its existence [16]. These results limitize decay width data to be near  $4 < \Gamma_{\mathcal{O}} < 50 \text{ MeV}/c^2$  and mass of  $\mathcal{O}$  near the range  $|M_{\mathcal{O}} - M_{J/\psi}| < 80 \text{ MeV}/c^2$  [17]. BES energy scan

shows absence of distortion for the channel  $J/\psi \rightarrow \rho\pi$  does not violate  $M_{\mathcal{O}} \simeq M_{J/\psi}$  shows lower limit to  $\Gamma_{\mathcal{O}}$ . Ref. [18] indicates that experiments shows that constrained mass is hundreds of  $\text{MeV}/c^2$  less than the mass of simplest vector glueball as resulted in lattice QCD without constituent quarks [19]. Recent data from BES and CLEO-c is unfavorable for this glueball hypothesis. For example, a major isospin violating channel  $\psi' \rightarrow \omega\pi^0$  [20]. This contradicts the concept that suppression pattern contradicts the dependence of spin-parity of final quarkonia. In  $\omega\pi^0$  to  $\rho^0\pi^0$  ratio is calculated in [21] as,

$$\frac{B(J/\psi \rightarrow \omega\pi^0)}{B(J/\psi \rightarrow \rho^0\pi^0)} < 0.0037 \quad (5)$$

which is less than PDG06 predicted value 0.08 [09]. Another unfavorable result for this hypothesis is  $\psi'$  suppressed decay into VT (vector-tensor) states, [22]. As hadronic VT decay mode conserve HHC mechanism while VP decay mode does not. Ref. [23] explains that glueball theory has some unanswered points:

1. Only  $\rho\pi$  and  $K^*\bar{K}$  are affected, but other channels  $\mathcal{O} \rightarrow 5\pi$  not, also  $\mathcal{O}$  couples dominantly to  $\rho\pi$  and  $K^*\bar{K}$ .
2. If such type of heavy  $1^{--}$  gluonium state is present, then why  $2^{++}, 0^{++}$  has not found, which are less heavy and easy to search? [3]

### 3.3 Intrinsic Charm Wave Function Explanation

The experimental evidence and discussions about intrinsic charm is termed as charm content in the nucleon. On the other part, a highly successful and well known constituent quark model gives phenomenological explanation that in the nucleon there are three quark constituents. If the two physical pictures are reconciled, one concludes that these constituent quarks are personally complicated objects having a light  $q\bar{q}$  pair, a sea of gluons, and a small non negligible  $c\bar{c}$  intrinsic charm quark. In extension, intrinsic components are extended from diagramic contribution which results in the multiconnection of two or more valence components in nucleon. This implies that vector mesons like  $K^*, \rho$ , etc. contain intrinsic charm quark. The extraordinary  $\rho\pi$  puzzle can also

be examined in other ways, like wavefunction with intrinsic charm in the final state hadrons [24]. In Brodsky and Karliner suggestion that decay channel  $J/\psi, \psi' \rightarrow \rho\pi$  is a result of  $\rho$  wavefunction of intrinsic charm quarks. They suggested that there is a nodeless wavefunction for  $c\bar{c}$  in the Fock state  $|u\bar{d}c\bar{c}\rangle$  of  $\rho^+$ . Which results in a large overlap of charmed states  $J/\psi$  and  $\psi'$ . However, this model is a challenge for the assumption that charmed meson must decay via intermediate gluons to light hadrons [5]. Brodsky and Karliner (BK) give explanation of Fock components of intrinsic charm  $|q\bar{q}c\bar{c}\rangle$  light vector state [25]. For example, if light cone is considered for Fock representation of  $\rho$  is: [3]

$$\rho^+ = \psi_{u\bar{d}}^\rho |u\bar{d}\rangle + \psi_{u\bar{d}c\bar{c}}^\rho |u\bar{d}c\bar{c}\rangle + \dots \dots \quad (6)$$

The wavefunction  $\psi_{u\bar{d}c\bar{c}}^\rho$  is maximized with minimal invariant mass; i.e; with equal rapidity with the constituents and for the spin configuration, whereas  $u\bar{d}$  is a pseudoscalar state, which minimizes the QCD spin-spin coupling. In [24], a rough estimate is being made about the decay rate of  $J/\psi \rightarrow \rho\pi$  by comparing it with analogue decay  $\phi \rightarrow \rho\pi$ , as

$$\Gamma(\phi \rightarrow \rho\pi) \approx 6 \times 10^{-4} \text{ GeV} \quad (7)$$

Initial and final state radial configuration could play an important role in  $J/\psi \rightarrow \rho\pi$  puzzle [24]. The decay rates of  $J/\psi \rightarrow \rho\pi$  and  $\phi \rightarrow \rho\pi$  from quark pattern is roughly scaled with  $R\left(\frac{c\bar{c}}{s\bar{s}}\right)$  with respect to phase space, while it is assumed that integration of quark wavefunctions gives similar results. This assumed decay rate  $\Gamma(J/\psi \rightarrow \rho\pi) \sim 10^{-6} \text{ GeV}$  is consistent with measured value  $10^{-6} \text{ GeV}$ . Fig. 1 indicates that, this approach favors to lowest invariant states produces a strong coupling constant  $g_{J/\psi \rightarrow \rho\pi}$ , means there is a natural overlapping between  $J/\psi$  and  $\rho\pi$  which prefers the  $J/\psi \rightarrow \rho\pi$ , as schematically shown in Fig. 1(b). And  $\psi' = \psi(2S)$  decay is naturally suppressed due to its mode in radial wavefunction also drawn schematically in Fig. 1(c). In  $J/\psi(1S) \rightarrow \rho\pi$  intrinsic charm in pion also favors this decay to enhance through constituent quark rearrangement figures [24].

#### 4. SUPPRESSED HYPOTHESIS

The hypothetical glueball faced many questions after its arrival. In [23], a point is raised that

suppressed helicity is not strong argument for charmonium decays. One argues that  $J/\psi \rightarrow \rho\pi$  is not enhanced one channel, but  $\psi' \rightarrow \rho\pi$  is suppressed one. Following is given detailed explanation for  $\psi'$  suppression [3].

#### 4.1 Sequential Fragmentation Scheme

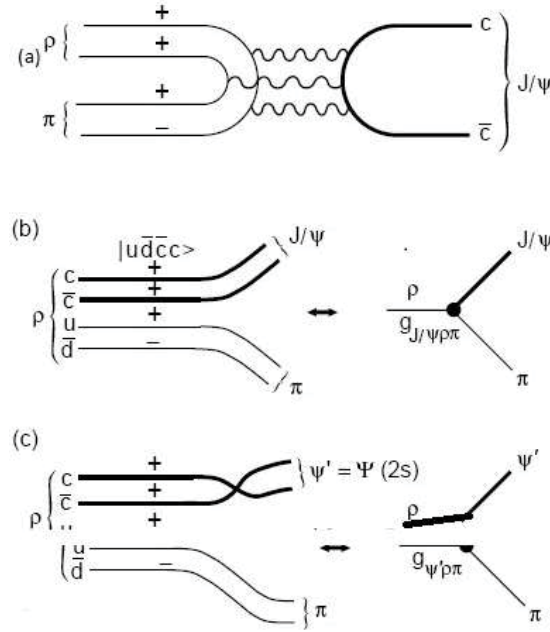
Karl and Roberts (KR) explained  $\rho\pi$  puzzle on the scheme of sequential fragmentation of quark pair creation [26]. The thought proceeds as the mesons produce sequentially; as a result, the amplitude to produce two quark pairs in their ground energy level is an oscillation of the system with total energy. The argument was that oscillatory function probability has minimum value near  $\psi'$  mass, which could be reason of  $\psi'$  suppressed decay. Also the data calculation just accommodates decays of  $J/\psi$  and  $\psi'$  to  $K^*\bar{K}$  and  $\rho\pi$ , it is difficult to explain  $\mathbf{Y}$  decays. Their calculation says that the oscillation probability of  $\mathbf{Y}$  resonance has a chance of the presence of  $\rho\pi$  for all channels like  $\mathbf{Y}, \mathbf{Y}', \mathbf{Y}'' \dots$  etc. So, the predicted decay width value of  $\rho\pi$ ,  $\Gamma(\mathbf{Y} \rightarrow \rho\pi) = 0.05 \text{ KeV}$  and branching fraction  $B(\mathbf{Y} \rightarrow \rho\pi) < 2 \times 10^{-4}$  [8].

#### 4.2 Exponential Form Factor Scheme

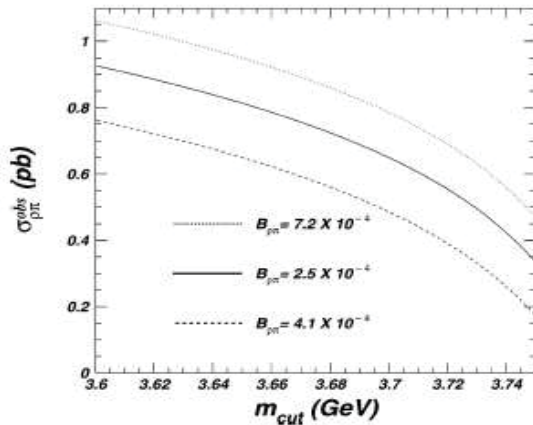
Chachian and Tornqvist explained  $\psi'$  to  $J/\psi$  suppressed ratio on the basis of non-relativistic quark model describes that hadronic form factor has an exponential decrease for these states [23]. There is a drastic suppression in two body decay rate of  $\psi'$  to  $J/\psi$  ratio. According to this model, it does not only estimates up mode for having small B, but also for VT channels, which is not in accordance with BES results [22].

#### 4.3 FockState Mechanism

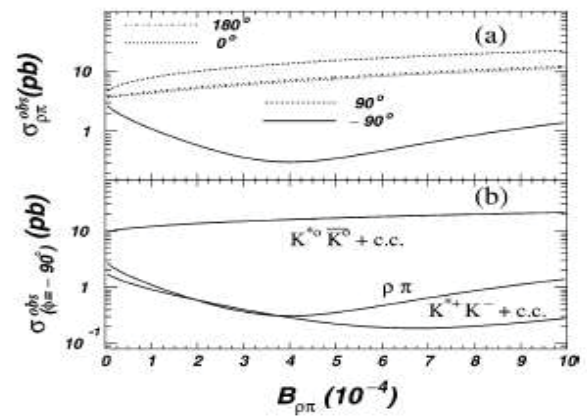
Prominent decay of  $J/\psi \rightarrow \rho\pi$  and suppression of  $\psi(2S)$  is caused by Fock components of light vector states and is naturally followed by intrinsic charm  $|c\bar{c}q\bar{q}\rangle$  Fock states. A basic tenet of QCD is that heavy quark mesons like  $J/\psi, \psi'$  necessarily decay into light mesons with the emission of gluons. OZI rule is the main motivation of this assumption which states that some hadronic states are suppressed, and QCD also corresponds to this assumption that there is some amplitude which possesses numerical suppression with the presence of intermediate gluons. Chen and Braaten (CB)



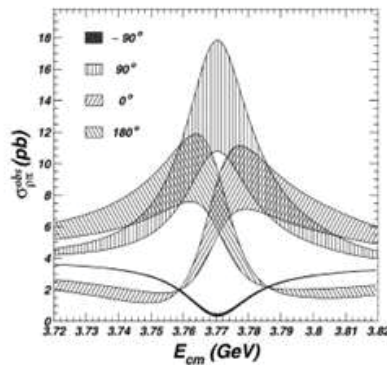
**Fig. 1.** Decay mechanism of  $J/\psi \rightarrow \rho\pi$ : (a) The annihilation of  $J/\psi \rightarrow \rho\pi$ ; (b) The induced coupling  $g_{(J/\psi \rightarrow \rho\pi)}$ . (c) The suppressed coupling of  $\psi' \rightarrow \rho\pi$  [24].



**Fig. 2.** (a) Observed cross section of  $\rho\pi$  for different angles [31]



**Fig. 3.** (a) Different phases of  $B_{(\psi^{\prime n} \rightarrow \rho\pi)}$  cross section. (b) Comparison of  $K^{*0}, \bar{K}^0, K^{*+}, K^- + c.c.$  for  $\phi = 90^\circ$  as a branching function of  $B_{\psi^{\prime n} \rightarrow \rho\pi}$  [31]



**Fig. 4.** Center of mass energy variation of  $\rho\pi$  cross section for different angles ranging from  $-90^\circ, +90^\circ, 0^\circ, 180^\circ$  respectively [31].

suggested a solution for  $\rho\pi$  puzzle, they were in argument that  $J/\psi \rightarrow \rho\pi$  channel is a Fock state  $|u\bar{d}c\bar{c}\rangle$ , where  $c\bar{c}$  charmonia represents color octet state  ${}^3S$  decaying via  $c\bar{c} \rightarrow q\bar{q}$ , while  $\psi' \rightarrow \rho\pi$  is a suppressed decay mode with small energy gap for the masses of  $\psi'$  and  $D\bar{D}$  value. CB predicted branching fraction values for many VP channels of  $\psi' \rightarrow \rho\pi, K^*\bar{K}$  as shown in Table 1. There is a clear obedience for many VP channels but  $\omega\pi$  is deviated from predicted value. Where the values may be different from recent branching fractions for  $K^*\bar{K}$  and  $K^{*0}\bar{K}^0$ , etc. of BES [27] and CLEO-c. In Table 1 CB's prediction has little bit effect on angular distribution value, which are in general  $1 + \alpha \cos^2\theta$ , with  $-1 < \alpha < +1$ . CB's proposal was that  $\alpha$ 's value for any two body decay of  $\psi'$  should be equal to or less than corresponding  $J/\psi$  decay. But this supposition needs further explanation in future with huge data samples. In Table 2 predicted and experimental data is compared with 90% confidence level [9].

#### 4.4 S-D Wave Mixing Formulae

$\psi'$  is a D-wave ( $c\bar{c}1^3D_1$ ) charmonium state, where leptonic decay width shows its mixing with S-wave nearby  $\psi(2^3S_1)$ . So, Rosner was in point of view that  $\psi' \rightarrow \rho\pi$  suppression is due to the fact that there is a mixing of S and D matrix elements with this mechanism.

$$\langle \rho\pi | \psi' \rangle = \langle \rho\pi | 2^3S_1 \rangle \cos\phi - \langle \rho\pi | 1^3D_1 \rangle \sin\phi \sim 0 \quad (8)$$

$$\langle \rho\pi | \psi'' \rangle = \langle \rho\pi | 2^3S_1 \rangle \sin\phi + \langle \rho\pi | 1^3D_1 \rangle \cos\phi \sim \langle \rho\pi | 2^3S_1 \rangle \sin\phi \quad (9)$$

So,  $\phi$  represents mixing angle of pure  $\psi(2^3S_1)$  and  $\psi(1^3D_1)$  states [28], its leptonic width fitting of  $\psi''$  and  $\psi'$  lies near  $(12 \pm 2)^\circ$  [29], consistent with other results of [30]. If coupling and mixing of  $\psi''$  and  $\psi'$  completely cancels out  $\psi' \rightarrow \rho\pi$  amplitude ( $\langle \rho\pi | \psi' \rangle = 0$ ), so  $\psi' \rightarrow \rho\pi$  will be more prominent decay instead of  $\psi'' \rightarrow \rho\pi$ , defined by the factor  $\frac{1}{\sin^2\theta}$ , in [29], solid results shows that

$$B(\psi'' \rightarrow \rho\pi) = (4.1 \pm 1.4) \times 10^{-4} \quad (10)$$

In [31] results for  $\psi''$  in  $e^+e^-$  annihilation indicates that continuum contribution is important. According to Rosner's point of view, Born order

cross-section of  $\psi'' \rightarrow \rho\pi$  is,

$$\sigma_{\psi'' \rightarrow \rho\pi}^{Born} = (4.8 \pm 1.9) \text{ pb} \quad (11)$$

This value is comparable with continuum cross-section value,

$$\sigma_{e^+e^- \rightarrow \rho\pi}^{Born} = 4.4 \text{ pb} \quad (12)$$

Experimental analysis shows that there is phase coherence between electromagnetic and strong decay amplitudes in charmonium annihilations. So, all that decay is coherent sum of two body decay amplitudes. This assumption refers to the interference of strong amplitudes with continuum amplitude either fully destructively in case of  $\rho\pi, \omega\eta, K^*\bar{K}$  or fully constructively, e.g.  $K^{*0}\bar{K}^0$ . The destructive interference shows that cross-section values are less on upper points of resonance. Recently, experiments on  $\rho\pi$  [32] and other VP states represent such interference phenomena. This result supports eq. 10. But to understand  $B(\psi'' \rightarrow \rho\pi)$ , present experimental values are not enough, this resonance must be checked out [33]. Rosner's work is a good task of  $\tau$ -charm physics. Furthermore, Mo, Wang and Yuan elaborates wave mixing (S+D) scheme to proton-proton final states [3].

## 5. OTHER DESCRIPTIONS

Some other provoking explanations are given in this section for  $\rho\pi$  puzzle.

### 5.1 Final State Interaction (FSI)

Final state interaction may play an important role in charmonia decays [34]. This process can be induced with strong interactions at minimum energy state, and is governed by non pQCD, which is not described within present theoretical framework. Phenomenological interpretations are needed to solve the mysteries. Ref. [35] states that FSI and direct  $J/\psi$  decay to pseudoscalar and vector meson results that both reasons contribute to data and there should be destructive interference for data explanation. Proper evaluation of such effects is difficulty in this path. Final state interaction is another cause of suppression, which may be the reason of small R-value, also enhances

the branching ratio  $R$  of  $J/\psi \rightarrow \rho\pi$  [34], but because theoretical predictions depends on phenomenological parameters, so this assumption is not fine. So, one cannot convince that FSI is final answer. If we consider the thought that  $J/\psi$  is not a pure charmonia  $c\bar{c}$  bound state, so it would be a great challenge, as  $c\bar{c}$  bound state is considered for charmonia from very beginning. If we consider it really true, all the previous work, parameters, potential models, data fittings etc. may be reconsidered. Pretty interesting factor, Suzuki [36], Zhao [37] do not accept small branching ratio  $R$ -value as a ‘‘puzzle’’ because they think that FSI plays an important role in  $\psi' \rightarrow$  hadrons [3].

### 5.2 OZI Process

OZI process is based on Standard Model straightforwardly. The decay width of the forbidden OZI process is calculated for ( $J/\psi \rightarrow ggg \rightarrow \rho\pi$ ) and the width is found proportional to  $(\frac{m_q}{m_{J/\psi}})^2$ . This factor is known as hadronhelicity suppression factor. The suppression factor for  $J/\psi \rightarrow \rho\pi$  should be less than 0.1% numerically and same for  $\psi'$ . Isospin violating subprocess  $J/\psi \rightarrow \rho\pi$  is recalculated for a test. This process is also dominated sometime by another subprocess  $J/\psi \rightarrow \gamma^* \rightarrow \pi\pi$  due to the violation of isospin in electromagnetic process. Conclusion indicates that there is proportionality between transition amplitude and  $(\frac{m_u - m_d}{m_{J/\psi}})$ , i.e; this mass difference in quarks results in isospin violation. Numerical results  $J/\psi$  for  $J/\psi \rightarrow ggg \rightarrow \rho\pi$  are consistent with data as electromagnetic subprocess is also responsible for isospin violation and consistent with data. Above observation reveals that EM and OZI subprocess have same magnitude but theoretical explanations are model dependent. Relative ratio of EM contribution and OZI is approximately proportional to  $\frac{\alpha_s}{\alpha_k} \sim 1.1$ . Where  $\frac{1}{\pi}$  is a factor of loop integration,  $\alpha \sim \frac{1}{137}$  is an electromagnet constant, and for  $J/\psi$  energy level,  $\alpha_s$  is nearly 0.3k, which is numerically equal to  $\mathcal{O}(1)$  responsible for quark pair production in case of vacuum. Numerically, this value is equal to the running constant  $\alpha_s$ . Since near the charmonia region, non pQCD plays an important role. Where the value of  $\alpha_s$  has an uncertainty [35].

### 5.3 Large Phase Mechanism

Suzuki explained FSI for  $J/\psi$  decay in another way. His research was based on amplitude analysis of  $J/\psi \rightarrow 1^-0^-$  decay channel. Results of his efforts were in argument that a large phase is present between a photon and three gluon amplitude. But large phase cannot be explained on the basis of pQCD interactions, it should be done on the long distance range of strong interaction, termed as hadrons rescattering in an inelastic range. After this Suzuki researched for another decay mode of  $J/\psi \rightarrow 0^-0^-$ , and again the result was large phase [38]. He also found that inclusive decay rate of  $J/\psi$  are in agreement with exclusive decay rate. He concluded that relative suppression for  $\frac{\psi' \rightarrow 1^-0^-}{J/\psi \rightarrow 1^-0^-}$  originates for  $\psi'$  not for  $J/\psi$  [38].

### 5.4 Mass Reduction Scheme

Ma proposed QCD factorization concepts for the radiative decays of  $1^-0^-, \eta$  and  $\eta'$ , and resulted theoretical values were consistent with CLEO-c experiment. The uncertainty in this analysis comes from relativistic corrections of charmonium mass. Ma was in argument that uncertainties may be reduced by quarkonium masses rather than quark mass. Such reduction provides a modified value in the original 12% value [3],

$$Q_h = \frac{B(J/\psi \rightarrow \rho\pi)}{B(\psi' \rightarrow \rho\pi)} = \frac{M_{J/\psi}^8 B(J/\psi \rightarrow e^+e^-)}{M_{\psi'}^8 B(\psi' \rightarrow e^+e^-)} = (3.6 \pm 0.6)\% \quad (13)$$

Although the result is much different from experimental value listed in table I [3].

### 5.5 Discussion

In this brief history of  $\rho\pi$  puzzle, theorists concentrate some special channels like  $\rho\pi, K^*\bar{K}$  e.g; Soni’s and Hou’s efforts. As long as the theory developed and experiment progressed, scientists provide more comprehensive mechanisms of charmonia decay modes. e.g, Feldmann and Kroll did so. Research, review and analysis of all previous work on  $\rho\pi$  puzzle demands for more comprehensive details. Quantitative explanation of all charmonium decay channels are under observation. Fock state representation is of severe importance, many

scientists explained this concept e.g; BK [25], CB [22], and FK [39].

## 6. PROSPECTS OF FINDING OF $\psi'$ to $J/\psi$ DECAY RATIO

Different approaches are used to estimate the puzzle, including one theoretical analysis [1], and other from experimental efforts [40].

### 6.1 Theoretical Efforts

The  $\psi'$  to  $J/\psi$  ratio R for hadronic decay is compared with pQCD, known as “12% rule”. pQCD [1] explains  $J/\psi$  as bound state of charm and anticharm quark. Dominate decay of  $J/\psi$  is processed to be through three gluons into light hadrons. For annihilation the separation between  $c\bar{c}$  is much smaller than their size of the order of  $\frac{1}{mc}$ . So, the annihilation amplitude for  $\psi'$  to  $J/\psi$  (S-wave mixing) is proportional to wavefunction at  $\psi(r=0)$  [1]. They decay width into light hadron is proportional to  $|\psi(0)|^2$ . Similarly, decay width to  $e^+e^-$  is also proportional to  $|\psi(0)|^2$ . Which is just according to 12% rule as given by eq. (1). Some correction factors are also applied to this approximation. If first order correction is applied to the branching fraction for  $J/\psi \rightarrow e^+e^-$  so that correction could be 50% of least term. If we suppose  $\alpha_s(m_{J/\psi}) \sim 0.2$  [41], ignoring relativistic effect. In this case, the mass difference between  $\psi'$  and  $J/\psi$  is about 20% and  $\langle \frac{v^2}{c^2} \rangle \sim 0.24$  for  $J/\psi$ , where this correction factor is at lowest order [42]. While finite size of decay vertex reduces three gluon decay width of  $J/\psi$  [43]. Where non perturbative effect is ignored, anyhow, the question arises, is 12 % rule is hallmark for checking compatibility of experimental data [3].

### 6.2 Experimental Method

The first experimental assumption for  $\psi'$  to  $J/\psi$  decays were based on estimation that both of these states are categorized into radiative decays, electromagnetic decays, hadronic decays (ggg) and decaying into lower mass charmed states like ( $c\bar{c}X$ ) [40]. So, the relation

$$B(\text{ggg}) + B(\gamma\text{gg}) + B(\gamma^*) + B(c\bar{c}X) = 1 \quad (14)$$

One gets,

$$B(\text{ggg}) + B(\gamma\text{gg}) = 1 - B(\gamma^*) + B(c\bar{c}X) \quad (15)$$

Table 3 shows the calculations of  $B(\gamma^*)$  and  $B(c\bar{c}X)$ . So, the values of

$$B(J/\psi \rightarrow \text{ggg}) + B(J/\psi \rightarrow \gamma\text{gg}) = (73.3 \pm 0.5)\% \quad (16)$$

$$B(\psi' \rightarrow \text{ggg}) + B(\psi' \rightarrow \gamma\text{gg}) = (18.9 \pm 1.3)\% \quad (17)$$

So, by calculating ratio, one gets,

$$\frac{B(\psi' \rightarrow \text{ggg} + \gamma\text{gg})}{B(J/\psi \rightarrow \text{ggg} + \gamma\text{gg})} = (25.7 \pm 1.7)\% \quad (18)$$

Above result is in accordance with [39]. The first order pQCD calculation for the decay rates of  $\gamma\text{gg}$  and  $\text{ggg}$  is as follows in [41].

$$\frac{\Gamma(J/\psi \rightarrow \gamma\text{gg})}{\Gamma(J/\psi \rightarrow \text{ggg})} = \frac{16}{5} \frac{\alpha}{\alpha_s(m_c)} \left(1 - 2.9 \frac{\alpha_s}{\pi}\right) \quad (19)$$

Considering  $\alpha_s(m_c) = 0.28$  this ratio goes to 0.062. Similar calculations can be obtained for  $\psi'$  decay then,  $B(J/\psi \rightarrow \text{ggg}) \cong (17.8 \pm 1.2)\%$ . While the results of eq. (18), the 25.7 % ratio is according to recent results of CLEO and BES [10]. This ratio of eq. (18) in literature is termed as, hadronic excess of  $\psi'$  has some suppressed decay modes, but ( $\psi' \rightarrow \text{ggg}$ ) is raised with respect to 12 % rule as predicted in  $J/\psi$  decays [3].

## 7. DEBATES ON $\rho\pi$ PUZZLE

### 7.1 Theoretical Aspect

Non-relativistic potential model deals Charmonium physics upto a good approximation involving  $\rho\pi$  puzzle. However, detailed examination is required to explain these assumptions. As section 6.1 indicates, several correction factors are needed for the ratio of  $J/\psi$  and  $\psi'$ , where non perturbative QCD effects are neglected, which is hard for charmonium physics. It is noted that non-linear or some non-perturbative effects are not consistent with experimental data. But such effects are hard to include in Non-Relativistic Potential Model. It gives good results for interference of strong and electromagnetic amplitudes of  $\psi'$  annihilation to pseudoscalar mesons [44]. If S-D wave mixing scheme is believed to be true for  $\rho\pi$  puzzle, then D-wave matrix element into light hadrons will be greater. So, potentials model or other theories accommodate this D-wave mixing scheme. It



shows that current charmonium physics is still incomplete [45]. Phenomenology produces sound basic results, which can be verified by experiments and lead to

**Table 1.** Theoretical and experimental comparison of  $\rho\pi$ ,  $K^{*0}\bar{K}^0 + c.c.$  and other VP states [3].

VP	X = 0.64 (Theoretical)	Experimental.
$P\pi$	0 – 0.25	0.13±0.03
$K^{*0}\bar{K}^0 + c.c.$	1.2 – 3.0	3.2 ± 0.08
$\Omega\eta$	0 – 1.6	<2.0
$\omega\eta'$	12 – 55	19 <sup>+15</sup> <sub>-13</sub>
$\Omega\pi$	11 – 17	4.4 <sup>+1.9</sup> <sub>1.6</sub>

**Table 2.** Comparison of predicted values in ref. [39] and experimental values of ref. [9] for R of  $\psi$ , with upper limits at 90% confidence level.

VP	Predicted value	Experimental value
$P\pi$	1.3	3.2 ± 1.2
$K^{*0}\bar{K}^0 + c.c.$	5.1	10.9 ± 2.0
$\Omega\eta$	1.2	<1.1
$\omega\eta'$	6.3	3.2 <sup>+2.5</sup> <sub>2.1</sub>
$\omega\pi^0$	3.8	2.1 ± 0.6

**Table 3.** Experimental results for branching ratios of  $J/\psi$  and  $\psi'$  for decay through virtual photon [3].

Decay Mode	B( $J/\psi$ )	B( $\psi'$ )
$\gamma^* \rightarrow hadrons$	(13.50±0.30) %	(1.66 ± 0.18) %
$e^-e^+$	(5.94 ± 0.06) %	(7.35 ± 0.18) × 10 <sup>-3</sup>
$\mu^+\mu^-$	(5.93 ± 0.06) %	(7.3± 0.8) × 10 <sup>-3</sup>
$\tau^+\tau^-$	-	(2.8 ± 0.7) × 10 <sup>-3</sup>
$\gamma^* \rightarrow X$	(25.37 ± 0.35) %	(3.41 ± 0.27) %

### 7.2 Phenomenological Trails

In the very beginning of  $J/\psi$  discovery  $J/\psi$  was considered as hydrogen atom of QCD, and it was supposed that  $J/\psi$  will play same role in QCD for quark-antiquark interaction as hydrogen atom plays in QED. But further meticulous efforts are needed to understand complications of charmed mesons comparable with hydrogen atom. For this purpose, extensive phenomenological efforts are needed. Phenomenology is good to review experimental facts with true physical meanings. Also with short experimental aspects,

other experimental findings. Phenomenology is in good relationship with theory and if found correct, it may be used for theoretical processes. Phenomenology may guide QCD for advance effective models for the solution of generic problems [3].

### 7.3 Experimental Perspectives

Physics based on hypothesis, theory and experiment. These hypotheses are facts (assumptions) to check physical aspects and theoretical developments. In early stages of  $\rho\pi$

puzzle, meager experimental reports were presented, which were later ruled out by accurate results. Experimental facts on  $\rho\pi$  puzzle incase of  $\psi''$  are still incomplete, even summing up all charmed and charmless channels in PDG [9]. This situation forbade us from building solid basis for elementary explorations. However,  $Q_h$  value estimation gives some clues for exploration of charmodynamics. In these efforts many suppressed modes are explored, especially  $\rho\pi$  puzzle in case of  $\psi''$ . Anyhow, systematic  $\psi'$  experimental study is in wait to be done. In addition not only  $\psi'$  to  $J/\psi$  ratio is in line to be studied, also  $\eta'_c$  and  $\eta_c$ ,  $J/\psi$  and  $\psi''$  [46] and many resonances of different channels are awaiting. BaBar and Belle observed new charmed states, leads to new theoretical predictions in charmonium spectroscopy [47]. Large data sample is observed by CLEO-c and even larger data sample of BESIII [48] will open new era for charmed meson study [48].

## 8. SUMMARY

This was a review paper for studying  $\rho\pi$  puzzle. Although no satisfactory final explanation exists in experimental and theoretical efforts, but some bright ideas have been presented. The ways of ( $\psi'$ ,  $J/\psi$ ) branching fractions ratio were presented. Theoretical and experimental study shed light on potential model, new phenomenology, although systematic measurements are crucial to perform for charmed systems. Charmonium physics keeps clear boundaries for perturbed and non-perturbed systems of QCD. Recent observed charmed hadronic decays may be new challenges to existing theoretical concepts of decay amplitude schemes. Efforts are in any of above case will help for charmonium physics development. In my point of view new experimental analysis will be more successful rather than following empirical theoretical concepts. Although, we may not obtain revolutionary understandings of existing concepts but from any effort, a new chapter of physics may be opened from " $\rho\pi$  puzzle".

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