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Case Study

Towards Software Testing as a Service for Software as a Service Based on Cloud Computing Model

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Abstract: Cloud computing is an advance technology paradigm that permits ubiquitous access to shared pools of higher level advance services from different sources at single point services that is quickly administered with minimal managing effort through the Internet. As Cloud computing relies on shared resources to attain consistency, monitoring and potency issued to store, manage and processes data on the internet. For improving and providing facilities in Cloud, service models such as Infrastructure as a Services (IaaS), Platform as a Services (PaaS) and Software as a Services(SaaS), are used. Consequently, efforts must be done as not enough work exists which can describe maintenance and assessment of applications test suite on cloud environment especially in services model SaaS assessment and for testing in cloud use Testing as a Services (TaaS) as a third-party service to end users. As these models are providing two different services environments, their evaluation is a difficult task for engineers. In this research, we use testing layers in SaaS model to increases the efficiency of software testing requests while using SaaS applications by introducing archetype of TaaS layers merging SaaS over cloud for software testing. Consequently, this research provides prototyping in cloud-based testing for SaaS. The case study is used as an empirical evaluation method for our approach to test software in TaaS infrastructure with properly and precisely defining five layers using cloud environment, by introducing to accomplish the requirements testing and evaluation through SaaS. The results show that our proposed framework improves the testing activities and makes the evaluation process more efficient for engineers.

Keywords: Cloud computing, Software Testing and Quality, Software as a Service (SaaS), Testing as a Service, (TaaS), Scheduling and Dispatching.

1. INTRODUCTION

Cloud computing is an archetype of computing; a new way of rational about information industry but not any explicit technology. In cloud environment (CE) users do not care about how it works or about its physical infrastructure; they only care about the services and services quality. CE provided more scalability and elasticity, more availability and reliability, more manageability and interoperability, more accessibility and portability, and more performance and optimization. CE provides three different types of service models i.e. infrastructure as a service (IaaS), platform as a Service (PaaS) and services as a services (SaaS) [1].

These services provide different level of services from development to users. IaaS provides users services of resource management and system monitoring interfaces. e.g. Amazon EC2, Open Stack (Open Source); whereas, PaaS provides users capability for applications deployment on IaaS using programming languages and tools supported e.g. Microsoft Windows Azure, Google App Engine and SaaS provided running applications which accessible from various client devices e.g. web browser, web-based email, Google Apps etc. instead of handling or controlling the underlying cloud infrastructure.

SaaS provides ready to use services, which demand strong testing for high quality and reliability [2]. The testing in SaaS is done through testing as services (TaaS), in which testing facilities are acquired from a third party [3]. There are three types of TaaSi.e. Functional (integrated, regression

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and automated testing), performance (deal with multiple users thru virtualization) and security (check susceptibility in application and web portal) [1-2]. TaaS is flexible and readily accessible. It guarantees information authenticity and reduces operational costs; is easily maintainable and functions based on pay as per use. There are some back draws, however. For example, different clouds are used for SaaS and TaaS, so performance issues can occur. Problem of job scheduling, resource management and dynamic arrangement of processes can arise as well [3-7].

Therefore, TaaS is still a new paradigm for researchers and industrial practitioners to deal with challenges of TaaS [3, 7-8].Consequently, in our study we mainly focused on TaaS and its framework to elaborate requirements for supporting computerized and automated cloud based testing relevant to SaaS applications. So, we addressed four major issues in this paper. These issues are; (Performance Maintenance) Different testing tasks submitted to TaaS platform by test tenants, processes, directly allocating separate testing environment to different tenants will acquire significant cost and time. A solution is not increased the performance by decreasing the time and cost in terms of various techniques; (Clustered tasks and Scheduling**)** the levels of agreements for example limit limitations are acquired by different tenants. Besides this problem, there is ambiguity of tough processes. Such as, due to communication errors in cloud testing and system break down, the priority of processes are dynamically amend and slack times will modernize. In addition, testing services is hosted on a group of virtual machines with dissimilar aspects by TaaS cloud; (Oversee resource testing and task position) Monitors has provided by TaaS platform that can provide information related to status including scheduler that maintain the task queue to choose the processor or virtual machines for test implementation; (Dynamically arranging cloud processes, processors and Virtual Machines (VM) produces great flexibility and elasticity. In this case un availability of VM due to fully occupation of existing VMs, request that will send by scheduler to build a new VM to process a new job. When Virtual machine skilled its tasks, the VM will be discharged authority sending by monitor.

Additionally, we manage these issues with our proposed framework as; Increasing the performance of testing over cloud and manage tasks more efficiently; By merging TaaS and SaaS and its supporting test structure, it improves the performance in all levels of testing; The tasks are scheduled and clustered in a way that the testing goal and objectives can be achieved efficiently; All tasks and testing resources must be managed and According to scheduling decisions organizes cloud resources which include the creation, maintenance and migration of processes and processors at runtime.

1.1. Objectives

This paper evaluated the software testing in TaaS infrastructure with appropriately defining five layers using cloud environment come into existence to congregate the requirements in SaaS testing and evaluation. The two algorithms that are dispatching and scheduling, were introduced to enhance usage of tasks and requirements. In this work, we are introducing a model over cloud of TaaS layer integrating SaaS.

1.2. Research Questions

Following research question constructed to improve testing performance using TaaS as SaaS.

RQ1: Is software testing in TaaS is successfully conducted?

RQ2: Is SaaS validation is improving the performance of software testing?

RQ3: Is Software testing will be improved by defining layers?

1.3. Related Work

In this section, we review existing literature to identify, classify, assess and understand the entirety of searching contents relevant to cloud computing services models i.e. TaaS and SaaS. In literature, we have found that there were many researchers who had taken into consideration in the field of software testing over cloud, but they did not focus on increasing the performance in terms of cost and time. By studying the literature review, we have figured out the problems depicted in Table 1.

1.3.1 Testing in Cloud Environment

By uniting the two research field this paper defines on an organized report of published attained outcome. Concerning major assistance, gaps,

Table 1. Identifying Issues of Current Software Testing

ISSUES	DESCRIPTION
Less focused on Software testing	By reviewing previous literature, there were not focused on increasing the performance of software testing in terms of enhancing cost and reducing time.
Indistinct definition of SaaS based Testing	In the past, not clearly defined the prototype of SaaS based testing over cloud infrastructure.
Inept environment of software testing	There were no security measurements to conduct the software testing effectively.
Lack of Resource and Task management mechanism	There was not an effective mechanism to control and manage the resources and tasks over TaaS layers and SaaS testing structure.

opportunities trends, potential research guidelines and challenges, a general idea is being offered [1-7]. Soon the software tests and Cloud compute and are expected to be widely accepted research areas and concepts. For cloud established techniques of software testing are being used. On the contrary, there is a great evaluation in cloud compute and for software testing research, the new opportunities and challenges are being emerged.

In this paper the arrangement of present research study was presented, searched the association of software test with dissimilar procedure structure of cloud compute and acknowledged gaps in the literature. It will provide a great advantage to researchers to explore new strengths and select their research path. It has been witnessed that approval testing provides an opportunity for testing. For advance research, managing test of tasks is among the possible fields [8-12].

To sum up , interoperability testing requirements is which provides services composition by merging services deliverance structures. In cloud computing, we mainly focus on satisfying these gaps for having a wide-ranging verification and validation model. The issue will be addressed that enhance cloud as a proposal for unit testing and for extra proliferated utilize over the cloud, we will also spotlight existing computerized test tools.

1.3.2 TaaS on Clouds

The phenomena of Cloud computing TaaS for clouds, applications that are cloud based and for SaaS which results enhance into new challenges and business features resulting in different scenario related to service models, requirements and testing techniques [7, 13-14]. A widespread discussion group on TaaS in cloud computing is evaluated by this paper. The engineers and managers raised the questions that are answered by this paper which defines conceptual discussions based on TaaS, which includes its objectives, scope, values and direction, goals, testing environments, unique features as well as required techniques. In county, it checks original challenges, issues and evolving requirements [14-15].

TaaS is appropriate a major investigate topic in both software engineering and cloud computing examine groups. Due to the advancement of cloud-based knowledge and TaaS, for unlocking problems and difficulties on TaaS, techniques, infrastructures and mechanization solutions further research results are needed. Besides this, it also emphasizes on discussion based on TaaS and necessities and different aspects. Furthermore, it spotlights the main dissimilarities between cloud based TaaS and conformist software testing [25, 17, 19].

Therefore, a comprehensive study was conducted to find out the effects of different TaaS practices benefits in SaaS environment to improve the testing performance by proper load balancing, job scheduling, dynamic provisioning and monitoring.

2. MATERIALS AND METHODS

This paper merged the TaaS infrastructure with SaaS over cloud. It provides the general view of how the application is sent to a number of layers and receives by the clouds for the testing process. It emphasizes on the importance of the cloud computing structure through application environment. This paper prototyping in cloud based Testing for SaaS. This paper introduced SaaS agents to increase the performance of testing. Each agent performed individual tasks that communicate with TaaS layers over the cloud. This paper follows scheduling and clustering algorithms, data mining technology. This paper increased the performance by decreasing the time and cost in terms of various techniques.

2.1 Proposed Work

This paper emphasized on the importance of SaaS validation and performance in cloud based TaaS infrastructure. It emphasizes on the important features of the cloud computing prototype using application situation. It provides the general view of how the application is sent to several layers and receives by the clouds for the testing process. In proposed framework we merged TaaS layers with SaaS for improving testing services at one place instead of using third party services for testing in a cloud based environment as shown in figure 1.

2.1.1. Architecture of Cloud TaaS

The TaaS architecture layers in cloud are: Contributor layer; Management layer for tasks testing; Management layer for resource testing; Test layer including testing service, testing service composition and so on and Testing Database layer.

2.1.2. Working on TAAS Tasks

In this paper there are two clouds. One cloud has TaaS Infrastructure and the other has SaaS

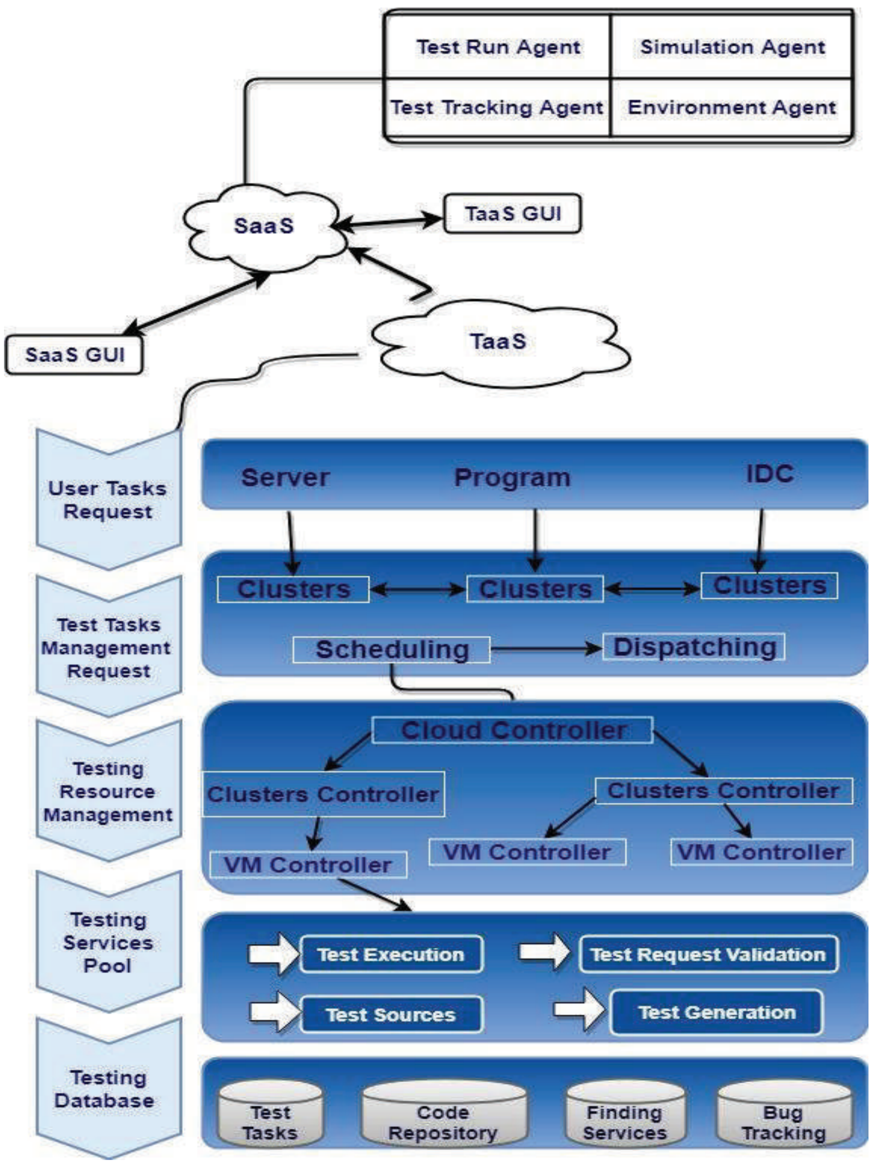

Fig. 1. Systematic Model for Software Testing using Cloud

2.1.2.1 User's test requests

This layer is the first layer. In this process and provides access to the TaaS platform. This layer is divided into two parts i.e. Test service tenants and contributors. This layer gets the tenants test requests by using IDE or any interface and ahead it to the next layer. They contribute layer publishes or deploys their services to TaaS platform. This paper increased the performance by decreasing the time and cost in terms of various techniques.

2.1.2.2 Management layer for tasks testing

The second layer receives the test request from tenants in the capability checking process. Capability checking means to check the TaaS platform's request. The second layer has clusters that used data mining techniques to store the selected data more effectively. The scheduler arranges the test tasks and allocates the test tasks to appropriate virtual machine. The scheduler also sends the scheduling test task to monitor in the third layer to monitor testing resources.

2.1.2.3 Management layer for resource testing

The second layer's scheduler offers monitor to maintain task queue to select processes and scheduler. It checks the physical machines, where as VMs gives resources according to request submitted by task. It allocates resource according to the job appeal. There are three sub layers of this layer: The preprocessing, the resource managing level, the computing node level (specific applications establish through the VM).Then the monitoring information is passed to the cloud controller. Each cloud controller controls the tasks identified by the tenants. It controls all the cluster controllers, for example, querying the cluster etc.

2.1.2.4 Testing service pool layer

Group of testing services for different forms of testing is hosted by Service pool, for example, test source service, unit testing, including test execution service, and result validation service. This layer presents matching rules, fault tolerance technology and scheduling test-task. This layer has four service pools i.e. Test completion, Test outcome validation, Source code testing and Generate code testing. This layer focused on providing services of test task execution properly. After testing, layer validates the results and source code. At the end generates the source code.

2.1.3 Testing Database Layer

It maintains and stores the test task request of tenants, bug tracking results. It played a very important role to conduct the software testing. This layer consists of four databases:

- Test tasks
- Code repositories
- Bug tracking

Each database stores and manages test requests more efficiently**.** These databases test the requests, stores the requests, test the services and also keep the track of bug in the requests.

2.1.4 SaaS Infrastructure

This paper also emphasized on SaaS infrastructure. In SaaS, there are four agents. The cloud based virtual test environment refers to deploy under-test Saas and its sustaining test structure which are: Test runs Agent, Test Tracking Agent, Simulation Agent and Environment Agent. The SaaS agents evaluated the test task requests and collaborate with seas to increase the security over the cloud. TaaS services emphasized on current TaaS before SaaS deployment. Amazon'sEC2 used in this project as the cloud infrastructure to sustain the environment that has based on virtual test and Server, even though the planned and on-going TaaS infrastructure with three-layers shows that they can be deployed on dissimilar cloud infrastructure:

2.1.4.1 User interface (UI) layer

This interface has two user interfaces:

I) SaaSUI and
II) TaaS UI

2.1.4.2 Test space layer

Cloud that has contains SaaS testing deployed through it and its behind test structure based on virtual test environment refers by the Test space layer, as well as the Tracking agent, Environment agent, Simulation agent and Test agent.

2.1.5. Merging TaaS Environment with SaaS Over Cloud

This paper proposed the software testing by using TaaS with SaaS. This increases the efficiency of software test requests. This paper prototyping in cloud based Testing for SaaS. This paper explains the layers of TaaS in detail focus on SaaS services and evaluate the software requirements in a TaaS environment over the cloud based system. In this work, we are introducing a model of TaaS layers integrate SaaS over cloud for software testing. The TaaS environment merges with SaaS environment over the cloud. It emphasized on the importance of the cloud computing prototype through application atmosphere. This paper introduced SaaS agents to increase the performance of testing. Each agent performed individual tasks that communicate with TaaS layers over the cloud. By merging the TaaS with SaaS infrastructure increases the efficiency and effectiveness the software testing requests from users or clients. TaaS handles the user's requests with SaaS testing agents and pass it in all the layers and then stores in database repositories.

3. RESULTS AND DISCUSSION

To evaluate and understand the efficiency in the real time, multiple browser platform and types and different forms of hardware used in multiple operating systems. These hardware types evaluate and reconfigure the effectiveness in real time. For evaluation of our study we adopted case study evaluation method.

3.1. A Case Study

At San Jose State University, we offered students in a testing class (CMPE287) to investigate Orange Human Resource Management (HRM) system as the ideal Software as a testing function. To perceive the developed TaaS method (Cloud based TaaS) for SaaS validation. Orange HRM is presented as hosted SaaS application which is an open source. For both small and average enterprises, it can be easily located up and configured as their HRM service structure. These are the following functional services included by Orange HRM:

a) Identify and way organizational structure
b) pay-scale index
c) Depart management
d) Private information management
e) Worker service structure

For both features, by means of two black boxes efficient testing techniques there is need to design scheme stage test, which are:

i) Decision Table Testing
ii) Equivalence panel.

They require generating test scripts to generate test cases:

i) Test scripts based on functions
ii) Performance test scripts.

The test tools like Jemeter and Selenium supports Cloud based and with these test tools it can carry out test scripts. It is mandatory for the student groups to use cloud based TaaS system in the case study to do following steps:

i) Build a test task and its practical test room to set up Orange HRM SaaS program.
ii) To migrate and supervise test scripts into the test space.
iii) To execute automated GUI-based functional testing GUI-based test agent is used.
iv) Jemeter is used as a presentation test tool, large-scale by connecting automated presentation testing and scalability evaluation are conducted using pre-loaded performance and auto-test scripts.
v) As a testimony test computerization outcome is reported.

The results of case study defining efficiency of software testing using applications in table 2 Orange HRM system elements which we investigated on different parametric comparison and most of efficiency parameter satisfied by our framework. The performance testing results our approach depicted in figure 2; in which x-axis describes the elements of selected application and y-axis illustrates the elements performance percentage after testing in cloud environment.

We use the validation model metrics for testing performance which used in exiting literature to support SaaS and TaaS services i.e. Computing Resource Utilization Meter (CRUM), for resources usage monitoring, System Performance Meter (SPM) monitoring the system performance, System Load Meter (SLM), to evaluate system load for network load, and data access load [4]. The results of these performance testing validation in table 3 which describes that our approach feasible for testing activities while using SaaS as TaaS.

4. CONCLUSION

With the advancement in technological industry cloud computing is one of the environment

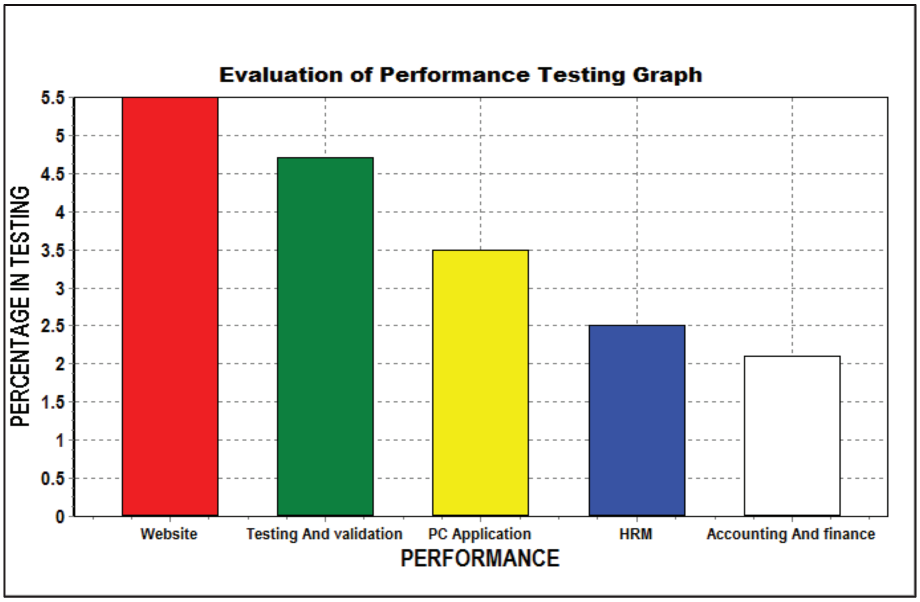

Fig. 2. –Evaluation of Performance Testing using Graph

Table 2. Defining Efficiency of Software Testing Using Applications

✓ = "Exists" X = "Does not exists"

	Increased Performance	Cost Effective	Reduce Time	Increased Task Resource Management
Website	✓	✓	✓	X
Testing & Validation	✓	✓	✓	✓
PC Applications	✓	X	✓	✓
HRM	✓	✓	✓	✓
Accounting & Finance	✓	✓	✓	✓

Table 3. Report of Performance Testing Operation

Report					
Test Evaluation model	**No. of user Access**	**Test No.**	**No. of Test Scripts**	**Start Time**	**End Time**
CRUM	600	Test#1	32	18:03:34	19:15:02
SPM	1600	Test#2	32	19:12:03	19:22:12
SLM	2000	Test#3	32	19:33:01	19:32:00

architype in which infrastructure, platform and software are uses as services. Mostly used service model is SaaS for using complete and running application, web services, systems etc. and testing of these SaaS services is the difficult for engineers to test and maintain them with low cost and time. But there have some open issues and challenges in SaaS and TaaS which come across from literature that is lack of resource management and monitoring interfaces. In our study we merge TaaS and SaaS Platform to help quality engineers for automatic test cases production, execution, collection and fault tolerance to reduce load, job scheduling, monitoring and management within limited time and less efforts. The Orange HRM system as case study to empirically investigate our model performance testing to reduce testing time, customer pleasure, avoid misunderstandings, provide easy data flow and less hard work. The results depicted that merging TaaS with SaaS context upsurges the proficiency and efficacy for software testing requests from clients. TaaS switches with SaaS testing agents for dealing with client's requests and pass to all the layers then saved in repositories. As such, this area has high potential for additional research, tool and technique development to deal with large multi-tenant clients and efficient dynamic provision of fault free services.

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Case Study

Spatio-temporal Assessment of Productivity in Pehur Main Canal System, Pakistan

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Abstract: This study aimed at assessment of productivity in Pehur Main Canal system on Spatio-temporal basis. The water appropriation is based on warabandi scheduling system. The data for this study was collected from official record of irrigation department and field survey. Productivity is assessed on a scale of zero to one (0-1), one represents maximum productivity and zero indicates nil productivity. The analysis revealed most of productivity values closer to zero. Temporally productivity is better in Rabi season than Kharif Miana (dry summer season). On spatial scale, the productivity in both Rabi and Kharif Miana seasons is random in most of secondary canals. In majority of canals, the productivity is better in the middle and tail sections then the respective heads and tails. This situation shows lack of operational maintenance of the irrigation scheme. Findings of this study can be used to rectify the operational management of the irrigation system to enhance productivity.

Keywords: Pehur Main Canal system, Spatio-temporal Productivity, Irrigated agriculture, Area Productivity, Kharif Miana.

1. INTRODUCTION

Irrigated agriculture provides 45% of the world's total food supplies, and without it, it will not be possible to feed our planet's population [1, 2, 3, 4, 5, 6]. Irrigation is the main source of increased food production worldwide and especially in Asia [7, 8, 9, 10, 11, 12]. Irrigated agriculture plays a fundamental role in the economic development and social uplift of the non industrialized nations in the world [13, 14]. According to estimate the world population will increase from the base of 6.1 billion in year 2001 to 9.3 billion by 2050. In the mean time, the population of 49 least developed countries will increase by three folds in size [15, 16, 17, 18]. This population growth will pressurize irrigated agriculture to considerably increase food production in the future [19, 20, 21,].The estimated 60% of additional food requirements through 2050 will have to be met by production on irrigated land [22, 23]. Therefore the future food security of our planet entirely depends upon irrigated agriculture [24, 25, 26, 27].

The productivity of irrigated agriculture is related to output in response to inputs added to the system [28, 29, 30, 31, 32, 33]. A number of indicators are used for the measurement of productivity. The irrigated agriculture has a number of outputs which are area irrigated, crop produce, the economic value of the crop produced [34, 35, 36, 37, 38, 39]. The productivity indicators are easy to asses both temporally and spatially [40, 41, 42, 43]. The productivity can be calculated in response to certain inputs or in gross terms by measuring outputs. The important inputs in the irrigated agriculture are water, land and financial resources [44, 45, 46, 47, 48]. The productivity is more relevant when it is measured in response to inputs which are scares. The researchers [49, 50, 51, 52, 53, 54] have listed a number of indicators for the measurement of productivity. These indicators can be summarized as total area irrigated, total crop production and total economic benefits in gross terms, and total irrigated area, total production and total economic benefits per unit of water applied,

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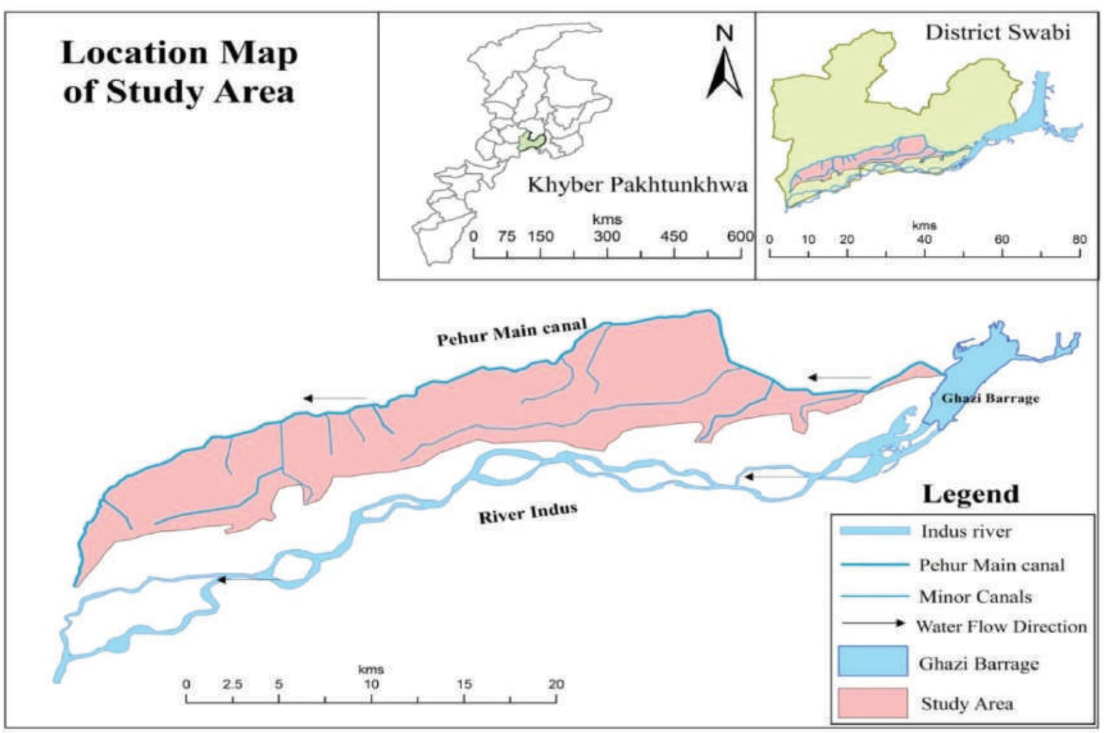
cropped area or total area available in net terms. The water applied for irrigation at multiple levels is measured in the irrigation system, i.e. from the head of a canal up to field level of the cropped area [55, 56, 57, 58, 59]. In all performance assessment studies of irrigation schemes, productivity indicators are included which are easy to quantify [60, 61, 62, 63].

Productivity is the ultimate goal of each and every economic activity. Agriculture is one of the basic economic activities and irrigation in arid and semi-arid regions significantly enhances food production [64, 65, 66]. Arid and semi-arid regions have a scarcity of fresh water resources therefore, it is necessary to use these limited resources as efficient as possible to increase their utility [67, 68]. Although irrigated agriculture is the most promising means of increased food production but inefficient use of water can cause damage to soil in addition to wastage of this precious resource. It is therefore necessary to assess the Spatio-temporal productivity of irrigated agriculture to point out areas where improvement can be made to enhance food production.

The Pehur Main Canal (PMC) system in District Swabi of Khyber Pakhtunkhwa was selected for the assessment of productivity. The study area is located along the southern boundary of district Swabi [Fig 1]. The area runs parallel to River Indus along its right bank. The canal takes its water supply from the Ghazi barrage and distributes it over the command area through main canal and twelve (12) distributaries and minors [69, 70]. There are three distinct cropping seasons in the study area. These are winter (Rabi), dry summer (Kharif Miana) and rainy (Kharif). In Rabi season little rainfall is received and loss of water through evapotranspiration is least so the demand for irrigation remains low. The rainfall is low in the dry summer season while the consumptive loss of water is high due to evapotranspiration so cropping is restricted and requiring frequent irrigation [68, 71]. During Kharif season maximum rainfall is received from monsoon. Due to abundant rainfall, the crops does not need frequent irrigation and only one or two protective irrigations are sometime required. The dams are filled to their capacity and the rivers are often flooded so the rainy (Kharif) season is not included in this study [36, 68, 72].Consequently, the Rabi and dry summer crops were considered



Source: Map in Arc GIS from Toposheets and Google earth map

Fig. 1. Location of the study area, District Swabi, KP, Pakistan

for the assessment of productivity spatially in head, middle and tail sections of the main and secondary canals and temporally during Rabi and dry summer Seasons.

2. MATERIALS AND METHODS

2.1 Measurement of Productivity

The water allocation plan followed in the study area is based on warabandi scheduling system. The water is provided for fixed time duration proportional to the size of the land holding [68, 73]. The main objective of warabandi system is to distribute available water resources equally over as large area as possible. The total area irrigated is the variable of interest in the measurement of productivity. The measurement of productivity must be in relation to certain standard which in this case is the targeted area to be irrigated. The productivity is dependent on the water distribution amongst various allocation units along the distribution channels (main, secondary and tertiary canals) [34, 35, 36, 37, 53, 60]. The proposed measurement of productivity is to use the indicators in comparison to certain standards. The standards can be target values, or the values associated with land or water, or the values associated to maximum output management strategy in response to certain inputs [39, 40, 41, 44, 62]. The proposed indicators for the measurement of area productivity are given in the following equation.

$$\mathrm{Prg} = \frac{\mathrm{OAa}}{\mathrm{OAt}}$$

Where the Prg = Area productivity, OAa = actual output (total area irrigated during an irrigation season i.e. Rabi or Kharif Miana, OAt = targeted output (total area planned to be irrigated during an irrigation season). Productivity in different sections of the canal can be measured by considering allocation units in these sections.

2.2 Data Collection

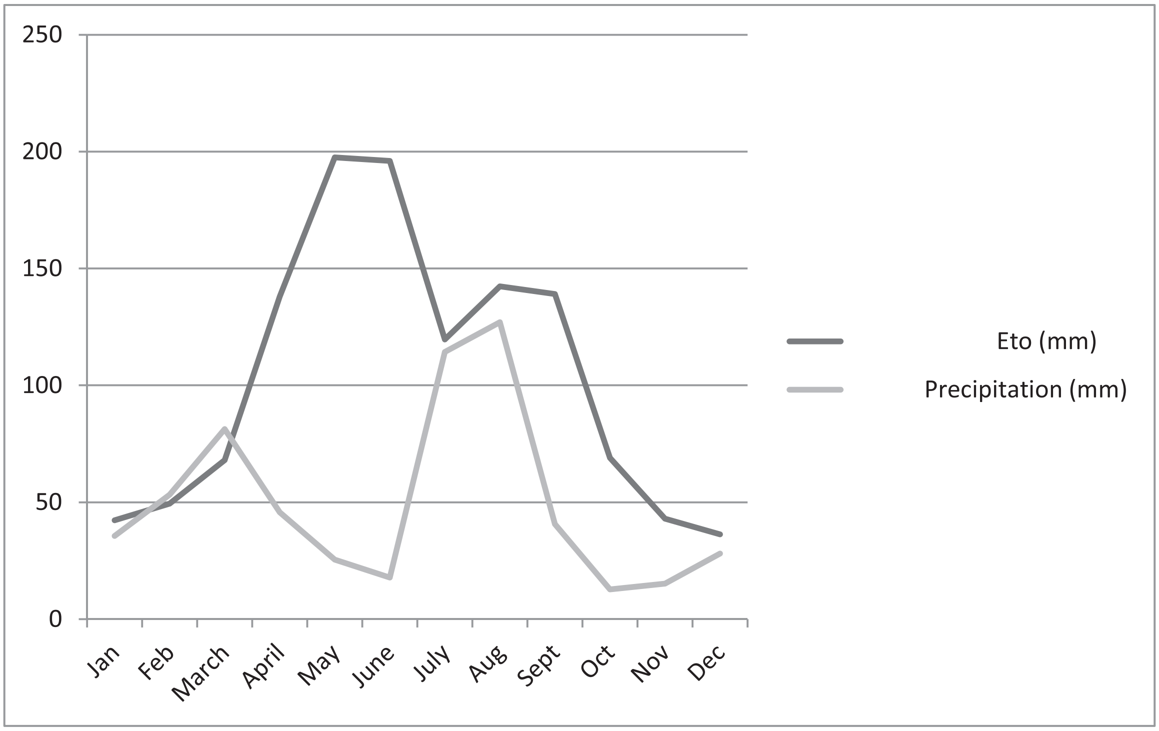
The data for this study was collected both from primary and secondary sources. The sub-divisional offices of Provincial Irrigation Department (PID) keep records of the irrigation activities. Among the records kept are Outlet Registers (*Mogawar*) maintained by the irrigation staff. The 'Outlet Registers' refer to designed discharge, area to be irrigated, area actually irrigated, crops assessed in each growing season, water tax (*abiyana*) collected in each growing season. The allocated discharge flow of each outlet was obtained from the allocation plans. For delivered volume of water an extensive survey was conducted from 1st October 2013 to 15th January and from 1st to 31st March 2014 for Rabi season while from 1st April to 30th June 2014 for Kharif Miana season. The canal system remains closed for desiltation and necessary repairs from 16th January to 28th February each year. The total 250 outlets were divided into ten groups of 25outlets in each. Only one group of outlets was surveyed on daily basis so that each group had its turn on every tenth day. The flow data was recorded and compiled for each individual outlet for both seasons. The data was analyzed for assessment of reliability in both seasons and in different sections of main and secondary canals [68, 74].

3. RESULTS AND DISCUSSIONS

Irrigated agriculture is an economic activity employing bulk of labour force especially in the developing countries. The strategy of each farmer is to increase the crop produce with minimum possible input resources. The resources of interest in an irrigation scheme are land and water. With limited water resources in the study area, the farmers are always in pursuit of increasing productivity. The productivity is measured on a scale of 0-1(zero to one), where 1 (one) represents highest productivity and 0 (zero) indicates nil productivity. In the PMC irrigation system the overall productivity remains low. The Rabi season registers a slightly higher productivity than the dry summer season. The Spatio-temporal analysis of productivity is discussed in following pages.

3.1 Productivity of Pehur Main Canal system during Rabi season

The Rabi season extends from October of one year to March of next year. Both evapotranspiration and rainfall are low during Rabi season [Fig. 2]. The irrigation demand remains low owing to a small gap between precipitation and potential loss of water through evapotranspiration. Due to low irrigation demand irrigation interval is increased from 07 days to 14 days. Locally this setup is called week



Source: Sugarcane Research Institute, Mardan.

Fig. 2. Crop-Water requirements in the Study Area

Table 1. Productivity Comparison of Pehur Main Canal system (Spatio-temporal)

	Name of canal	Rabi			Kharif Miana		
		Head	**Middle**	**Tail**	**Head**	**Middle**	**Tail**
H	PMC (head-section)	0.435	0.152	0.489	0.261	0.057	0.241
E	Topi Minor	0.353	0.359	0.149	0.192	0.248	0.098
A	Zarobi Minor	0.471	0.570	0.412	0.255	0.258	0.168
D	Kotha Distributary	0.465	0.367	0.628	0.289	0.214	0.240
-	Kaddi Minor	0.740	0.440	0.513	0.348	0.242	0.238
M	PMC (Mid-section)	0.473	0.384	0.510	0.225	0.323	0.370
I	Zaida Minor	0.553	0.549	0.421	0.214	0.227	0.221
D	Sheikh Dhari Minor	0.487	0.854	0.679	0.431	0.722	0.358
D	Zakarya Minor	0.400	0.319	0.282	0.412	0.206	0.165
LE	Lahore Minor	0.578	0.349	0.326	0.463	0.245	0.198
T	PMC (Tail-section)	0.617	0.561	0.322	0.209	0.181	0.131
A	Thanodher Distributary	0.513	0.347	0.074	0.203	0.123	0.036
I	Bazar Minor	0.361	0.134	0.048	0.122	0.024	0.009
L	Manki Minor	0.558	0.525	0.205	0.149	0.160	0.126
-	Jahangira Minor	0.325	0.440	0.247	0.125	0.126	0.054
	Average	0.488	0.419	0.353	0.260	0.223	0.177

Source: GoKP (2012), Field Survey, 2013-14.

wise schedule (*haftawari*), this means that each secondary canal get water on alternate weeks. The farmers on secondary canals can irrigate their crops on every 14th day. The low demand for irrigation helps to irrigate more area therefore productivity comparatively remains high as compared to dry summer season (Kharif Miana). The overall average productivity of Pehur Main Canal system in Rabi season is, head (0.488), middle (0.419) and tail (0.353) [Table 1]. The productivity in different sections of the PMC system with respective canals is discussed below.

3.1.1. Productivity in Head Section of Pehur Main Canal system (*Rabi*)

The head of the PMC system comprises head of main canal, Topi minor, Zarobi minor, Kotha distributary and Kaddi minor canals. The head section of main canal is again stratified into head, middle and tail sections. The highest productivity during Rabi season is recorded in the head of Kaddi minor (0.74) located in the lower head instead of upper head section of the PMC system. Kaddi minor also has better productivity in the tail than the middle section. On the contrary the lowest productivity (0.149) is observed in the tail of Topi minor located at head of canal system. Kotha distributary records higher productivity at the tail and Zarobi minor has higher productivity in middle than its head [Table 1]. The inter-canal comparison shows no proper sequence of change from upper head to lower head of the system in all sections of the minor canals. Higher values of productivity are observed in the lower head of the system instead of upper head. The higher productivity of Kotha distributary in the tail section is attributed to Badri Lift Irrigation Scheme which supplements the water supply in the tail section of the distributary.

3.1.2. Productivity in Middle Section of Pehur Main Canal system (*Rabi*)

In the middle section of the PMC system the canals included are mid-section of main canal, Zaida minor, Sheikh Dhari minor, Zakarya minor and Lahore minor. The main canal has better productivity in lower mid-section than the upper sections (0.473, 0.384 and 0.510). The Sheikh Dhari minor canal has the highest productivity of the system in its middle (0.854) while its tail (0.679) performs better than its head (0.487). The Zaida, Zakarya and Lahore minor canals have higher productivity at the head steadily decreasing towards the tail [Table 1]. The comparison among canals of this part of irrigation system show irregular changes in productivity values from upper to lower mid-section. The head, middle and tail of all constituent canals show variations with no ascending or descending order. The random distribution of productivity values indicates that either the farmers interfere in the operation of main canal to divert more water into the secondary canals or the system is not maintained at recommended level of flow which results in irregular distribution of water among the secondary canals.

3.1.3. Productivity in the Tail Section of PMC system (Rabi)

The canals included in the tail section of the PMC system are tail end of main canal, Thanodher distributary, Bazar minor, Manki minor and Jahangira minor. The tail of main canal included is again divided into head, middle and tail section for analysis. The section of main canal has better productivity than the head and middle sections of the main canal (0.617, 0.561 and 0.322). All the minor canals in this part of the system show a normal behavior having higher productivity at the head decreasing towards the tail with exception of Jahangira minor canal which has better productivity in the middle than the head. The lowest productivity of entire system during Rabi season is in the tails of Thanodher distributary (0.074) and Bazar minor (0.048) [Table 1]. The inter-canal comparison demonstrates a normal behavior in the head sections of the main and minor canals with exception of Manki minor. All the minor canals retain a better productivity in the head sections than the middle and tail sections. The middle sections show steady decrease up to Bazar minor then an increase afterwards. The same pattern of productivity is prevalent in tail sections of main and minor canals.

3.2 Productivity of PMC system during dry summer season (Kharif Miana)

The Kharif Miana (dry summer season) is characterized by very hot and dry weather. The rainfall is normally very low and erratic. The evapotranspiration rate is highest during this season and a very wide gap between the monthly received precipitation and evapotranspiration exists [5, 11]. Therefore irrigation demand remains high during this season [Figure 2]. The irrigation interval in dry summer season is 07 days. The crops grown during this season are Tobacco (Virginia and local), Watermelons, Muskmelons, and Maize and fodder crops. All these crops need intensive irrigation throughout the growing season. Virginia tobacco is so much water dependent that from mid May onwards it has to be irrigated on every alternate 4th day. To manage this situation the framers normally leave half or two-third of their land fallow this season. The farmers frequently exchange warabandi turns to cope with this extraordinary situation. The water availability in the reservoir is satisfactory during this season but owing to very high evapotranspiration rate and the high demand of crops like Tobacco and Watermelons the cropping remain limited. Apart from irrigation a lot of people are dependent on canal water for their domestic water needs which increases many folds during this season. These conditions essentially limit the productivity during the Kharif Miana season. The productivity as a whole remains low during this season owing to high demand of crops for irrigation and limited availability of irrigation water. The average productivity values for the system are (0.260), (0.223) and (0.177) in the head, middle and tail respectively [Table 1]. The productivity in various sections of Pehur Main Canal system during Kharif Miana season is analyzed below.

3.2.1 Productivity in Head-Section of Pehur Main Canal system (Kharif Miana)

The lowest values of productivity in the head of the system are observed in the middle section of main canal (0.057) and tail of Topi minor canal (0.098). The highest productivity of this section is in the head of Kaddi minor located in the lower head of the system rather than the upper head. Kaddi minor canal has more productivity in the head decreasing towards the tail (0.348, 0.242 and 0.238). All other canals in head section of the system do not show normal behavior. Main canal has more productivity in the lower than middle head; Topi and Zarobi minor canals have more productivity in middle than head and Kotha distributary has high productivity in the tail than middle section. The inter-canal comparison show a decreasing trend of productivity from the upper to lower head of the system in the head sections of constituent canals while their respective middle and tail sections demonstrate a mixed trend [Table 1].

3.2.2 Productivity in middle Section of Pehur Main Canal system (Kharif Miana)

The productivity is better in middle and lower parts of this section of the irrigation system during Kharif Miana season. Individually highest productivity (0.722) is in the middle section of Sheikh Dhari minor which is also higher than the head section of the same minor canal. The part of main canal in this section has better productivity in the tail than respective middle and head sections (0.225, 0.323 and 0.370). The Zaida minor canal has better productivity in middle and tail than the head section. The Zakarya and Lahore minor canals have descending sequence of productivity from head to tail sections. Comparatively productivity is high in this section with respect to head and tail of the system but the inter-canal comparison does not show any logical progression of change from upper to lower mid-section. The head of Zaida minor canal located upstream has a lower productivity (0.214) while head of Lahore minor canal, located downstream has higher productivity (0.463) [Table 1].

3.2.3 Productivity in Tail-Section of Pehur Main Canal system (Kharif Miana)

The tail section of Pehur Main Canal system is least productive in Kharif Miana season. The highest productivity in this section of the system is (0.209) in the upper tail of main canal and the lowest is (0.009) in the tail of Bazar minor. The tail ends of Thanodher distributary, Jahangira minor while both middle and tail sections of Bazar minor have a very low productivity. The main canal, Thanodher distributary and Bazar minor canals show a usual trend of more productivity in the head decreasing towards tail. The inter-canal comparison show a decreasing trend in the heads of constituent canals while in middle and tail sections of these canals the sequence is broken by very low productivity in tail of Thanodher distributary and in both middle and tail of Bazar minor canal. The productivity in this section from upper to lower tail of the system ranges from 0.209 to 0.009 [Table 1]. The comparatively better productivity in Manki minor canal is due to its short span and less number of water users. Otherwise sever water shortages are felt in the tail sections of all minor canals in this part of the system.

4. CONCLUSION

Productivity is one of the major objectives of all irrigation projects. The productivity of Pehur Main Canal system is assessed on location and seasonal basis. It is measured on a scale of zero to one (0-

1). In ideal conditions the productivity values will be closer to one (1) and the average of all values from head to tail must also be closer to 1. Data analysis shows majority of values closer to 0 (zero). This situation indicates low level of performance of Pehur Main Canal system. On system level average productivity values in head, middle and tail sections are, Rabi (0.488, 0.419 and 0.353) and Kharif Miana (0.260, 0.223 and 0.177) [Table 1]. The average productivity show a normal trend of higher values at head decreasing towards tail in both seasons. Overall few productivity values are higher than half of scale, for instance four (4) in head, six (6) in middle and five (5) in tail of the irrigation system. These higher values are more frequent in middle and tail sections than head. Another aspect coupled with low level of productivity is its irrational distribution among different sections of the main, distributaries' and minor canals.

The data analysis reveals that on individual canals level some middle sections doing better than heads while tails better than middle and head sections. This illogical behavior indicates lack of operational maintenance of the Pehur Main Canal system. The canal systems which operate under the warabandi system have water receiving structures which are sensitive to the flow of water in the main, distributaries and minor canals. Canals flowing up to 70% of their design capacity or above the water drawing structures will behave normally. The water flow below 70% of the full capacity of the canal induces abnormal changes in drawing water by the outlets. The distribution of productivity indicates a lack of management on part of the irrigation staff and lack of organization among the farmers. These irregular changes indicate that system is not maintained properly which induces the water withdrawing structures to behave abnormally. This results in irrational distribution of water and hence the performance of the system becomes erratic.

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Review Article

Accident Analysis Techniques in the Industries: A Review

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Abstract: Since the industrial revolutions, accidents have become more common and frequent in all the industries. To find out the causes of these accidents, plenty of techniques have been used until now. A review of the existing techniques of accident causation has been done in this study which highlights the strengths and weaknesses of all these techniques that have been used to find out the contributory factors of these incidents. In addition, the industries where these techniques have been used, have been identified and it is emphasized that these techniques should be applied in the textile industry in Pakistan which is the most accident prone industry in recent times. This study also leads to the realization that there is a need for newer techniques such as structural equation modelling that address the complex nature of accidents and deal with the interactions of contributory factors of accidents in large scale industries. It is also emphasized that the government should take the initiative of making an institution that gathers the accidents data so that the exact causes can be known and dealt with accordingly.

Keywords: Accident causation analyses, Systemic analysis, Sequential analysis, Human information models, Structural equation modelling, Manufacturing industries, Industrial safety.

1. INTRODUCTION

There have been plenty of disastrous accidents happening or happened worldwide such as Bhopal incident, Deep Horizon, Piper Alpha in which severe injuries and diseases occurred to gazillions, plenty of lives were lost, sensitive equipment was destroyed, there was an innumerable loss of property and the environmental damages, which changed the living style permanently. These life threatening incidents or accidents led to absenteeism, lack of skilled workers, adverse effects on the environment, which directly affected the industrialists, and indirectly to the GDP of developed as well as developing countries. Some of these accidents are caused due to a simple and/or single stimulus, however mostly are the results of a combination of errors made by human beings, arise by the equipment failure and those that are the consequences of poor environmental conditions altogether. As most accidents are caused by a series of failures and to find the reasons of such incidents is an uphill task, there have been and still are significant researches going on for accident causation and to bring the reasons behind such catastrophes to limelight.

2. LITERATURE REVIEW

Initially techniques such as what if and Checklist have been used but they have not been sufficient in explaining the scenarios of incidents and have therefore become redundant. To date, different accident analysis and risk analysis techniques are being used to identify different reasons of such calamities, to prevent such disastrous events from happening and to take precautionary measures before they actually happen and claim the precious lives of individuals. Each of these analysis methods has its merits and demerits, and is there to be further explored. These accident analysis methods have been classified into three groups which are systemic analysis, sequential analysis and human information models [1].

2.1. Systemic Analysis

Systemic analysis views accidents because of uncontrolled system interactions. It includes methods such as Failure mode and effects analysis (FMEA), Failure mode, effects and criticality analysis (FMECA), System Theoretic Accident Model and Processes (STAMP), System Theoretic

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Process Analysis (STPA), Causal analysis using systems theory (CAST), Hazard And Operability Analysis (HAZOP), Humans factor analysis and classification system (HFACS), Accimap, Functional resonance analysis method (FRAM), Layers of protection analysis (LOPA) and Swiss Cheese Model (SCM).

2.1.1. FMEA & FMECA
Failure mode and effects analysis (FMEA) procedure consists of a sequence of steps used to indicate all the probable failures in a process [2]. Failure modes, effect and criticality analysis (FMECA) is an extension of FMEA which includes criticality as well that is the mathematical estimation of severity and occurrence. FMECA has a pivotal role in reliability systems engineering which illustrates the potential of a system to perform at the given conditions for a fixed amount of time [3].

2.1.2. STAMP, STPA & CAST
System theoretic accident model and processes (STAMP) is a method in which the failures or accidents are investigated in the way that why the barriers placed cannot stop the occurrence of these accidents and why these barriers are not sufficient enough to ensure the safety of the entire system [4]. Two new methods System theoretic process analysis approach (STPA) and Causal analysis using systems theory (CAST) have been devised from STAMP in order to refine the existing accident analysis and the hazard analysis. System theoretic process analysis approach (STPA) is a hazard analysis approach that embodies the idea of STAMP method. Leveson who developed STPA is of the view that safety is of prime importance and accidents do not occur owing to the sequence of failures but by the poor indication and inappropriate recognition of safety related constraints in a system. These constraints may be human error, design error in equipment or organizational problems [5]. Causal analysis using systems theory (CAST) is also based on the STAMP approach which assesses the whole accident process in case of an accident and indicates the key causal factors. In addition, CAST also focuses on why the accidents occur in the first place which helps in prevention of accidents in future [6].

2.1.3. HAZOP & HFACS
Hazard and operability analysis (HAZOP) in the early days has been used to indicate the abnormalities in the proposed design. It is developed to spot the hazards and propose the safety measures to avoid these hazards especially in the process industries [7]. Human factor analysis and classification system (HFACS) is a human factor accident analysis initially proposed solely for the aviation industry. It embodies the idea of Reason's model which states that active failures are the result of latent failures. [8].

2.1.4. AcciMap & FRAM
AcciMap approach is an accident analysis approach that is used as means of modeling the socio-technical context to identify the combination of events and decisions that produce an accident. It differs from the traditional accident models in the way that it describes the different causal factors that lead to a failure event and their inter relationships in a graphical form. Such analysis with causal diagrams guide us to the patterns that lead to the occurrence of accidents from which one can judge what elements are necessary for safe operations [9]. Functional resonance analysis (FRAM) is a methodology used both for risk analysis and accident causation modeling. It has the capacity to entail the incidents that have already happened before and for that it is used as an accident analysis approach [10].

2.1.5. LOPA & SCM
Layers of protection analysis (LOPA) are a risk analysis approach developed to reduce risks in the process industries by evaluating the adequacy of the layers of protection. In a process plant the processes that are more risk prone are selected, each process is then related to the probable failures on the basis of a person's knowledge, experience and the database available [11]. Reason's model commonly known as Swiss Cheese Model (SCM) is a systemic approach which shows system's defenses diagrammatically in such a way that the pits in the slices exhibit the breakage of defenses. These slices look exactly like the Swiss cheese, hence the name. When the pits are in line in the slices, it leads to the occurrence of disasters and the inevitable accidents [12].

2.2. Sequential Analysis

Sequential analysis is the type of analysis which explains the accident as the outcome of a sequence of events in a proper order. Various methods such as Fault tree analysis (FTA), Event tree analysis (ETA),

Table 1. Applications of accident analysis techniques in different industries

No	Methods	Applications	References
1	FMEA	Space industry, Chemical industry, Thermal plant, Paper mill ,Nuclear industry, Oil and gas industry.	[2, 19-22]
2	FMECA	Aerospace industry, Railway industry, Aviation industry, Food industry, Electric power plant.	[3, 23-25]
3	STAMP	Oil and gas industry, Chemical industry, Nuclear industry, Aviation industry.	[4, 26]
4	STPA	Aviation industry, Chemical industry, Oil and gas industry, Defense industry, Automobile industry.	[5, 27-29]
5	CAST	Maritime industry.	[6]
6	HAZOP	Space industry, Oil and gas industry, Chemical industry.	[2, 30, 31]
7	HFACS	Aviation industry, Mining industry, Chemical industry,Railway industry.	[8, 32-34]
8	AcciMap	Chemical industry, Aerospace industry, Oil and gas industry, Maritime industry	[9, 35-37]
9	FRAM	Aviation industry, Construction industry, Chemical industry, Rail industry, Oil industry.	[10, 38, 39]
10	LOPA	Chemical industry,Oil and gas industry.	[40, 41]
11	SCM	Aviation industry,Chemical industry, Railway industry.	[12, 42, 43]
12	FTA	Chemical industry, Nuclear industry,Steel plant, Mining industry.	[13, 44-46]
13	ETA	Nuclear industry, Defense industry, Automobile industry, Chemical industry, Mining industry.	[14, 46, 47]
14	DEA	Oil industry, Chemical industry, Petrochemical industry.	[15, 48]
15	CA	Oil industry, Chemical industry, Electric power plant,Gas industry.	[16, 49-51]
16	CREAM	Electric industry, Maritime industry, Aviation industry.	[17, 52]
17	SPAR-H	Oil and gas industry, Petroleum industry, Nuclear industry, Chemical industry.	[18, 53-55]

and Domino effects analysis (DEA), Consequence analysis (CA) are included in this group.

2.2.1. FTA & ETA

Fault Tree Analysis (FTA) is a sequential methodology that has made its mark in the nuclear industry and is most often used as a tool for risk assessments and in the accident investigations. According to this methodology, an enabling event gives rise to a initiating event which has the capacity to cause an accident[13].Event tree analysis (ETA) is another sequential methodology which is said to be developed in 1974 during a safety assessment of a nuclear power plant. During this study, it was noticed by the WASH-1400 nuclear power plant team that with the help of fault tree analysis, risk analysis of the plant can be accomplished but the fault tree obtained would be very big and unmanageable. Therefore event tree analysis was introduced to present the table in much more viable form [14].

2.2.2. DEA & CA

Domino effect analysis (DEA) as the name suggests is the analysis of chain of events that lead to accidents or have the capability to cause an accident in the future. This kind of analysis is used to analyze such situations in which an explosion/ fire/toxicity in one unit cause secondary and tertiary incidents in other units and the process continues [15]. Consequence analysis (CA) is a sequential analysis which assesses the consequences in case of an accident. It is a risk analysis methodology which determines the effects of a likely failure event on human, equipment and facility and tells about the possible consequences they may have to face [16].

2.3. Human Information Models

Human information models are the type of models, which explain accident as the cause of human errors, unsafe acts and unsafe conditions. They include methods such as Cognitive Reliability Error Analysis Method (CREAM) and Standardized Plant Analysis Risk-Human Reliability Analysis (SPAR-H).

2.3.1. Cognitive Reliability Error Analysis Method

Cognitive Reliability Error Analysis Method (CREAM) is a human information model, which involves technical factors, factors of individuals and of the whole organizations. It is used as both an accident analysis and risk analysis technique in which actions of single actors are specifically addressed with the help of control modes. It can predict human error as well and can be used single handedly for accident investigations or can be collaborated with any other method for interactive systems[17].

2.3.2. Standardized Plant Analysis Risk-Human Reliability Analysis

Like CREAM, Standardized Plant Analysis Risk-Human Reliability Analysis (SPAR-H) is also a human reliability analysis method which has been initially developed in the nineties for nuclear power industry to determine the chances of human errors related to the workers' actions [18].

Table 2. Applications of accident analysis techniques in different industries

Methods	Strengths	Weaknesses	References
FMEA	-Indicates all the probable failures. -Supposes a failure mode and determines the worst case effects.	-Consequences are described mostly instinctively. -Unable to take complex failure modes into consideration.	[2, 56, 57]
FMECA	-Includes criticality which is the estimation of severity and occurrence. -Determines process reliability.	-An extensive knowledge of the issue under investigation is needed. -The implementation phase is difficult.	[3, 24, 25]
STAMP	-Investigates and assesses the minute things such as duties of the staff in addition to the large ones. -Mentions the causes of human performance and component failures.	-Cumbersome to use a STAMP model, takes a lot of effort and is unsuitable for a novice.	[4, 58]
STPA	-Is concerned with the safety constraints in a system. -Considers many of the systemic factors including the interactions.	-Analysis is too complex; a tool is needed for simplification. -The resulting tables are too large in size.	[5, 26, 27]
CAST	-Assesses the whole accident process in case of an accident and indicates the key causal factors. -Focuses on why the accidents occur in the first place.	-Detailed data about the system is needed which might not be available publically. -The recommendations based on CAST may also not be feasible or may take a long time to be implemented.	[6]
HAZOP	-It not only determines the hazards; it demonstrates the probability and consequence of an event. -Spots the hazards and proposes the safety measures to avoid these hazards.	-Depends solely on human knowledge and a whole team is required for a considerable long amount of time. -Does not include the interactions among various parts of the system.	[2, 7, 26, 59]
HFACS	-Takes into account all kind of errors. i.e. active as well as latent ones. -Multiple accident cases and scenarios can be easily entertained.	-Cannot be applied outside aviation industry satisfactorily. -The failure beyond the organization's premises such as government role cannot be incorporated.	[8, 60]
AcciMap	-Describes the different causal factors and their inter relationships in a graphical form. -Causal diagrams guide us to the patterns that lead to the occurrence of accidents.	-Training of AcciMaps and sufficient pertinent knowledge is essential in using this methodology. -The reliability can be challenged and also lacks taxonomical support.	[9, 58, 60]
FRAM	-Used both as a risk analysis tool and as an accident investigation tool. -Application is structurally easy.	-Demands vast knowledge about human factors and an extensive theoretical background with a big chunk of time to learn it in the beginning.	[58]
LOPA	-Includes all preventive and mitigative measures. -Includes its own calibration and contains the use of corporate criteria in a lucid way.	-Does not entail the common cause failures (CCF). -Takes considerable amount of time, requires a lot of resources and expertise of professionals.	[61]
SCM	-Considers the interactions between latent factors and the unsafe acts. -Shows system's defenses diagrammatically in such a way that the pits exhibit the breakage of defenses.	-Oversimplifies the causation analysis more than enough. -Is never aimed to be a detailed accident analysis model.	[12, 43, 60]
FTA	-Provides insights into the operation. -Enables the analyst to determine major contributors to TOP event frequency. -Takes different systems into consideration such as emergency systems, operations.	-Model is incomplete, only deals with the listed mechanisms. -There is a major uncertainty in the frequency of an event.	[44]
ETA	-Starts from one event and discovers the probabilities. -Used to quantify the chances of the end event in terms of different outcomes.	-Deals with only one starting event at one time. -A professional with practical knowledge and vast experience is required.	[62, 63]
DEA	-Analyzes chain of events that lead to accidents.	-Has a limited scope and only contains the clear causes of an accident.	[15]
CA	-Determines the effects of a likely failure event on human, equipment and facility and tells about the possible consequences they may have to face.	There is a great deal of uncertainty with many of these models. a potential error in terms of magnitude is anticipated in these consequence analyses.	[16, 44]
CREAM	-Involves technical factors, factors of individuals and of the whole organizations. -Addresses single actors as well and predicts human error.	-Lacks theoretical background and has limited ability to deal with the psychological factors.	[17, 64]
SPAR-H	-Determines the chances of human errors with performance factors.	-Prediction of human error probabilities may not be suitable.	[18, 55]

2.4. Structural Equation Modeling

Recently a statistical technique known as structural equation modeling has been used effectively in accident causation in order to identify and address the significant factors that contribute to the occurrence of these accidents. Structural equation modeling is a technique that hypothesizes how a construct/factor is defined by a set of variables and what is the link between constructs themselves [65] N and is preferred because of its ability to deal with complex theoretical models using multiple group models [66]. This technique has been effectively used to analyze 320 coal mines accidents in China which determined the lead causes that led to these minor and major accidents [67]. Since Pakistan is a developing country and is yet to implement zero accident vision, plenty of accidents occur in different industries yearly. Data of accidents has been collected online from February 2012 to April 2017. As it can be seen from the last five years' data, majority of accidents in Pakistan occurred in textile and garments industries therefore it is pertinent to use structural equation modeling in textile industry to know the reasons behind these accidents and to identify the primary and secondary causes that lead to them.

Table 3. Summry of injuries occured in industries

Sr #	Industry/City	Data of occurrence	Number of injuries	References
1	Medicine factory, Kharak	Feb 6th 2012	9 dead ,16 wounded	[68]
2	Gas Cylinder Company, Karachi	May 21st ,2012	1 dead,6 injured	[69]
3	Ali Enterprises Textile, Karachi	Sept 12th ,2012	289 people dead	[70]
4	Shoe factory, Lahore	Sept 11th ,2012	25 people dead,8 injured	[71]
5	Tissue paper and diaper factory, Karachi	Oct 5th ,2012	2 injured	[72]
6	Aslam Industry and Medical Gases, Rawalpindi	Jan 7th ,2013	3 dead,2 injured	[73]
7	Winboard factory, Faisalabad	Jan 9th ,2013	1 dead,5 injured	[74]
8	Layyah Sugar Mills, Layyah	Jan 13th ,2013	8 injured	[75]
9	Plastic factory , Lahore	Mar 30th ,2013	8 injured	[76]
10	Shoe making factory, Lahore	Apr 24th ,2013	No casualties	[77]
11	Thermopol factory, Lahore	Nov 15th ,2013	Valuable goods burned	[78]
12	Dawood exports, Faisalabad	Dec 26th ,2013	9 dead,8 injured	[79]
13	Fine Gas Company, Lahore	Mar 15th ,2014	4 dead,17 injured	[80]
14	Saad Garment factory, Karachi	May 15th ,2014	1 dead	[81]
15	Garments factory, Karachi	May 16th ,2014	1 dead	[82]
16	Garments factory SITE, Karachi	June 16th ,2014	No casualties	[83]
17	Garment factory Karachi	July 22nd , 2014	No casualties	[84]
18	KBI Textile Mills, Karachi	Dec 7th ,2014	No casualties	[85]
19	Food Factory, Karachi	Apr 13th ,2015	6 dead	[86]
20	Garment factory, SITE Karachi	May 4th ,2015	13 injured	[87]
21	Dye factory, Karachi	May 30th,2015	No casualties	[88]
22	Towel factory, SITE Karachi	July 23th ,2015	No casualties	[89]
23	Garment factory, Lahore	Sept 4th ,2015	4 dead,18 injured	[90]
24	Flour Mill, Gujranwala	Sept 5th ,2015	5 dead,30 injured	[91]

25	Garment factory, SITE Karachi	Sept 9th ,2015	1 dead	[92]
26	Plastic factory, Lahore	Nov 5th,2015	41 dead,103 injured	[93]
27	Steel Mill, Lahore	Nov 10th ,2015.	5 dead,3 injured	[94]
28	Arfan Steels, Lahore	Jan 5th ,2016	8 injured,	[95]
29	Plastic Factory, Gujranwala	Jan 6th ,2016	1 dead,30 injured	[96]
30	Daud Steel Mill, Swabi	Jan 8th ,2016	1 dead	[97]
31	Kashmir Sugar Mills, Jhang	Jan 19th,2016	8 dead,11 injured	[98]
32	Kims Biscuit factory, Hattar	Feb 25th ,2016	1 dead	[99]
33	Textile Factory, Karachi	Apr 4th , 2016	1 dead,4 injured	[100]
34	Plastic factory, Lahore	Apr 17th ,2016	No casualties,	[101]
35	Garment factory, Lahore	Apr 17th ,2016	No casualties	[101]
36	Plastic factory, Karachi	May 15th ,2016	5 dead	[102]
37	Fauji Cement, Fateh Jang	May 31st ,2016	No casualties,	[103]
38	Pharmaceutical factory, Karachi	June 24th,2016	No casualties	[104]
39	Paper Mill, Okara	July 1st ,2016	2 dead,3 injured	[105]
40	Cold Storage factory, Karachi	July 4th ,2016	6 dead,3 injured	[106]
41	Garments factory, Lahore	July 23rd ,2016	1 dead,4 injured	[107]
42	Cakes and Bakes Factory, Lahore	Sept 17th ,2016	3 dead,2 injured	[108]
43	Chemical factory, Karachi	Oct 4th ,2016	3 dead	[109]
44	Indigo Textile mills, Karachi	Oct 5th ,2016	3 dead,2 unconscious	[110]
45	Ciaton Engineering Company, Karachi	Oct 22nd ,2016	3 dead, 4 injured	[111]
46	SITE Textile factory, Karachi	Oct 25th ,2016	No casualties	[112]
47	Ashraf Garments, Lahore	Nov 11th ,2016	3 dead	[113]
48	Sugar mill, Rahim Yar Khan	Mar 18th ,2017	2 dead,2 injured	[114]
49	Shoe factory, Lahore	Apr 9th ,2017	No casualties	[115]
50	Cotton factory, Gujranwala	Apr 28th 2017	No casualties	[116]

3. CONCLUSION

This study is unique in the sense that a thorough review of existing accident causation techniques has been done and their strengths and weaknesses have been stated. The industries where these techniques have been applied worldwide have been pointed out and it is declared that for complex and integrated systems there is a need for newer techniques, which not only address the contributory factors but also address the interactions among these accident causation factors. Recently a statistical technique, structural equation modeling has been used in accident causation analysis in order to determine the causes and their interactions that lead to such catastrophes and it is emphasized that this technique should be used more in order to check it's feasibility and usage in accident causation[67].Keeping in view the recent statistics of accidents in industries in Pakistan, it is recommended that the textile industries specifically should be analyzed with the above mentioned techniques or more preferably with structural equation modeling to find the causes of the accidents in order to make the environment more stable and hazard free. Moreover, data has been gathered from the best available resource i.e. the online newspapers' archives and the online news. In some developed countries, there is a separate accident database and a separate institution for accident data, which files the data that contains the actual occupational accidents, their reasons, their causes and the damage that they caused and the lives they affected. In Pakistan there is no such institution so the data has been gathered using the sources of web, however it is a common practice in some developed countries as well to collect data of occupational accidents with the help of newspapers and TV channels such as BBC, Reuters, The Guardian, The Times of India and many more [117] . Accident data in biodiesel industries has also been gathered for a database with the help of documented sources such as the Herald, the Telegraph, CTV News [118], however it is essential to have an institution that logs the accidents data and the causes of these failures. In Pakistan there is no such institute currently therefore it is highly recommended for the government to make a separate organization that logs and documents all these details on daily basis.

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Research Article

Investigation of Radiolysis Induced Dosimetric Parameters of a Synthetic Dye for Gamma Dosimetry

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Abstract: Present work deals with the study of radiation-induced dosimetric parameters of aqueous solutions of Sandalfix Orange C2RL (SO) dye within 0.1-100 kGy gamma dose range. A UV/VIS spectrophotometer was used for the spectrophotometric analysis of irradiated and un-irradiated sample solutions.Sample solutions were irradiated by Cs^{137}γ-ray source within 0.1-100 kGy dose range and sample solutions were analyzed in three different dose ranges i.e., 0.1-0.9kGy (low dosimetry), 1-9kGy (intermediate dosimetry) and 10-100 kGy (high dosimetry), respectively. The effect of gamma radiation on the response of co-factors of absorbance i.e., change in absorbance (ΔA) and relative absorbance (Ã) was discussed.The value of molar extinction coefficient was maximum at 430nm and decreased with respect to absorbed dose (D).

Keywords: Radiolysis, Molar extinction coefficient, Relative absorbance, Change in absorbance, Dosimetry, Gamma radiation, Sandalfix orange C2RL, SO dye

1. INTRODUCTION

Now-a-days the chances of exposure to Ionizing Radiations (IRs) have been increased. Therefore, radiation measurement is an active research area of present era. Radiation Dosimetry (RD) plays an important role in the quality control of radiation processing [1]. A quantitative study in RD requires information of the amount of energy absorbed by the IR. IR is a special type of advanced oxidation processes (AOPs) which can yield nearly equal amounts of oxidizing species (•OH and H_2O_2) and reducing species through water treatments [2]. Irradiation can change the chemical properties of a chemical dosimeter; which may respond linearly, exponentially or logarithmically etc. to IRs under appropriate conditions. Dyes either natural or artificially produced are being used to impact color as well as for dosimetric purposes like aqueous solutions [3-10]. Researchers have used different colors i.e., direct yellow 12 [4], alizarin yellow GG [2], sandalfix red C4BLN and sandalfix yellow CRL [8].

Dye dosimeters work on the principle of chemical dosimeters; and are being used for the estimation of absorbed dose of gamma radiation [11-15]. The absorbance (A) of the aqueous solutions of Sandalfix Orange C2RL (SO) dye was found to decrease linearly and logarithmically for low dosimetry (0.1-1 kGy) and high dosimetry (10-100 kGy) ranges, respectively. However, an exponential and logarithmic increase in %decoloration (%Đ) was observed within 0.1-1 and 10-100 kGy, respectively [11]. Absorption spectra of SO dye showed that the absorbance (A) of irradiated solutions was decreased with respect to absorbed dose (D). Mean absorbance (Ā) of irradiated sample solutions followed a linearly decreasing function with respect to absorbed dose (D) within low dosimetry range while logarithmically decreasing response of Ā was observed for both intermediate

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and high dosimetry ranges [12]. The decoloration (Đ) of aqueous solutions of congo red (CR) dye showed a linear relationship with absorbed dose [16].

Dye dosimetry is an area of interest for the researchers to produce eco-friendly and inexpensive dosimeters which have the capacity to work within wide dose range. The value of molar extinction coefficient for solutions is required in many scientific, engineering and chemical disciplines involving photon interactions [17].Calculating the molar extinction coefficient (ε) of material(s) is important in the field of radiation physics [18]. Sandalfix Orange C2RL (SO) dye is inexpensive and easily available in market. The proposed plan of this work is to investigate the radiation-induced dosimetric parameters of the aqueous solutions of SO dye within the selected dose range.

2. MATERIALS AND METHODS

The sample solutions of Sandalfix Orange C2RL (SO) dye (Molecular Weight: 1034.27 amu; Molecular Formula: $C_{31}H_{20}ClN_7Na_4O_{16}S_5$) were prepared by following the procedures as reported by Hayat et al., [11-12]. The prepared acidic sample solutions (having pH value 4, 5 and 6) were stored in dark. A UV-Visspectrophotometer *(Lambda 25 1.27, PerkinElmer, USA)* was used for the determination of characteristic wavelength (λ_{max}); absorbance (A) of all the sample solutions was determined at this λ_{max}. Cuvettes (path length of 1 cm) were used to keep the solutions in the object beam. Figure 1 shows the molecular structure of SO dye.

3. RESULTS AND DISCUSSION

Effect of IR on an exposed material depends upon the composition of that material and energy transferred by the incident photon.Water radiolysis causes the production of species i.e., hydrated electron, H_2O_2, H_2, OH^-, H^+ and •OH radical etc. The production of these primary species depends upon the linear energy transfer value of radiation [11]. The electrophilic attack by •OH radicals occurs at the carbon (C) atoms where the naphthalene ring and the azo group link up. This reaction leads to the destruction of the chromophore. So, the products formed do not have enough light absorption and consequently absorbance of the sample solutions decreases. The probability of partial or total saturation of N=N also exists in this reaction [19]. The absorbance (A) of SO is decreased with respect to absorbed dose (D) caused the increase in response of both co-factors of absorbance i.e.,change in absorbance (ΔA) and relative absorbance (Ã).

Response curves are obtained by plotting ΔA and Ã against absorbed dose (D). Change in absorbance (ΔA) can be calculated by using equation 1 [20].

$$\Delta A = A_0 - A_i \quad (1)$$

Where, A_0 and A_i represent the absorbance of un-irradiated and irradiated sample solution,

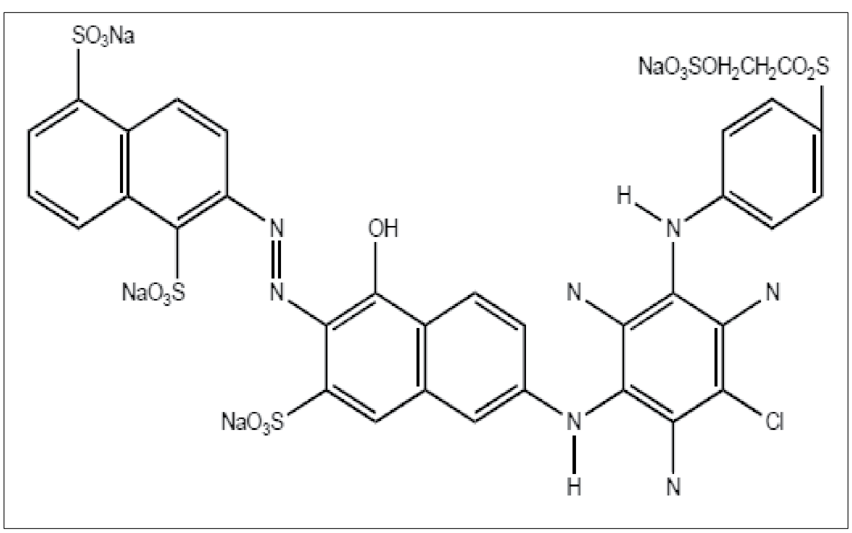

Fig. 1. Structure diagram of SO dye

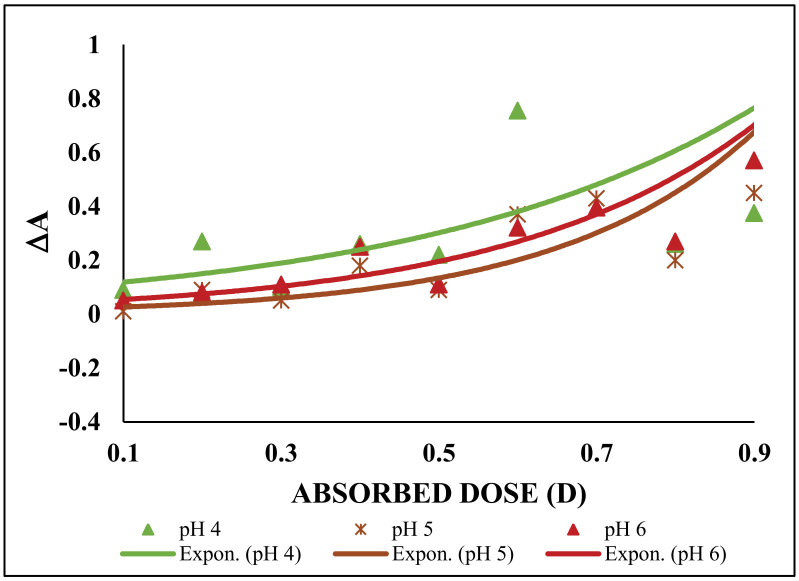

Fig. 2. Response of change in absorbance (ΔA)within 0.1-1 kGy dose range

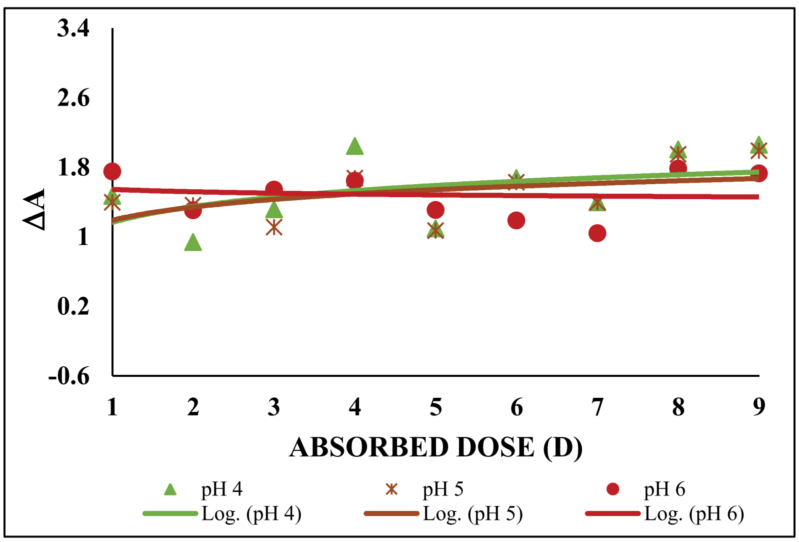

Fig. 3. Response ofchange in absorbance (ΔA)within 1-10 kGy dose range

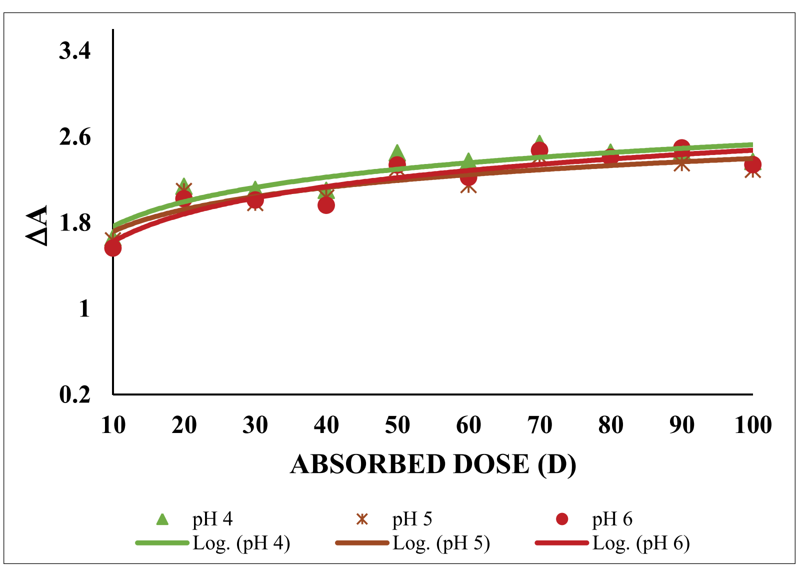

Fig. 4. Response of change in absorbance (ΔA) within 10-100 kGy dose range

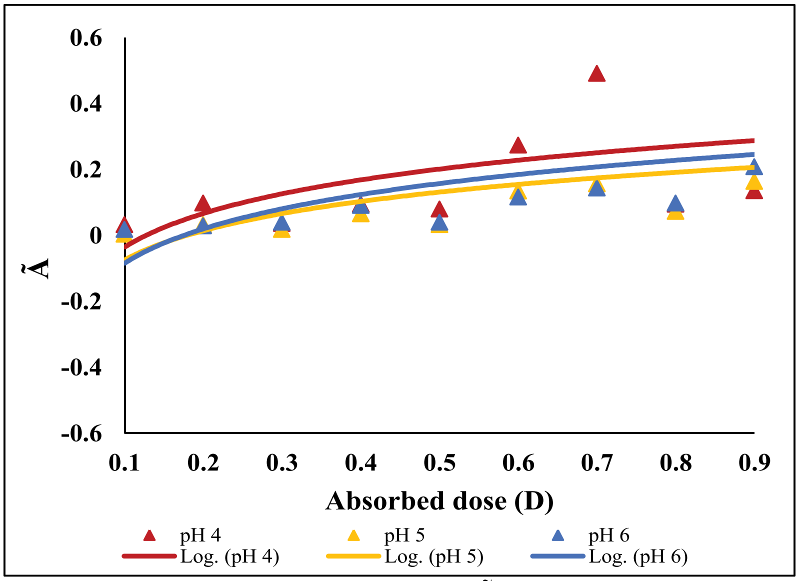

Fig. 5. Response of relative absorbance (Ã) within 0.1-1 kGy dose range

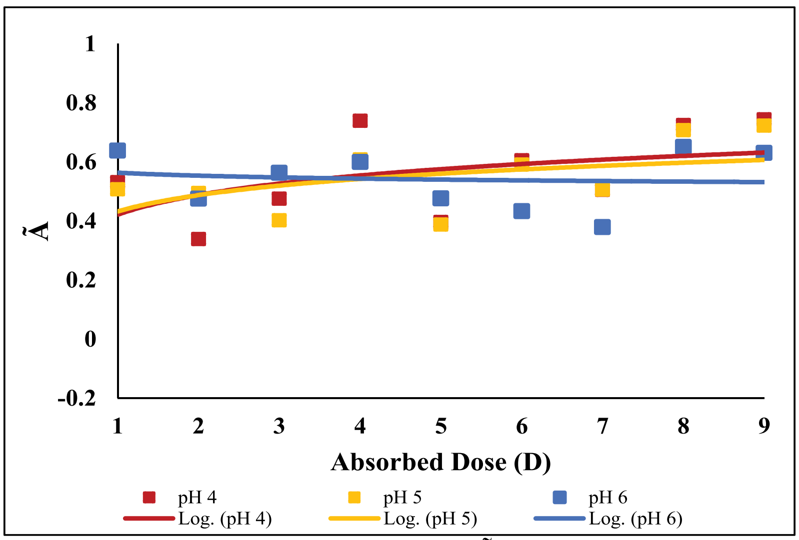

Fig. 6. Response of relative absorbance (Ã) within 1-10 kGy dose range

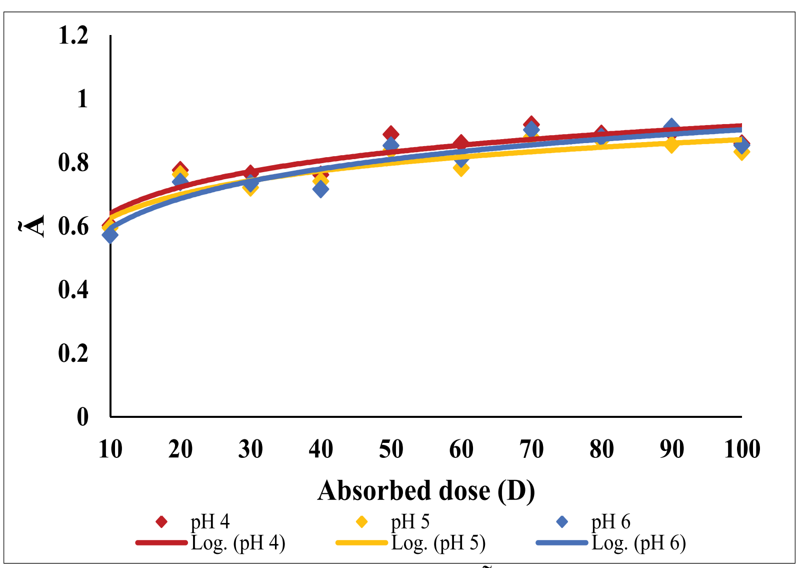

Fig. 7. Response of relative absorbance (Ã) within 10-100 kGy dose range

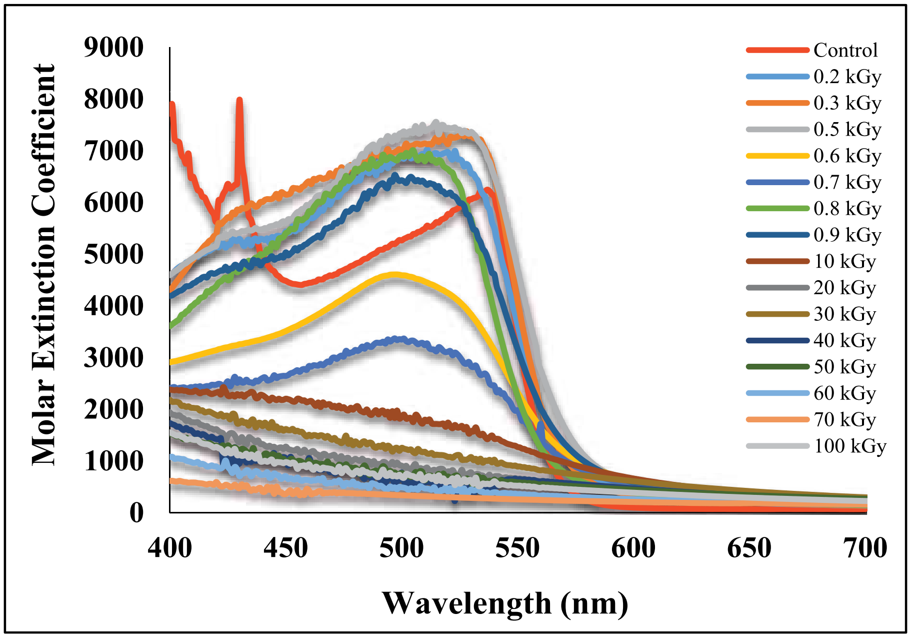

Fig. 8. Value of molar extinction coefficient (ε) of control and irradiated sample solution versus wavelength (nm)

respectively. Figure 2, 3 & 4 illustrate the change in absorbance (ΔA) of irradiated acidic sample solutions of SO dye with respect to absorbed dose (D) at characteristic wavelength (nm) i.e., 430 nm.

The ΔA is increased exponentially with respect to absorbed dose (D) within low dosimetry; while in both intermediate and high dosimetry ranges, the ΔA is increased logarithmically with respect to absorbed dose (D). Results give the evidence of photo-degradation of SO dye and the formation of defects and clusters in the material upon irradiation. Sensitivity to incident gamma photons is decreased with decrease in initial absorbance (A) [1]; so increase in ΔA is observed. The relative absorbance (Ã) of the sample solution can be calculated by using equation 2[21].

$$\tilde{A} = (\Delta A/A_0)_{430} \quad (2)$$

Where,ΔA and A_0 represent the change in absorbance and absorbance of un-irradiated sample solutions, respectively, at characteristic peak (λ_{max}=430 nm).

Figures 5, 6 and 7 represent the relative absorbance (Ã)of irradiated acidic sample solutions of SO dye within low, intermediate and high dosimetry ranges, respectively, which show the signs of dye degradation at different irradiation doses. The response of Ã tends to follow a good logarithmic function with respect to absorbed dose (D) as the dose increases from low to high dosimetry range. The value of Ã is found to increase with respect to absorbed dose (D)and follows a logarithmic function within 0.1-100 kGy dose range.

The probability of a medium (solution) to absorb a photon is directly proportional to the concentration of the solute in the solution and to the thickness of the sample solution as given in equation 3 (Beer-Lambert law).

$$A = \varepsilon c l \quad (3)$$

$$\varepsilon = A/cl \quad (4)$$

Where, A is absorbance, *l* is the path length of Cuvette, *c* is the concentration of solute and *ε* is the molar extinction coefficient. The value of *"ε"* depends upon the wavelength of the incident radiation; and is maximum where the absorption is most intense [17]. Figure 8 represents the value of molar extinction coefficient (ε) with respect to wavelength (nm). For control sample solution; the value of *"ε"* is maximum at 430 nm because the absorption is most intense at this wavelength. However, the value of *"ε"* is decreased with respect to absorbed dose (D)for irradiated sample solutions

Table 1. Value of molar extinction coefficient (ε) within low dosime

Dose (kGy)	Molar Extinction Coefficient		
	pH4	pH5	pH6
0.1	5286.89	5348.36	5327.87
0.2	4959.02	5245.90	5204.92
0.3	5102.46	5389.34	5512.29
0.4	4979.51	5122.95	4938.52
0.5	5081.97	5409.84	5614.75
0.6	4040.98	4610.66	4877.05
0.7	2799.18	4764.34	4938.52
0.8	4907.79	5081.97	4815.57
0.9	4774.59	4877.05	4436.48

Table 2. Value of molar extinction coefficient (ε) within intermediate dosime

Dose (kGy)	Molar Extinction Coefficient		
	pH4	pH5	pH6
1	2688.52	2926.23	2188.52
2	3606.56	2725.41	2922.13
3	2991.80	3520.49	2594.26
4	1272.54	1887.30	1926.23
5	3532.79	3356.56	2938.52
6	2159.84	2311.48	3204.92
7	2836.07	2758.20	3200.82
8	1778.69	1631.15	2094.26
9	1331.97	1485.66	2397.54

Table 3 Value of molar extinction coefficient (ε) within high dosimetry

Dose (kGy)	Molar Extinction Coefficient		
	pH4	pH5	pH6
10	2397.54	2327.87	2430.33
20	1254.10	1432.38	1479.51
30	1321.72	1575.82	1538.93
40	1411.89	1467.21	1502.05
50	788.93	971.31	823.77
60	993.85	1120.90	975.41
70	807.38	973.36	907.79
80	834.02	852.46	1407.79
90	807.38	782.79	676.23
100	956.97	956.97	866.80

of SO dye. The results are in agreement with the Beer's law and the statement of Singh et al., (2002) [18]. Tables 1, 2 and 3 give the value of *"ε"* within low, intermediate and high dosimetry conditions, respectively.

4. CONCLUSION

The aqueous solutions of SO dye showed sensitivity to gamma radiation. It was found that, except low dosimetry range (0.1-0.9kGy), both co-factors of absorbance i.e., change in absorbance (ΔA) and relative absorbance ($\tilde{A}$) followed the logarithmic function with respect to absorbed dose (D). The response of ΔA followed an exponentially increasing function with respect to absorbed dose (D) within 0.1-0.9kGy dose range while a logarithmic relationship was found between ΔA and absorbed dose (D) for both intermediate and high dosimetry ranges. Response of $\tilde{A}$ followed a logarithmically increasing function within 0.1-100 kGy dose range. The value of molar extinction coefficient *"ε"* was decreased with respect to absorbed dose (D). The investigation of dosimetric parameter revealed the sensitivity of SO dye towards gamma radiation and its degradation upon irradiation.

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Research Article

An Efficient, Cost Effective and User Friendly Approach for MCQs Treatment

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Abstract: The ongoing era is called the technology era and every one want to have automatic system means that the work is done just on one click. The inclusion of human being in any work may become problematic, fraudulent and un-sophisticated. We keep the need of current time and decide to develop such a system which is capable of to automatically grade Multiple Choice Questions (MCQs) paper. Manually grading/marking of a paper is time consuming, boring and complicated task. Moreover, it may possible to make fraud by fraudulent examiner during paper marking. In this paper, we have proposed a novel approach for automatic paper grading. The proposed approach is user friendly and efficient, which will mark a candidate's answers automatically and return, within a very short period. The proposed system for MCQs marking consists of a camera and computer, and can accept any type of marking on bubbles.

Keywords: Paper Marking, Mark Reader, Image Processing, Image Segmentation

1. INTRODUCTION

As compared to automatic system the manual marking/grading of a paper is complicated and time-consuming task. The examiners are also uninterested by using this manual method. Fraudulence, wrong calculation and mistakes may also be possible in this method. Due to such types of mistakes may lead deserved students to bottom. In order to avoid such types of errors and to save examiner's time it may be better to have an automatic system. We want to solve this crucial problem, to save examiner's as well as student's time. Due to writing details in subjective test a student, waste a lot of time. While the objective (MCQs) test requires limited time for paper attempting.

We developed a system, which automatically grade MCQs type test automatically. The proposed system is user friendly, save candidate's time and inexpensive. The proposed system provides full permission to candidates on circle marking. The candidates are allowed to mark MCQs options by his/her own styles. So it saves a lot of candidate's time by providing such type of freedom. The proposed algorithm requires camera (mobile/digital/scanner) and computer. These recources are affordable for interested user or organization. The algorithm has some novel characteristics like saves the candidate's time, required low cost and user friendly. We also use some of the best image enhancement filters and techniques like histogram equalization that increase the accuracy rate.

Most of the existing systems in markets are working through dedicated scanners which are also known on Optical Mark Reader (OMR) [1,2]. Besides paper grading this technology is also use for survey, questionnaire etc. This system requires some conditions that should be followed by candidates like circle marking so it is not user friendly. The system also requires high cost which is not affordable for everyone. Nutchanat and Akawee [3-7,9-15] have suggested a system which is based on cross mark in grid form sheet instead of filling circle or square. A camera based work is presented by Al-Marake and Kosolapow [4,15]. Another system presented by Jantschi [4] is an online system instead of manual marking using a database of MCQs for testing of different subject. There exist other systems [3,5,7,12,13] which use scanner for image acquisition and pre-defined answer sheet (a marked master copy).

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In the above systems, the concept of 'master copy' requires image registration. Image registration is a process to bring similar objects into same co-ordinate position of input sheet and master copy for answers comparison. On circle marking the requirement of the existing systems is to shading or by specific way (Fig. 1). There are also few systems which uses cross symbols for selecting the correct option. We will use camera instead of scanner which will increase the speed and decrease the cost of the system because a camera can read multiple papers at a single time and have low cost as compared to OMR scanner. The price of normal camera is about Rs 7000 to Rs 15000 [1]. We also studied about different work related with image enhancement like noise removal, enhancing and smoothing filters in [8,10,11,14].

The filling of circles or marking answers in a specific way wastes time and any mistake in marking the answer in that specific way may lead to wrong marking and loosing of marks. The available systems are very expensive and the normal price is about Rs 40,000 to Rs 250,000 [1,2]. The available OMR technology is working on dedicated scanner. Normally the speed of scanner is low as compare to camera. These OMR scanners require the control environment like, specific thickness of paper, have specific colour of paper and completely filling the bubbles. These systems are also required technical and experienced people to operate it.

2. PROPOSED SYSTEM

In our new work we propose a system which understands any tick mark symbol (Fig. 2). This system will give more freedom to the user and will be able to save the user time. Our proposed system is user friendly, efficient and cost effective as compared to the available systems.

We design a special type of answer sheet of 100 questions in Corel-Draw. The using of Coral draw is not necessary the algorithm can also work on any other designing tool. Then import that sheet to MS word and mail merged with MS Access database for inserting user information (Fig. 3).

Basically the sheet consists of three sections. The first section for user information that is mail-merged from database. The second section is used for marking roll number which assigned by the test

Fig. 1. Symbols supported by existing systems

Fig. 2. Symbols supported by propose system

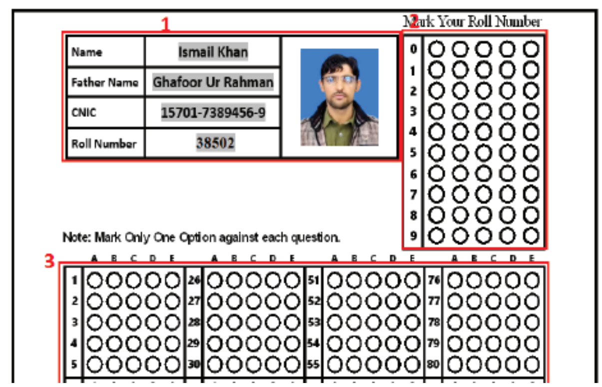

Fig. 3. User information through mail merging

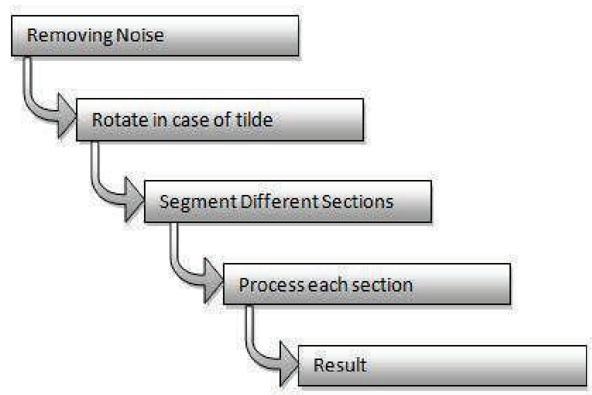

Fig. 4. Basic image processing steps

conductor. The third section is the important portion in which user will mark the MCQs options. This algorithm use basic image processing techniques for solving the problem. A user takes image of MCQ based answer sheet and give as input to the algorithm. The algorithm process the sheet for further information as shown in Fig. 4.

In the above steps when the algorithm reached to segmentation the paper is segmented into three sections. These sections consist of User information,

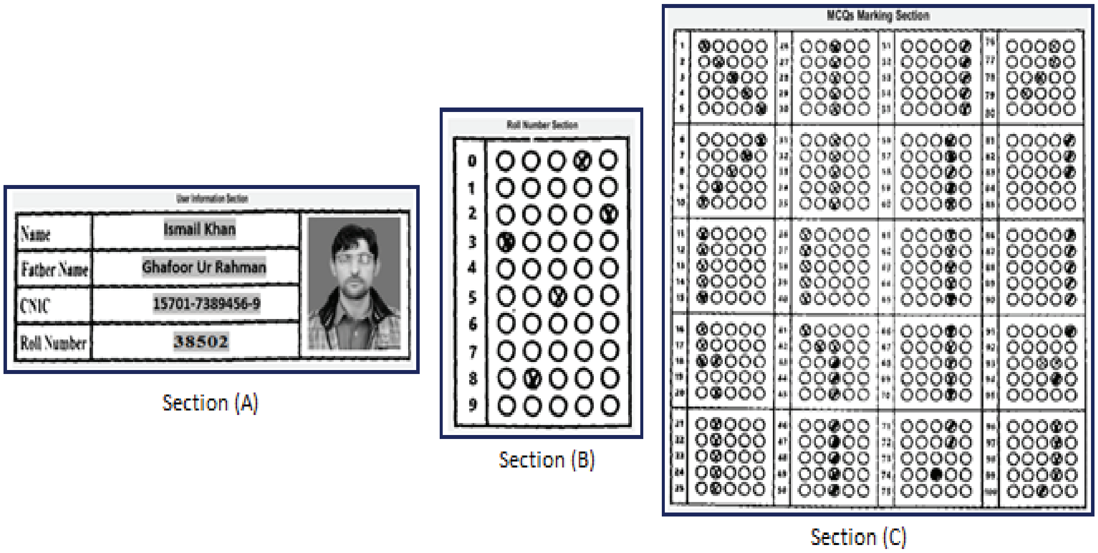

Fig. 5. Extracting three sections from paper

Roll number and MCQs sections (Fig. 5).

We have prepared a database in MS Access to store the user information and questions of subject with options. From the database the test conductor will prepares questions paper. The questions paper and MCQs answer sheet will be given to students to mark the answer. When the students complete this task the operator will capturing the answer sheets through camera and give the images as input to Automatic Paper Marking System (APMS). The AMPS process the sheet in a few steps. First it crops the extra area which not require. Now the required area processed for further information. The cropped area is subdividing into three sections like in Fig. 5. Then each section is processed separately. We apply different filters on each section like noise removal, enhancement and smoothing the image [8, 10-11, 14].

The system will check the paper and store the result back into database for future information. The following are the basic steps of the processing and recognition of bubbles area:

a) Image acquisition
In our proposed system we used camera for image acquisition.
b) Image preprocessing
The standard image preprocessing techniques are applied like converting to binary, edge detection, rotating, object separation, object detection and recognition.

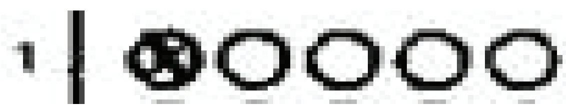

Fig. 6. Separating Question

c) Segmentation
Separate each question along with its options (Fig. 6).
d) Recognition of marked option(s)
This is the step where we recognize the options differently. Our method uses sum of the numbers of pixels and decide whether pixels have been added to the option or not. This is an efficient method to know whether an option has been marked.
e) Check whether the marked option is correct or not. Matching with database entries.
f) Result

We set the ratio of black and white pixels (2:3) 45% and 55% of bubble recognition. When the black pixels ratio exceeds from 40% then it should be conceived that it is marked otherwise unmarked. We used the region props algorithm which measure properties of image regions like extracting Bounding box, Centroid, Euler number, Perimeter, filled area and so on. This algorithm are designed using Matlab, MS Access and follow the basic image processing steps like image acquisition, preprocessing, segmentation, features extraction and result. The MS Access database is used for storing the questions with multiple choice answers. The system interacts with database for different types of queries.

3. RESULTS

For evaluating the performance of the algorithm, we performed a series of experiments. We investigated the user friendliness of the proposed algorithm, its performance on low quality images and its implementation cost.

In traditional systems, the users have to fill the circles for marking the answers, which takes a lot of time. In our experiment we have provided freedom to the users and they can use different options for marking the answers. The option includes filling the circles, tick marking the circles or crossing the circles. For comparing the time needed to mark the answers, we selected 40 students. Half of the students were asked to mark the paper using the traditional style i.e. filling the circles while the other rest of the students were left free to mark the answers. They were allowed to mark the answer by either filling the circles, tick marking the circles or crossing the circles. In this experiment we only considered the time needed for marking the answers and ignored the thinking time for finding the answers.

The result of this experiment is shown in Fig. 7. The results show the proposed algorithm saves 80% time in answers marking. We also tested the algorithm in terms of input image quality. We have

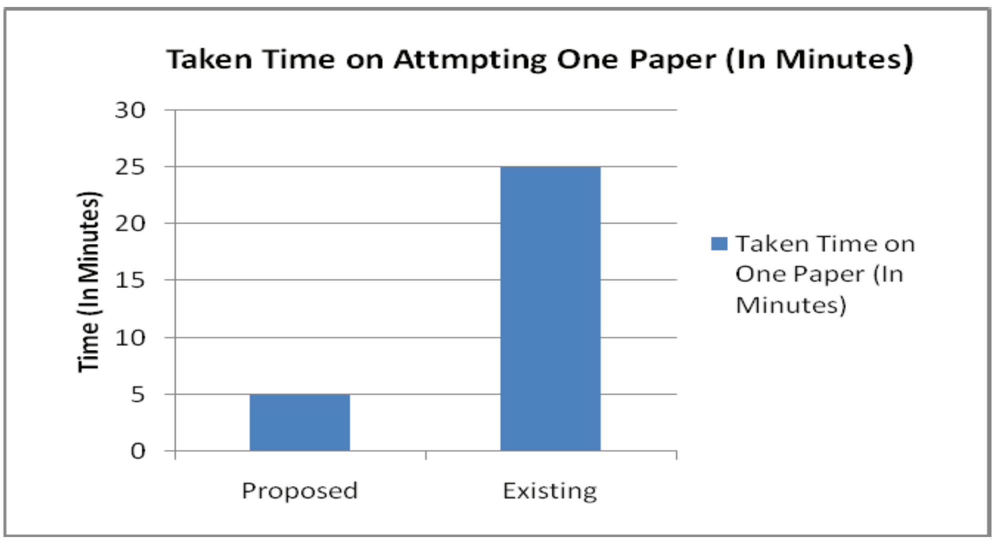

Fig. 7. Time comparison between proposed and existing system

Table 1. Total number of tested papers with results

S.No	Number of Paper	Number of Questions	Result	Image-Quality	Mark-Type of circle
1	30	100	94.5%	Bad	Cross
2	30	100	96.3%	Bad	Filled
3	30	100	97.7%	Moderate	Cross
4	30	100	98.8%	Moderate	Filled
5	30	100	99.6%	Good	Cross
6	30	100	100%	Good	Filled
	Total: 180		**Average: 586.90/6=98%**		

Table 2. Cost comparison between proposed and existing system

System Type	Requirements	Price (without computer of laptop)
Proposed	Computer or laptop, Camera	R.s (7000-15,000) [15]
Existing	Computer or laptop , OMR Scanner	R.s (40,000 – 2,50,000) [1-2]

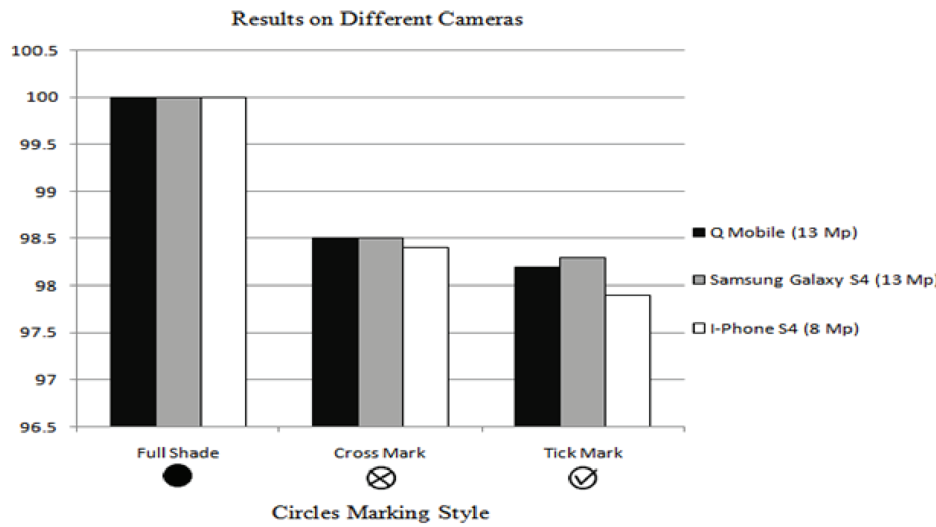

Fig. 8. Results of using different cameras

used cameras of different resolution including Mobile phone camera and digital cameras. In this experiment we scanned 30 papers. Fifteen papers were scanned using digital cameras and fifteen papers were scanned using mobile phone camera. In both cases equal papers were marked by filling the circles, tick marking and crossing the circles. The results of this experiment are shown in Table 1 while its graphical representation is shown in Fig. 8. The proposed algorithm also needs limited resources as compared to the existing approaches. A comparative study of the resources needed is shown in Table 2.

4. DISCUSSION

The proposed algorithm is cost effective and user friendly. For implementation, it needs only a computer and a camera so it is affordable for every organization. The system is easy to operate and even inexperienced people can also operate it. The system gives freedom to the users for answer marking and it saves candidate's time. The results show that the performance is not good when image quality is bad and the circles are not filled. However the performance is good when circles are filled even if the image quality is not good a shown in Fig.7. The minimum requirement of the proposed system is 5 megapixels camera.

5. CONCLUSION

In this paper, an algorithm is proposed for automatically grading Multiple Choice Questions (MCQs) paper. Manually grading/marking of a paper is a time consuming, boring and complicated. The proposed algorithm save a lot of user time, require low cost and is user friendly. The algorithm accepts any type of mark on bubbles that give more freedom and save a lot of user's time. The results show the proposed algorithm saves 80% time in answers marking. The proposed algorithm required only a camera and computer. The performance of the algorithm is satisfactory (95%) even if the input image quality is not good. The performance can be improved for bad images by investigating image enhancement techniques for improving the quality of the images.

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Resarch Article

Effect of Adding High Strength Concrete Topping on Flexural and Shear Behavior of Hollow Core Pre-stressed Slabs

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Abstract: This paper presents an experimental study regarding the effect of adding extra topping on flexural& shear capacity of hollow core pre-stressed concrete slabsand mainly focuses on how to increase shear and flexural capacity of slabs on site with the help of in-situ concrete topping. There may arise a situation where a depth greater than 200 mm and less than 265 mm may be required due to particular applied loading. Such extra depths can only be produced by applying in-situ concrete topping layer. High strength concrete toppings of 32 mm and 65 mm thicknesses were used on slabs of 200 mm original depth and were also compared with slab of 265mm depth. Out of four topped slabs tested in the program, two were tested under flexure and the other two under shear loading. Two control slabs of 200 and 265 mm but without extra topping were also tested in the same way. The ultimate load capacity, shear capacity, deflection, crack pattern, crack width and modes of failure were investigated for each slab. From the findings, it was concluded that the addition of concrete topping could enhance the flexural and shear capacity of hollow core units. For flexural loads, the increase was up to sixty percent for 65 mm topping and twenty-three percent for 32 mm topping. For shear testing, the increase was almost hundred percent for 65 mm topping and fifty-six percent for 32 mm topping. It was also observed that both flexural and shear strength of topped slabs was almost equal to the strength of un-topped slabs of same original depth. In cost comparison, the topped up slab came out to be 8.45% expensive as compared to hollow-core slab of similar depth

Keywords: Hollow core pre-stressed slab, Topping, Cracking, Deflection, Shear strength, Flexural strength

1. INTRODUCTION

Concrete is one of the most widely used, versatile and economical material in today's construction industry. Concrete as a structural component can be seen in buildings and bridges in various forms. For the development of an overall efficient and safe structure, it is required to understand the response of these forms under different types of loading conditions [1]. Although concrete is being used as a primary material since early 1960s, the form in which it is used has been changing with time. Pre-stressed hollow core slabs are the most common form of precast flooring used world-wide, with an annual production of about 20 million cubic meters in Europe. These represent about 40 to 60% of the overall precast flooring market [2] and are mainly used in the construction of car parking, industrial and residential buildings and sports stadiums. A typical hollow core slab is a monolithic pre-stressed or reinforced concrete unit with the overall depth divided into upper and lower flanges linked with vertical webs, constituting longitudinal holes, the cross section of which remains constant throughout. Scott presented a load test on hollow core slabs with concrete topping and came up with an answer that the composite action between the slab unit and additional topping was observed until the failure load [3]. Bayasiand Kaisar [4] used RCC topping but results showed that certain number of steel studs was needed to reduce failure potential, which was obviously the result of inadequate stress transfer.

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Flexural behaviour of hollow core slab units with concrete topping was also studied by Dowell and Smith [5] who clarified that the topping acted satisfactory in flexure and there was no shear slip at the interface of original slab and additional topping during the application of load. Rehman et al.[6] did research on hollow core slabs of different depths and concluded that for certain span lengths and prestressing tendons, the failure mode changed from pure flexure to flexure-shear for more than 200 mm deep slabs. Silfwerbrand [7] studied the bond between old and fresh concrete and concluded that the bond strength with proper workmanship and treatment of the interface was 3 MPa.

Micallef [8] performed research to assess the shear capacity of pre-stressed hollow core floor units. The hollow core slabs were tested and analyzed under the application of a knife edge load at a distance of 575mm from the support. The failure load was then compared with the respective safe calculated and load deflection curves were drawn to assess the mode of failure. Brooand Lundgren [9] presented a detailed analytical study of finite element analysis on hollow core pre-stressed concrete slabs subjected to shear and torsion. He found out that shear failure starts with a bending crack that turns into an inclined crack and ends with a shear displacement along the crack. Hawkins and Ghosh [10] concentrated on the shear strength of hollow-core slabs andobserved that if bond slip of the tendon occurred, it was only after the formation of the shear crack. In some of the test specimens, it was also noted that bond slip of the tendons was not the cause of shear failure. Girhammer and Pajari[11] studied the effect of additional concrete topping on the shear capacity of hollow core slab units. They found that the bond at the interface of original slab and additional topping adequate and noted the increase in shear capacity to be around 35 percent. Ibrahim et al. [12] did an experimental study on the shear behaviour of precast concrete hollow core slabs with concrete topping and concluded that the hollow core unit surface condition and longitudinal joint affect the stiffness and shear-flexure strength of the slabs. The optimum hollow core unit surface condition which can produce highest stiffness and shear strength is rough and wet conditions, while the longitudinal joint between hollow core unit panels reduces the slab shear strength. Eom et al. [13] did a research on evaluation of Shear Strength of Non-prestressed Reinforced Concrete Hollow-Core Slabs and concluded that the shear strength of hollow-core slab was degraded as the void ratio increased but hardly affected by other factors including the effective width of web. Pachalla and Prakash [14]studied Load resistance and failure modes of glass fiber reinforced polymer (GFRP) composite strengthened hollow core slabs with openings and concluded that GFRP strengthening is an effective and cost viable technique for restoring the strength and stiffness of precast prestressed hollow core slabs due to openings. Hwang et al. [15] did research on flexural capacity of Precast Concrete Triple Ribs Slab (TRS) and their results revealed that TRS had enough flexural strength and ductility to resist the design loads and its strength can be suitably predicted by using code equations.

Usually, at the time of production, surface of hollow core slab units is not a level and smooth surface. Instead, it is turned smooth and level at the site using screed or concrete topping. The thickness of such screed or topping is usually too small to have any effect on the flexural capacity of the slab units. In Pakistan hollow core slabs are available in two standard depths i.e 200 mm and 265 mm. There may arise a situation where a depth greater than 200 mm and less than 265 mm may be required due to particular applied loading. Such extra depths can only be produced by applying in-situ concrete topping layer. All the existing research tends to increase capacity of slab units by increasing slab depth at the time of manufacturing whereas research focuses on how to increase shear and flexural capacity of slabs on site with the help of in-situ concrete topping. However, the bond between old and fresh concrete should be sound. In this research, effect of additional topping on flexural strength of 200 mm depth slabs has been investigated. The results obtained were compared with corresponding hollow core slabs of 200 mm and 265 mm original depths.

2. EXPERIMENTAL PROGRAM

2.1. Testing Arrangement

The test specimens consisted of a total number of six slabs. Two were control slabs abbreviated as CONT200, CONT265 having 200 mm and 265 mm depth respectively. The rest of all four slabs

Table 1. Description of test slabs

Sr. No.	Slab	No. of Slabs	Slab depth (mm)	Topping depth (mm)	Topping type
1	CONT200	1	200	-	-
2	CONT265	1	265	-	-
3	HST32	2	200	32	High strength
4	HST65	2	200	65	High strength

were of 200 mm original depth. Two of these slaps were topped with 32 mm and the other two with 65 mm high strength concrete layer. The control slab of 200 mm depth (CONT200) was tested to study the effect of changing additional topping depth on its flexural and shear strengths. The control slab of 265 mm depth i.e., CONT265 was tested to compare its flexural and shear behaviour with 65 mm topped slabs with the same total depth of 265 mm. The surface of the slabs was roughened with steel brush before the application of topping for all the specimensto create a good bond between the old and the new concrete. Table 1 summarizes the detailed description of slab specimens tested in research program.

Hollow core slabs had a width of 990 mm with five 155 mm diameter holes running along 2400 mm length of slab. The pre-stressed strands of 10 mm diameter were used and an initial induced pre-stress was of 110 MPa. The area of steel used was 235.65 mm^2 in both CONT200 and CONT265. The cross sections of 200 mm and 265 mm slabs are shown in Figures 1 and 2 respectively whereas the testing arrangement for the application of load in flexural loading and shear loading are shown in Figures 3 and 4 respectively.

Tests were performed after 28 days of the application of topping. The hollow core pre-stressed slabs were tested under flexural and shear test conditions. Schematic diagrams of flexural and shear testsare shown in Figures 5 and 6. The load increment of 10 kN applied by a hydraulic jack was selected until the failure of slabs. Dial gauges were fixed on underside of the slab right below the point of application of load for the measurement of deflection. The development of first and subsequent

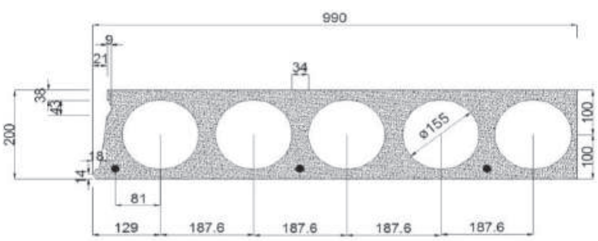


Fig. 1. Cross section of hollow core slab with 200 mm depth

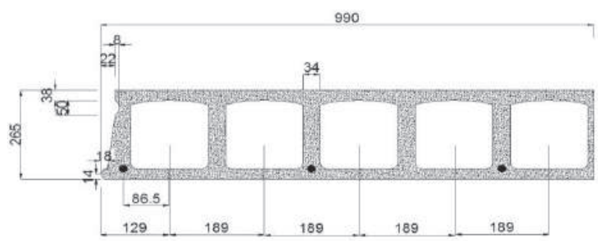


Fig. 2. Cross section of hollow core slab with 265 mm depth

Fig. 3. Testing arrangement for slabs in flexural loading

Fig. 4. Testing arrangement for slabs in shear loading

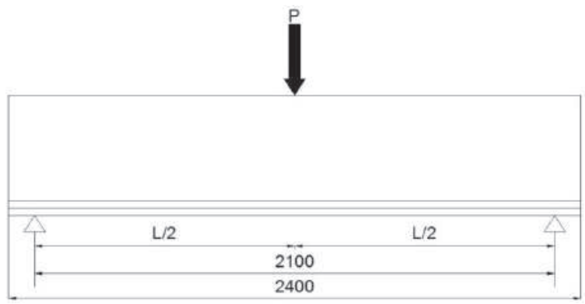


Fig. 5. Schematic diagram of flexural test

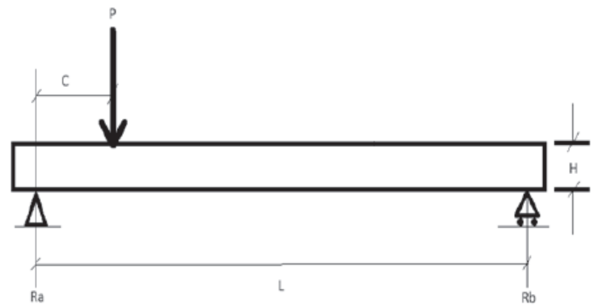


Fig. 6. Schematic diagram of shear test

cracks appeared up to failure of slabs was carefully monitored and the respective loads were noted.

2.2. Compressive Strengths of the Toppings

Three cylinders and three cubes were cast for each mix of concrete toppings and their compressive strength tested at 28 days age. Table 2 shows the average compressive strength values for 32 mm and 65 mm topped concretes. The compressive strength test was performed in accordance with the ACI code ASTM C42/C42-M-04 for cylinders and British Code BS EN 12390 for cubes. However, the 28 days compressive strength of concrete used in the hollow core slabs was 50N/mm^{2}as provided by the manufacturer.

Table 2. 28 days Compressive Strength of topping concrete

Sr. No.	Specimen	28 day's cylindrical strength (N/mm^2)				28 day's cubical strength (N/mm^2)			
		1	2	3	**Average**	1	2	3	**Average**
1	HST32	48.5	49.1	48.8	48.8	53	53.3	53.3	53.2
2	HST65	53.6	53.2	52.2	53	55	58.4	55.6	56.3

2.3. Bond Strength at the Interface

In order to check the bond strength at interface of old and fresh concrete, 3 cores were cut from each topped slab and tested for direct tensile strength in the Universal Testing Machine. According to Silfwerbrand, the tensile bond strength varies between 1 to 2 MPa for normal strength concrete [7] but in this case the tensile bond strength came out to be more than 2 MPa for all specimens which proved that the bond between the old and the new concrete performed well during testing. Table 3 presents the strength of cores and Figure 7 shows the testing setup used for direct tensile strength test in Universal testing machine.

Table 3. Tensile strength of cores

Sr. No.	Specimen	Tensile strength (N/mm^2)			
		1	2	3	Average
1	HST32	2.1	2.3	2.2	2.2
2	HST65	2.05	2.2	2.2	2.15

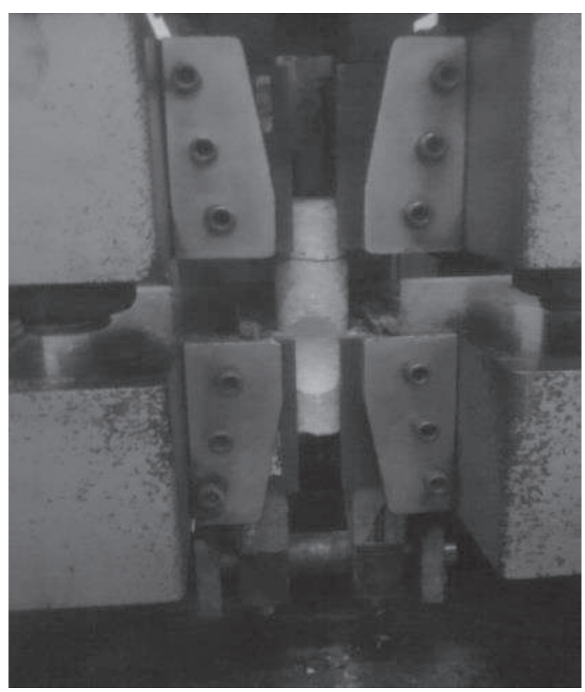


Fig. 7. Direct tensile strength test on composite slab cores

3. RESULTS AND DISCUSSION

3.1. Flexural Capacity

3.1.1. Ultimate Moment

Experimental ultimate moments were calculated as a product of failure load noted during flexural testing and its perpendicular distance from the support. Theoretical ultimate moment capacity was also calculated using the formula given by Elliot [16]:

$$M_u = f_{pb} A_{ps} (d - d_n) \qquad \text{(eq.1)}$$

M_u	= Theoretical ultimate moment capacity (kNm)			
f_{pu}	= Ultimate tensile strength in tendons		=	1770 N/mm^2
f_{pb}	= Design stress in tendons	= $0.87 f_{pu}$	=	1540 N/mm^2
A_{ps}	= Area of prestressing steel tendons		=	235.65 mm^2
d	= y^t(distance to centroid) + e(eccentricity) mm			
d_n	= Depth of compression zone			
	$= 2.47\{[A_{ps} f_{pu}/(b d f_{cu})][f_{pb}/f_{pu}]d\}$			
	$= 2.47\{[(235.65 \text{ x } 1770/(990 \text{ x } 200 \text{ x } 50)]\ [1540/1770]200\}$		=	18 mm
b	= Breath of Slab		=	990 mm
d	= Depth of slab		=	200 mm
f_{cu}	= Compressive strength of hollow core slab concrete		=	50 N/mm^2

Table 4. Experimentation results for all slabs tested

Specimen	Cracking Moment (kNm)		Maximum Midspan Deflection (mm)		Failure loads for slabs tested under flexural loading (kN)			Failure loads for slabs tested under shear loading (kN)		
	Theo-retical	Experi-mental	Theo-retical	Experi-mental	Load at first crack	Yielding point	Breaking point	Load at first crack	Yielding point	Breaking point
CONT200	48.96	52.25	57	61	75	85	100	110	118	140
HST32	64.11	63.08	63	59	90	102	112	170	180	213
HST65	87.37	84.1	94	93	120	131	139	225	232	260
CONT265	87.5	88.1	99	107	125	135	140	228	237	258

Table 4 shows a comparison between theoretical and experimental ultimate moment capacities for the slabs under flexural loading. Comparison shows that experimental results of ultimate moment capacity are well in accordance with the theoretical values calculated by using equation 1.Comparison of topped slab HST32 with control slab CONT200 shows that there was an increase in ultimate moment capacity when 32 mm topping layer was applied over 200 mm thick slab. Also there was considerable increase in moment capacity in case of HST65 when 65 mm thick topping layer was applied over 200 mm thick control slab. Furthermore, comparison of topped slab HST65 with control slab CONT265 proves that ultimate moment capacity is in the same range for both the cases.

3.1.2. Cracking

During the application of flexural loading on pre-stressed hollow core slabs, cracks appeared near mid-span as shown in figure 8. The cracks started from the bottom of the slabs and propagated vertically upward. The cracks appeared gradually with the increase of load and when these reached in the upper half of slabs, these became diagonal and travelled towards the point of application of load. The change in direction occurred abruptly and slabs failed completely when the cracks reached at the top of slabs. For theoretical calculation of cracking moments, the following equation was used as mentioned by Foubert [17].:

Table 4 shows the comparison of theoretical and experimental cracking moments for all the slabs. Again the experimentally observed and theoretically calculated cracking moments are found having nearly same values. Cracking moments behaved in a similar pattern as noted in case of ultimate moment for the tested slabs.

3.1.3. Cracking at the Interface of Slab and Topping

After failure, cracks appeared at the interface of slabs and additional toppings for both the HST32 and HST65. This shows that topping interacts monolithically with the slab and no shear slip occurred during the application of load similar to thefindings of Dowell and Smith [5].

3.1.4. Deflection

During testing, deflections were measured using dial gauges placed under the slabs right below the point of application of load. For calculating the theoretical values of deflection, (eq.3) derived by Bhatt, was used [18]. This equation is valid for flexural loading only because it takes into consideration the maximum bending moment that occurs at the mid-

(a) Crack patterns of 265mm depth slab

(b) Crack patterns of 200mm depth slab with 32mm topping

(c) Crack patterns of 200mm depth slab with 65mm topping on top

Fig. 8. Crack pattern of slabs under flexural loading

$$M_{cr} = \left[\frac{P_e}{A_g} + \frac{(P_e)(e)(y_b)}{I_g} + 0.6\lambda\sqrt{f_c'}\right]\frac{I_g}{y_b}$$

(eq.2)

M_{cr} = Cracking moment

P_e = Effective prestressing force after applicable losses

A_g = Gross area of the concrete section (including reinforcement)

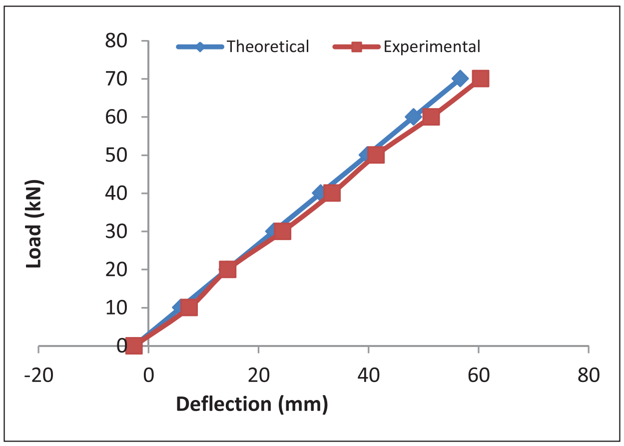
e = Eccentricity from the neutral axis to the internal reinforcement

y_b = distance from the neutral axis to the bottom fiber in tension

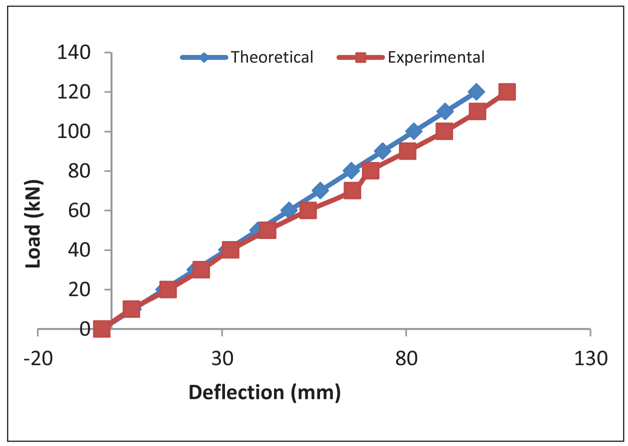
I_g = gross moment of inertia

λ = density factor of concrete

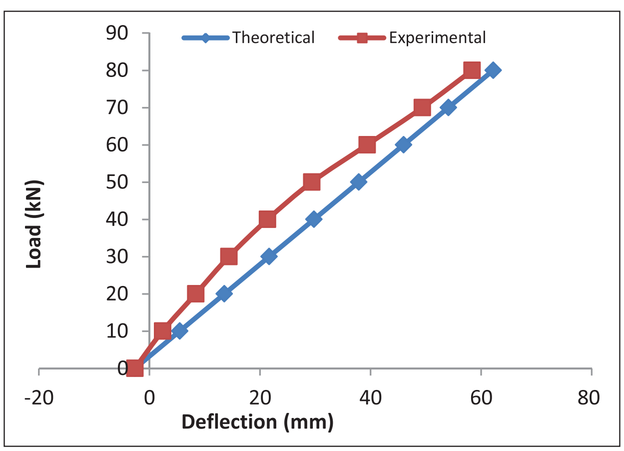
f_c' = Specified concrete strength



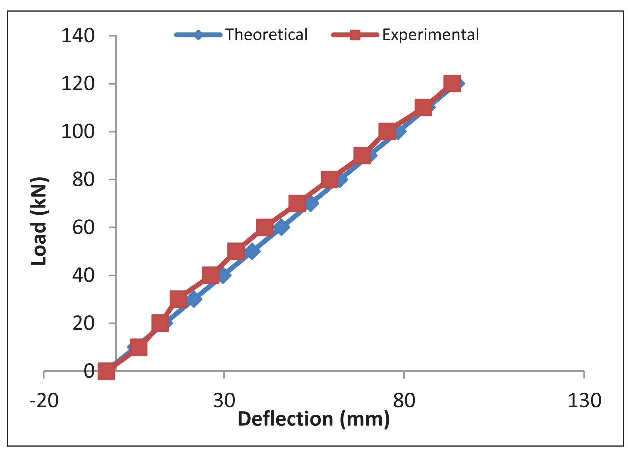
(a) Load-deflection curves for CONT200 up to cracking under flexural loading



(b) Load-deflection curves for CONT265 up to cracking under flexural loading



(c) Load-deflection curves for HST32 up to cracking under flexural loading



(d) Load-deflection curves for HST65 up to cracking under flexural loading

Fig. 9. Load-deflection curves for slabs tested

span of slabs. Topping was considered as a part of slab when calculating moment of inertia. Also, this equation was only applicable before the appearance of first crack.

Figure 9 gives comparison between experimental and theoretical values of net deflection. In these figures, the theoretical values were obtained from equation 3 and 4 whereas experimental values

$a_e = \left(L^2/E_{c,t}I\right)\Sigma\, k\, M$ eq. (3)

a_e = Deflection (mm)

L = Span of slab = 2400 mm

M = Maximum bending moment in the span (N-mm)

k = Factor depending on the shape of bending moment diagram.

I = Moment of inertia (mm^4)

$E_{c,t}$ = Modulus of elasticity at an age t (days) derived from the following equations:

$E_{c,20} = 20 + 0.2f_{cu,28}$

$E_{c,t} = E_{c,20}\left(0.4 + 0.6\, f_{cu,t}/f_{cu,28}\right)$ Where $t > 3$days

For calculating camber due to pre-stressing, the following equation was used:

$a'_e = L'^2/\left(E_{c,3}I\right)(1/8)\left(-\eta_t P_{op} e_p\right)$ eq.(4)

Where:

a_e' = Camber (mm)

L' = Span of slab between supports = 2100 mm

K = Factor that depends on the shape of bending-moment diagram

I = Moment of inertia (mm^4)

$E_{c,3}$ = Modulus of elasticity at an age of 3 days which can be derived from the following equations:

$E_{c,20} = 20 + 0.2f_{cu,28}$

$E_{c,t} = E_{c,20}\left(0.4 + 0.6\, f_{cu,3}/f_{cu,28}\right)$

$-\eta_t$ = Constant equal to -0.89

P_{op} = Initial prestressing force = 110 $\frac{N}{mm^2}$

e_p = Eccentricity of tendons = 68.4 mm

By substituting all the values, camber can be calculated as:

$$a'_e = L'^2/\left(E_{c,3}I\right)\Sigma kM = L'^2/\left(E_{c,3}I\right)(1/8)\left(-\eta_t P_{op} e_p\right)$$

$$= 2100^2/(22.8 \times 10^3 \times 659.01 \times 10^6) \times (1/8) \times (-0.89 \times 235.65 \times 10^3 \times 68.4)$$

$$= -0.53 \text{ mm (upwards)}$$

were taken from testing. Net deflection means experimental/theoretical deflection minus camber due to pre-stressing.

It can be seen from Figure 9 that for all slabs, experimental and theoretical values of deflections were almost in the same range until the appearance of first crack. Also, the experimental deflections of topped slabs i.e., HST32 &HST65 came out to be less than theoretical values whereas in case of CONT200 and CONT265 the situation was different. This was due to the fact that the topping acted as part of slab which increased the distance'd' i.e., sum of the distance to centroid and eccentricity resulting in greater moment capacity. Also, when slabs of similar depths were compared, the topped slab, HST65 showed less deflection than that of CONT265. Table 4 shows comparison of all slabs and it can be seen that the topping would result in 15 to 20 percent less deflection as compared to un-topped slabs. Same sort of behavior was seen while comparing the theoretical values.

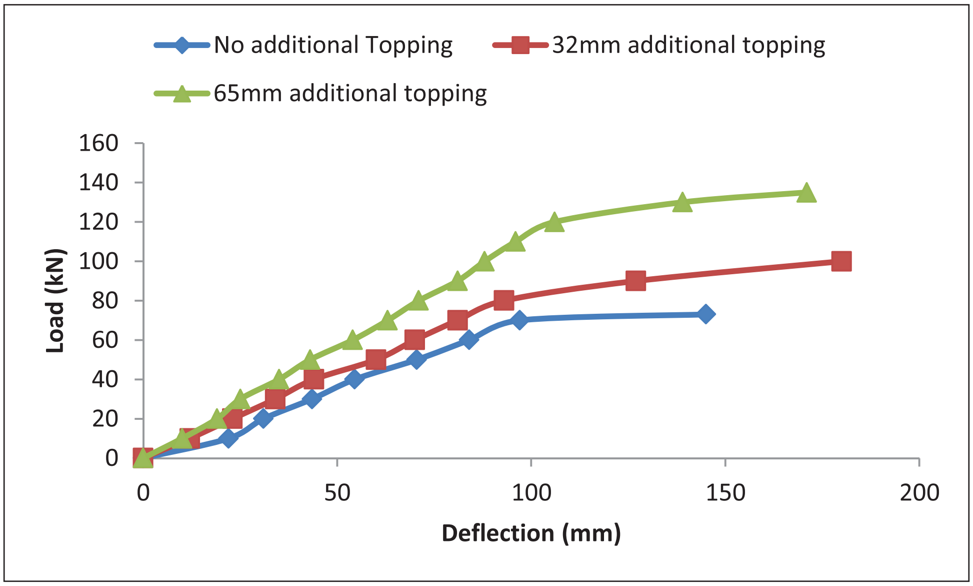


Fig. 10. Load-deflection relationship for high strength topping over 200 mm slab under flexural loading

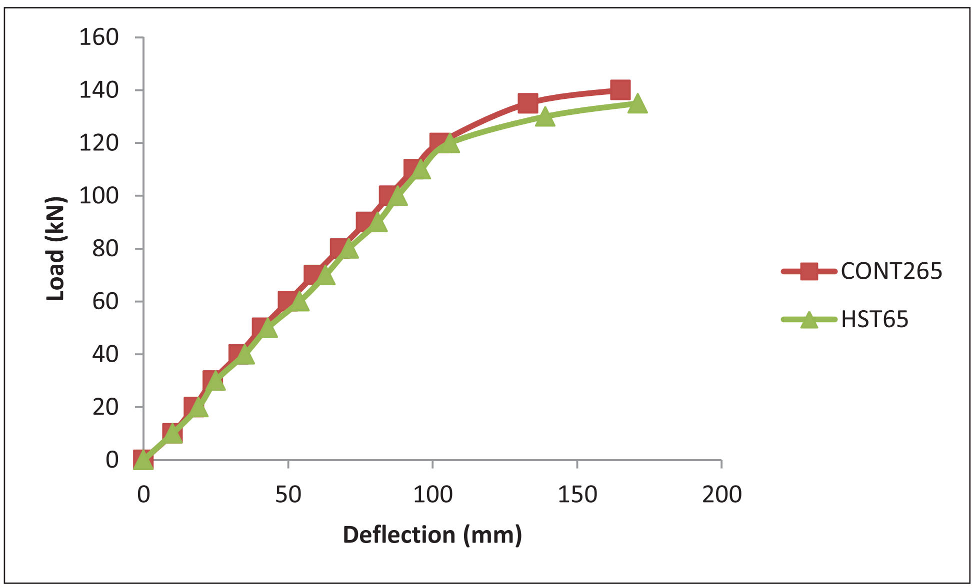


Fig. 11. Load-deflection relationship for CONT265 and HST65 under flexural loading

3.1.5. Effect of Topping Depth

Figure 10 shows load-deflection curves for 200 mm slab without topping and with toppings. It is evident from the figure that flexural capacity increased up to 60 percent with the increase in topping depth. When concrete failed and load transferred to the pre-stressed strands, deflection increased linearly with the increase in load. All slabs failed within 15 kN increase in load after first crack appeared. Also, in all slabs, vertical cracks were noticed initiating from the bottom of slabs directly under the point of application of load and propagating towards the top. This shows that all slabs failed under pure flexure.

A comparison of the first cracking, yielding point and breaking point loads for all slabs can be seen in Table 4. It can be noticed from the Figure that increase in flexural capacity is 23 percent for 32 mm topping and 60 percent for 65 mm topping as compared to 200 mm slab without topping. Also, slab of 200 mm depth with 65 mm topping (HST65) achieved flexural strength almost equal to

CONT265, with same total depth.

Figure 11 shows comparison of load-deflection curves for both slabs with the same total depth that is of 265mm. It shows that flexural capacity for two slabs was almost the same i.e 120 kN and corresponding displacements were 98 mm for HST65 and 105 mm for CONT265. It means that whether a precast slab of 265 mm depth is used or a 200 mm depth slab with 65mm topping is used, the total load they would carry will be the same. It is also proved that topping interacted monolithically with the slab and no shear slip occurred during the application of load due to proper roughening of the main slab surface at the interface with topping.

3.2. Shear Capacity

3.2.1. Cracking in the Slab Units

In case of shear loading, hairline cracking started to appear near supports and sudden failure occurred when cracks propagated y at an angle of 40 degree towards the point of application of load. After failure, a little increment in loading resulted in prominent cracks along the cores and web of slabs. Failure patterns can be seen in Figure 12.

3.2.2 Cracking at the Interface of Slab and Topping

After failure, cracks appeared at the interface of slabs and additional toppings for both HST32 and HST65. This shows that topping interacts monolithically with the slab and no shear slip occurred during the application of shear loading.

3.2.3. Deflection

In order to check the deflection of slabs under shear loading, dial gauges were installed under the slabs, right below the point of application of load. Figure 13 shows the load-mid-span deflection relationship between control and topped slabs. It can be noticed from the Figure that deflections under application

(a) Crack patterns under shear loading for 200mm depth slab from side

(b) Crack patterns under shear loading for 265mm depth slab from side

(c) Crack patterns under shear loading for 200mm depth slab from front

Fig. 12. Crack pattern under shear loading

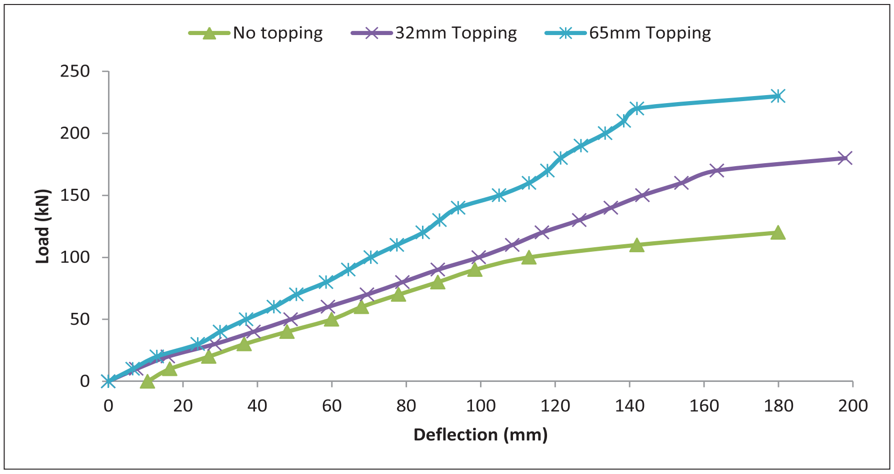


Fig. 13. Load-deflection relationship for high strength topping over 200 mm slab under shear loading

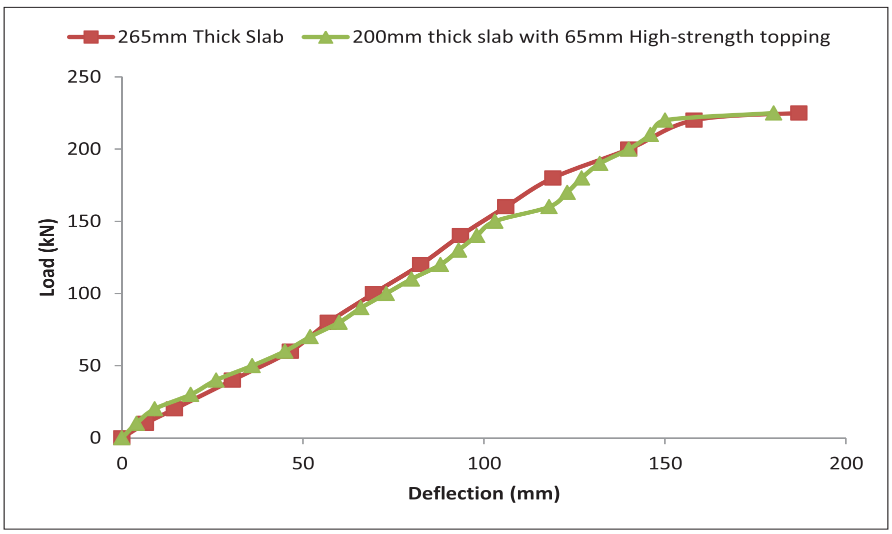


Fig. 14. Load-deflection relationship for CONT265 and HST65 under shear loading

of load were similar for all three slabs. However, the deflection in case of topped slab HST32 was found greater as compared to the other two slabs.

3.2.4. Effect of Topping Depth

Figure 13 shows load versus mid-span deflection curves for 200 mm slab without topping and with toppings. It is evident from the results that additional topping improved the shear strength of slab units linearly with the increase in topping depth. After failure of concrete, when load was transferred to the pre-stressed strands, the increase in strain was also linear with the increase in load for all specimens tested. During the course of loading, cracks appeared near the support and then propagated towards the point of application of load. It was noticed that all the slabs, with or without topping, reached their yielding point within 5 to 10 kN increase in load after the appearance of first crack which indicated an abrupt mode of failure. Almost all the cracks started to develop in between 35 to 40 degrees with the horizontalwhich is similar to the findings ofGirhammer and Pajari [11].

Different loads at which first crack appeared, as well as loads indicating yielding point and ultimate breaking point for the slabs can be seen in table 4. It can be noticed that increase in shear strength is linear with the increase in topping depth and increase in shear capacity is about 50 percent for 32 mm topping and capacity was more than doubled in case of 65 mm topping as compared to 200 mm depth slab with no topping i.e. control slab.

When compared with the control slab of 265mm depth, the failure load for the 65 mm high-strength concrete topped slab was almost the same as shown in Figure 14. This shows that whether a precast slab of 265mm depth is used or a 200mm depth slab with 65mm additional topping of high-strength concrete, the total load taken will be almost same. It was also noticed that additional topping interacted monolithically with the slab and no shear slip occurred during the application of load.

$$V_{co} = 0.67 b_v h \sqrt{(f_t^{\,2} + 0.8 f_{cp} f_t} \qquad \text{eq. (5)}$$

Where:

V_{co} = Ultimate shear capacity (kN)

b_v = Width of web (mm)

h = Depth of slab (mm)

f_t = Maximum principal tensile stress (N/mm^2)

f_{cp} = Compressive stress due to prestress (N/mm^2)

3.2.4. Comparison with Theoretical Values

For comparison of the values obtained from experimental testing, theoretical values were calculated using the formula derived by Elliot [16]. Additional topping was added to the depth of slab when calculating the ultimate shear capacity. The following equation was used:

Figure 1 to 14 shows the theoretical and experimental ultimate shear capacity values for ultimate loads in the slabs during application of the load. It can be seen from the Figure that most of the experimental values came out to be similar as compared with the theoretical values.

4. COST COMPARISON

The second main aspect that governs civil engineering projects is the economy after safety considerations. That is why, a cost comparison of factory manufactured control slab with that of topped slab of same depth is carried out.

The slabs were bought from the manufacturer at a rate of PKR 7592 per cubic meter. Total cost of

Volume of CONT265	=	(2.44 x 1 x 0.265) ft^3
	=	0.6466 m^3
Cost of CONT265	=	PKR 7592 x 0.6466
	=	PKR 4908
Volume of CONT200	=	(2.44 x 1 x 0.2) ft^3
	=	0.488 m^3
Cost of CONT200	=	PKR 7592 x 0.488
	=	PKR 3705
Cost of additional 65 mm topping		
Amount of cement used	=	150 kg
Cost of cement	=	PKR 10.2 x 150
	=	PKR 1530
Cost of sand	=	PKR 1.25 x 0.8 x 14.4
	=	PKR 14.4
Cost of crush	=	PKR 1.25 x 2.2 x 25.6
	=	PKR 70.4
Cost of HST65	=	PKR 3705 + 1530 + 14.4 + 70.4
	=	PKR 5319.8
Difference in cost	=	PKR 5319.8 – 4908
	=	PKR 411.8

control slab can be calculated as follows:

It can be seen that the cost of topped slabs comes out to be 8.45% more as compared to the factory manufactured slab of same depth.

5. CONCLUSIONS

1. High strength concrete topping can be used to increase the flexural and shear capacities of hollow core slab units. It is seen that up to 60 % increase in flexural capacity can be achieved with 65 mm topping and around 23 % with 32 mm topping. Similarly, increase in shear capacity is about 50 %for 32 mm topping and capacity is more than100% in case of 65 mm topping as compared to 200 mm depth slab with no topping i.e. control slab.
2. The increase in shear and flexural capacities of topped up slabs can be considered as linear with increase of topping depth. Thus, in a situation where the flexural or shear capacity required is greater than that of 200mm depth slab and less that than of 265mm depth slab, in-situ topping of the exact depth can be used to save cost.
3. The flexural and shear capacities of the concrete topped slabs were found equal to the capacities of the control slab of similar depth without topping.
4. Experimental and theoretical moment capacities fit well with each other.
5. Experimental mid-span deflection versus load curve in flexural testing coincides well with the corresponding theoretical curve which verifies Bhatt's outcomes [16].
6. The increase in flexural and shear capacities of the hollow core slab units was found linear with the increase in topping depth.
7. Under shear loading, cracks appeared at an angle of 30 to 40 degrees with the horizontal from the support to the point of application of load. On the other hand, during flexural testing, the cracks started from the bottom of the slabs and propagated vertically upward and when they reached in the upper half of slabs, they became diagonal and travelled towards the point of application of the load.
8. The cost of topped slabs comes out to be slightly more as compared to factory-manufactured slab of same depth. The increase in cost is 8.45 % compared to the price of factory manufactured control slab.
9. The bond between old and fresh concrete created by roughening the surface at interface, was found satisfactory and topping interacted monolithically with the slab, as no shear slip was noticed during the application of the load.

6. RECOMMENDATION

In Pakistan, hollow core slabs are available in two standard depths i.e 200 mm and 265 mm. There may arise a situation where a depth greater than 200 mm and less than 265 mm may be required due to particular applied loading. It is recommended that if the surface is prepared well and the bond between hollow core slab and in-situ concrete topping is satisfactory, concrete topping of varying depths can be used to enhance the shear and flexural capacity of hollow core slab units.

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Resarch Article

A Numerical Scheme for Solving Nonlinear Boundary Value Problems of Fractional Order $0 \leq \beta \leq \alpha < 1$

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Abstract: The primary objective of this research work is to find accurate numerical approximations for nonlinear fractional order boundary value problems (BVPs). To carry out this goal, central finite difference scheme of order four is used to approximate first- and second-order derivatives. Integrals are approximated using composite Trapezoidal rule in "the Caputo definition". The effectiveness of the proposed scheme is illustrated by solving nonlinear fractional order BVPs of order $0 \leq \beta \leq \alpha < 1$.

Keywords: Fractional differential equations, Boundary value problems, Trapezoidal rule, Central finite difference scheme.

1. INTRODUCTION

Fractional calculus provides a very supportive tool to describe natural phenomena more realistically by making beautiful and accurate modeling of physical phenomena [1]. There is much literature survey available which deals with the theory and applications of fractional differential equations [2, 3, 4, 5]. The applications of fractional derivative and fractional integral cover a broad field of complex systems including: chemistry, physics, visco-elasticity, signal processing, bioengineering, mathematical biology, and fluid mechanics, see, for example [6, 7, 8, 9]. Not only in applied mathematics, fractional calculus also has great applications in pure mathematics, see [10]. One of the hottest problems of fractional calculus is fractional differential equations with boundary conditions. These types of equations help to model many complex systems including: blood flow, thermo-elasticity, underground water flow, population dynamic, see, for example [11, 12, 13, 14, 15]. BVPs of fractional order are also applied in various physical processes of stochastic transport and many applications in the liquid filtration in a strongly porous medium, as described in [16].

Generally, numerical solution techniques are preferred when dealing with fractional models since the analytical solutions are available for a few simple cases. Following this numerous numerical techniques was developed to tackle fractional order boundary value problems

2. MATERIALS AND METHODS

Many algorithms have been developed and implemented for the numerical approximation of fractional order differential equations; see, for example [17, 18, 19, and 20]. Several new techniques are created for the solution of linear fractional order BVPs [21]. H. Demir have used shooting method to obtain solution of fractional order boundary value problem in [24]. Mohamed have investigated fractional Euler method and modified Trapezoidal rule in [25]. M.A. Anwar et al proposed the finite difference scheme for fractional order boundary value problem in [26]. Rahman dealt these boundary value problem by finite difference method in which discretization is done by 2^{nd} order finite difference scheme and Caputo operator [27]. In this paper, we develop a numerical scheme for the approximation of nonlinear fractional order BVPs with high accuracy.

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2.1. Derivative Approximation

We approximate the derivatives in the developed scheme (Section 2.3) using central difference formulae. The stencil of fourth order implicit compact finite difference scheme used to approximate the first and second-order derivatives for the interior nodes is:

$\{x_{i-1}, x_i, x_{i+1}\}, \quad for\ i = 2, 3, \ldots, n-2.$

Nodes for central difference scheme are shown in Fig 1. It means that if we are at location i, then we need one grid node to the left of it and one grid point to the right of it. It is noticeable that, mutual distance between nodes is equal to $h = \frac{x_n - x_0}{n}$. We consider the following implicit compact finite difference scheme

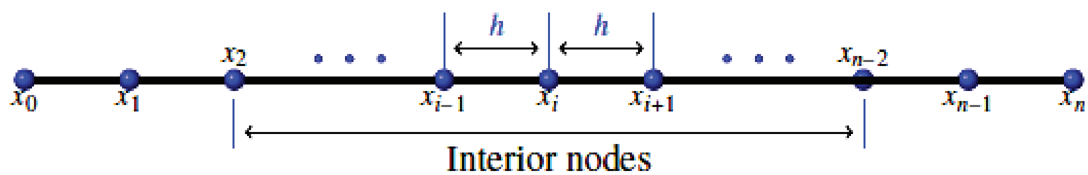

Fig. 1. Nodes for central difference scheme

$$\alpha_1 f''_{i-1} + f''_i + \alpha_2 f''_{i+1} = \frac{1}{h^2}\left(\beta_1 f_{i-1} + \beta_2 f_i + \beta_3 f_{i+1}\right). \tag{1}$$

We are interested in finding the values of unknowns in such a way that we can achieve fourth order accurate approximation of second-order derivative. On expanding equation (1) around x_1, we obtain the following system of algebraic equations:

$$\begin{aligned} &\beta_1 + \beta_2 + \beta_3 = 0 \\ &\beta_1 - \beta_3 = 0 \\ &-\frac{\beta_1}{2} - \frac{\beta_3}{2} + \alpha_1 + \alpha_2 + 1 = 0 \\ &-\frac{\beta_1}{6} - \frac{\beta_3}{6} - \alpha_1 + \alpha_2 = 0 \\ &-\frac{\beta_1}{24} - \frac{\beta_3}{24} + \frac{\alpha_1}{2} + \frac{\alpha_2}{2} = 0 \end{aligned} \tag{2}$$

By solving the system of equations (2), we obtain

$$\alpha_1 = \frac{1}{10},\ \alpha_2 = \frac{1}{10},\ \beta_1 = \frac{6}{5},\ \beta_2 = \frac{12}{5},\ \beta_3 = \frac{6}{5} \tag{3}$$

To find the approximations of first-order derivatives at the interior nodes, we consider the following model

$$\alpha_1 f'_{i-1} + f'_i + \alpha_2 f'_{i+1} = \frac{1}{h}\left(\beta_1 f_{i-1} + \beta_2 f_i + \beta_3 f_{i+1}\right). \tag{4}$$

After expanding equation (4), we get the following system of algebraic equations:

$$\begin{aligned} &\beta_1+\beta_2+\beta_3=0,\\ &\beta_1-\beta_3+\alpha_1+\alpha_2=0,\\ &-\frac{\beta_1}{2}-\frac{\beta_3}{2}-\alpha_1+\alpha_2=0,\\ &\frac{\beta_1}{6}-\frac{\beta_3}{6}+\frac{\alpha_1}{2}+\frac{\alpha_2}{2}=0,\\ &-\frac{\beta_1}{24}-\frac{\beta_3}{24}-\frac{\alpha_1}{6}+\frac{\alpha_2}{6}=0. \end{aligned} \tag{5}$$

Solving this system, we get

$$\alpha_1=\frac{1}{4},\ \alpha_2=\frac{1}{4},\ \beta_1=-\frac{3}{4},\ \beta_2=0,\ \beta_3=\frac{3}{4} \tag{6}$$

Similarly, for one sided approximation for boundary nodes and solving system of equations, we obtain the following coefficients:
for second-order derivative, we use the following scheme

$$f_1''+\alpha f_2''=\frac{1}{h^2}\left(\beta_1 f_1+\beta_2 f_2+\beta_3 f_3+\beta_4 f_4+\beta_5 f_5\right), \tag{7}$$

where,

$$\alpha=10,\ \beta_1=\frac{145}{12},\ \beta_2=-\frac{76}{3},\ \beta_3=\frac{29}{2},\ \beta_4=-\frac{4}{3},\ \beta_5=\frac{1}{12} \tag{8}$$

for first-order derivative, the proposed scheme is

$$f_1'+\alpha f_2'=\frac{1}{h}\left(\beta_1 f_1+\beta_2 f_2+\beta_3 f_3+\beta_4 f_4+\beta_5 f_5\right), \tag{9}$$

where,

$$\alpha=4,\ \beta_1=-\frac{37}{12},\ \beta_2=\frac{2}{3},\ \beta_3=3,\ \beta_4=-\frac{2}{3},\ \beta_5=\frac{1}{12} \tag{10}$$

2.2. Integral Approximation

We approximate the integrals in the developed scheme (Section 2.3) by the Trapezoidal method

$$\int_1^{x_{i-1}} \xi(s)ds \approx h\left(\frac{\xi_1}{2}+\xi_2+\cdots+\xi_{i-2}+\frac{\xi_{i-1}}{2}\right), \tag{11}$$

where, $\xi_j=\xi(x_j)$ and $h=x_i-x_{i-1}$ is a uniform step size.

2.3. Proposed Iterative Scheme

To describe the proposed iterative scheme, consider the following non-homogeneous nonlinear fractional order BVP

$$D^{-\alpha}y''+D^{-\beta}y+p(x)f(y)=g(x),\quad x\in(0,1),\ 0\le\beta\le\alpha<1, \tag{12}$$

with the boundary conditions:

$$y(0)=y(1)=0\,.$$

Where, $D^{-\alpha}$ and $D^{-\beta}$ are fractional orders derivatives in Caputo sense and is a nonlinear function. The fractional order differential equation (12) can also be written as

$$y'' = -D^{\alpha-\beta} y + D^{\alpha}(g(x) - p(x)f(y)). \tag{13}$$

For a given smooth function, we define

$$\begin{aligned} D^{-\alpha} w(x) &= \frac{1}{\Gamma(1-\alpha)} \int_0^x (x-s)^{-\alpha} w'(s)ds, \quad \alpha > 0 \\ &= \frac{1}{(1-\alpha)\Gamma(1-\alpha)} \left(x^{1-\alpha} w'(0) + \int_0^x (x-s)^{1-\alpha} w''(s)ds \right) \end{aligned} \tag{14}$$

We can write equation (13) with the help of equation (14) as

$$y'' = -\frac{1}{\Gamma(1-\gamma)} \left[\frac{x^{1-\gamma}}{1-\gamma} y'(0) + \int_0^x \frac{(x-s)^{1-\gamma}}{1-\gamma} y''(s)d \right] + \frac{1}{\Gamma(1-\alpha)}$$
$$\left[\frac{x^{1-\alpha}}{1-\alpha} \left(g'(0) - p'(0)y(0) - p(0)y'(0)\right) + \int_0^x \frac{(x-s)^{1-\alpha}}{1-\alpha} \left(g''(s) - p''(s)y(s) - 2p'(s)y'(s) - p(s)y''(s)\right) ds \right]$$

where, $\gamma = \alpha - \beta.$

We discretize $[0,1]$ for a given number of n nodes and compute a uniform step size

$$h = (1-0)/(n) = 1/(n)$$

Furthermore, we use central difference approximation of order four for the approximation of first and second-order derivative as given in Section 2.1. Whereas, integrals in our work are approximating by using composite trapezoidal method (11). The above equation can also be written as

$$\begin{aligned} y''(x_i) &= -\frac{1}{\Gamma(2-\gamma)} \left[x_i^{1-\gamma} y'(0) + I_1(x_i) \right] \\ &\quad + \frac{1}{(2-\alpha)} \left[x_i^{1-\alpha} \left(g'(0) - p'(0)y(0) - p(0)y'(0) \right) + I_2(x_i) + I_3(x_i) + I_4(x_i) + I_5(x_i) \right], \end{aligned} \tag{15}$$

where,

$$I_1(x_i) = \int_0^{x_i} (x_i - s)^{1-\gamma} y''(s)ds,$$

$$I_2(x_i) = \int_0^{x_i} (x_i - s)^{1-\alpha} g''(s)ds,$$

$$I_3(x_i) = \int_0^{x_i} (x_i - s)^{1-\alpha} p''(s)f \, ds,$$

$$I_4(x_i) = \int_0^{x_i} (x_i - s)^{1-\alpha} 2p'(s)f' \, ds,$$

$$I_5(x_i) = \int_0^{x_i} (x_i - s)^{1-\alpha} p(s)f'' \, ds.$$

Here, we implement a very robust iterative process. Equation (18) can also be written as

$$y_{n+1} = \Phi(y_n), \quad (16)$$

where, $y = [y_1, y_2, ..., y_n]^T$ is the nth approximation to the solution of discretized form of equation (18) and $\Phi(y)$ is the right-hand side.

3. RESULTS AND DISCUSSION

To quantify the quality, in terms of convergence and accuracy, of the above developed iterative scheme, we perform extensive numerical testing on a collection of test problems. In all our numerical testing, we approximate the numerical solution of five non-linear fractional order ($0 \leq \beta \leq \alpha < 1$) BVPs by solving iteratively equation (16) to obtain a sequence of presumably convergent vectors $y_0, y_1, y_2, ...,$ till $\|y_{n+1} - y_n\| \leq$ some specified tolerance.

Problem 1

Consider the following nonlinear fractional differential equation:

$$D^{-1/4}y'' + D^{-1/8}y + x^2y^3 = -\frac{1024}{663}x^{13/4}\sqrt{2}\Gamma\left(\frac{3}{4}\right)(32x^2 - 17)\pi^{-1} - \frac{2097152}{45886995}x^{\frac{41}{8}} sin\left(\frac{\pi}{8}\right)$$
$$\Gamma\left(\frac{7}{8}\right)\left(128x^2 - 133\right)\pi^{-1} + x^{17}\left(1 - x^2\right)^3$$

With boundary conditions:

$$y(0) = 0 = y(1)$$

Note that, the exact solution for this problem is $y(x) = x^5(1 - x^2)$.

Approximated solution and relative error of this problem are shown in Table 1. Analytic solution and approximated solution are shown graphically in Fig 2.

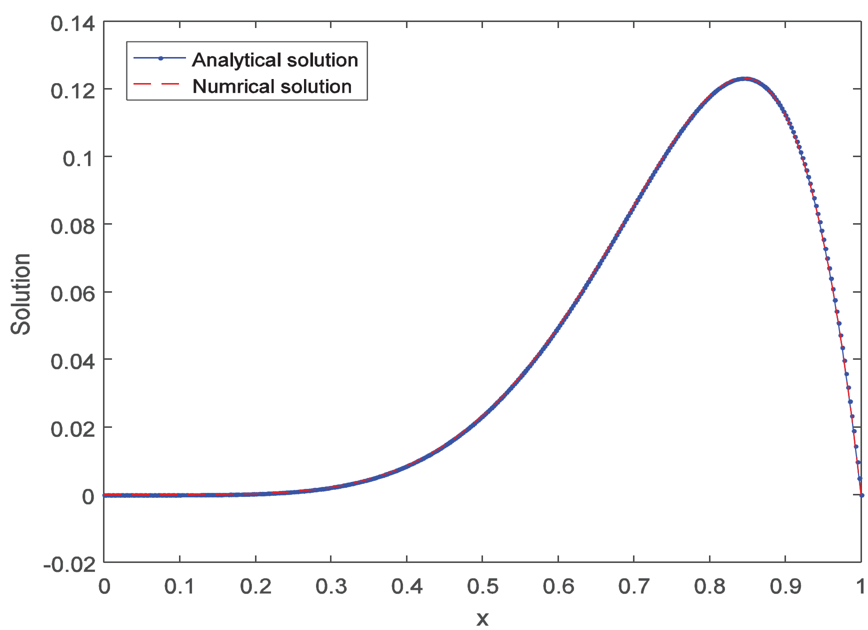

Fig. 2. Comparison between analytical and numerical solutions

Table 1. Approximate solution and relative error

X	Approximated Solution	Analytic Solution	Relative Error
0.2	2.98×10^{-4}	3.11×10^{-4}	4.05×10^{-2}
0.4	8.67×10^{-3}	8.70×10^{-3}	3.01×10^{-3}
0.6	4.94×10^{-2}	4.94×10^{-2}	7.75×10^{-4}
0.8	1.18×10^{-1}	1.18×10^{-1}	3.25×10^{-4}

Problem 2

Consider the following nonlinear fractional differential equation:

$$D^{-1/2}y''+D^{-1/10}y+x^2e^y=-\frac{5120}{3003}\frac{x^{9/2}(224x^2-143)}{\sqrt{\pi}}-\frac{8000000000}{583657942329}x^{\frac{61}{10}}sin\left(\frac{1}{10\pi}\right)\Gamma\left(\frac{9}{10}\right)$$
$$\left(5600x^2-5751\right)\pi^{-1}+x^2e^{10x^6(1-x^2)}\left(1-x^2\right)^3$$

With boundary conditions:

$$y(0)=0=y(1)$$

Note that, the exact solution for this problem is $y(x)=10x^6(1-x^2)$.

Approximated solution and relative error of this problem are shown in Table 2. Analytic solution and approximated solution are shown graphically in Fig 3.

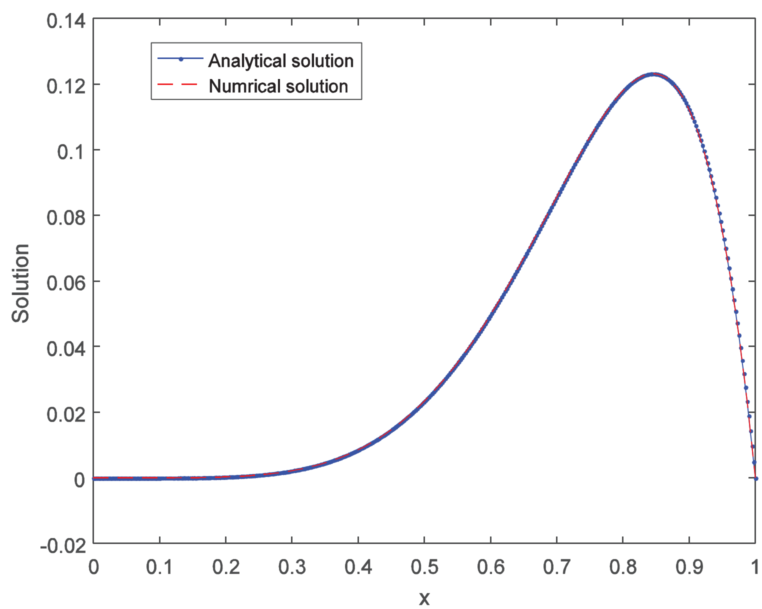

Fig. 3. Comparison between analytical and numerical solutions.

Table 2. Approximate solution and relative error

X	Approximated Solution	Analytic Solution	Relative Error
0.2	-1.14×10^{-4}	6.24×10^{-4}	1.18×10^{0}
0.4	3.35×10^{-2}	3.49×10^{-2}	4.16×10^{-2}
0.6	2.94×10^{-1}	2.96×10^{-1}	7.15×10^{-3}
0.8	9.40×10^{-1}	9.42×10^{-1}	2.39×10^{-3}

Problem 3

Consider the following nonlinear fractional differential equation:

$$D^{-1/2}y'' + D^{-1/10}y + x^2y^3 = \frac{128}{3003}\frac{x^{7/2}(896x^3-2912x^2+3432x-1287)}{\sqrt{\pi}} + \frac{40000000}{583657942329}x^{\frac{51}{10}} sin\left(\frac{1}{10\pi}\right)$$
$$\Gamma\left(\frac{9}{10}\right)\left(112000x^3-453600x^2+690120x-350811\right)\pi^{-1} + x^{14}(x-1)^3(1-(1-x)^3)^3$$

With boundary conditions:

$$y(0) = 0 = y(1)$$

Note that, the exact solution for this problem is $y(x) = x^4(x-1)(1-(1-x)^3)$.

The numerical results of this problem are shown in Fig 4 and Table 3.

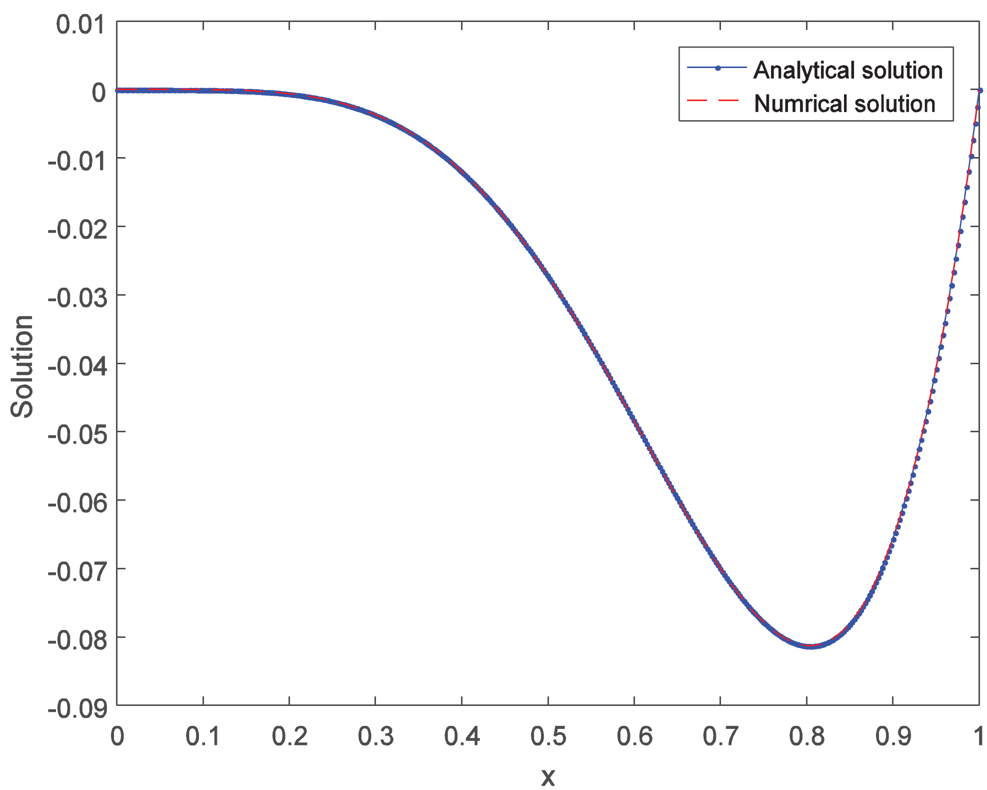

Fig. 4. Comparison between analytical and numerical solutions

Table 3. Numerical results of Problem 3

X	Approximated Solution	Analytic Solution	Relative Error
0.2	-6.07×10^{-4}	-6.32×10^{-4}	3.99×10^{-2}
0.4	-1.21×10^{-2}	-1.22×10^{-2}	4.44×10^{-3}
0.6	-4.82×10^{-2}	-4.83×10^{-2}	1.50×10^{-3}
0.8	-8.12×10^{-2}	-8.13×10^{-2}	7.48×10^{-4}

Problem 4

Consider the following nonlinear fractional differential equation:

$$D^{-1/2}y'' + D^{-1/10}y + x^2 ye^y = -\frac{128}{51051}\frac{x^{7/2}(21504x^5-91392x^4+152320x^3-123760x^2+48620x-7293)}{\sqrt{\pi}}$$
$$-\frac{40000000}{766342878277977}x^{\frac{51}{10}}sin\left(\frac{1}{10\pi}\right)\Gamma\left(\frac{9}{10}\right)$$
$$\frac{\left(144000000x^5-727200000x^4+1470560000x^3-1488942000x^2+755106300x-153538281\right)}{\pi}$$
$$+x^7(1-x)^5 e^{x^5(1-x)^5}$$

With boundary conditions:

$$y(0) = 0 = y(1)$$

Note that, the exact solution for this problem is $y(x) = x^5(1-x)^5$.

The numerical results of this problem are shown in Fig 5 and Table 4.

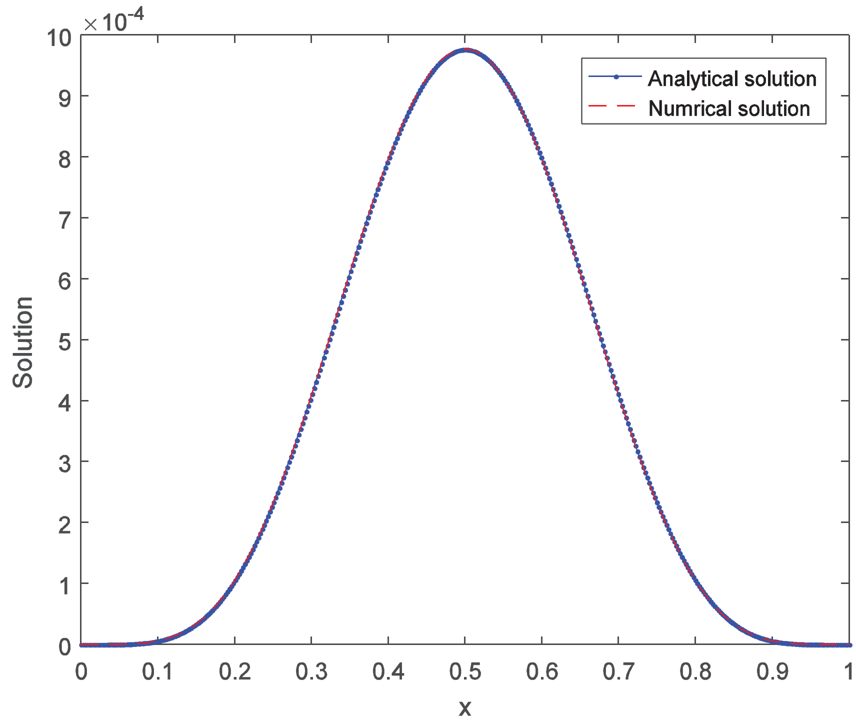

Fig. 5. Comparison between analytical and numerical solutions

Table 4. Numerical results of Problem 4

X	Approximated Solution	Analytic Solution	Relative Error
0.2	1.06×10^{-4}	1.06×10^{-4}	1.40×10^{-3}
0.4	7.99×10^{-4}	7.99×10^{-4}	1.19×10^{-4}
0.6	8.00×10^{-4}	7.99×10^{-4}	6.94×10^{-4}
0.8	1.06×10^{-4}	3.27×10^{-4}	3.27×10^{-4}

Problem 5
Consider the following nonlinear fractional differential equation:

$$
\begin{aligned}
D^{-1/2}y'' + D^{-1/10}y + x^2y^3 &= \frac{512}{969969}\frac{x^{9/2}(118272x^5-204288x^4+217056x^2-176358x+46189)}{\sqrt{\pi}} \\
&+\frac{800000000}{9451562165428383}x^{\frac{61}{10}}sin\left(\frac{\pi}{10}\right)\Gamma\left(\frac{9}{10}\right) \\
&\frac{\left(88000000x^5-177600000x^4+272053600x^2-275454270x+93129777\right)}{\pi} \\
&+x^{20}(1-x)^6(x^3-x+1)^3
\end{aligned}
$$

With boundary conditions:

$$y(0)=0=y(1)$$

Note that, the exact solution for this problem is $y(x)=x^6(1-x)^2(x^3-x+1)$.

The numerical results of this problem are shown in Fig 6 and Table 5.

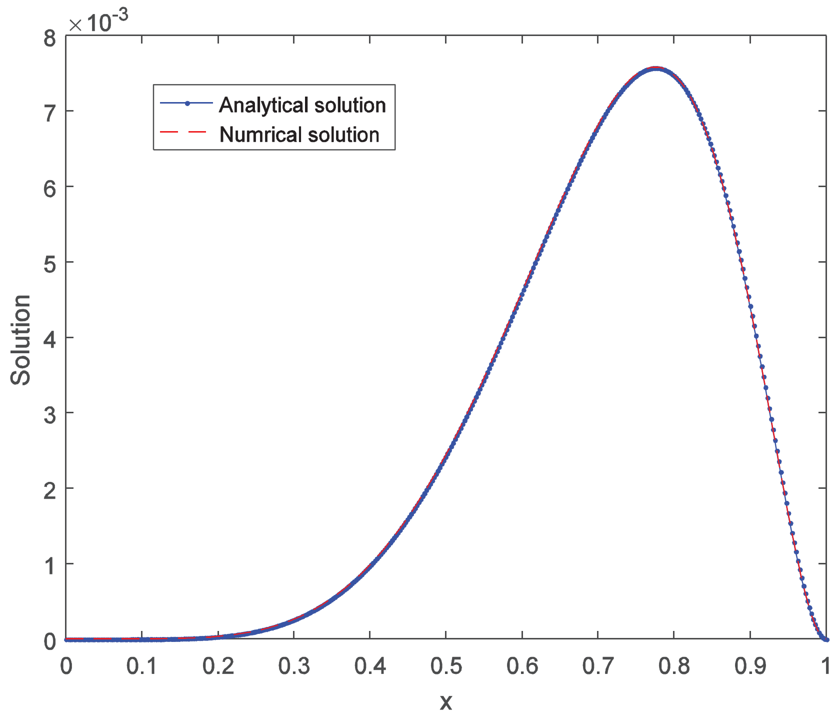

Fig. 6. Comparison between analytical and numerical solutions

Table 5. Numerical results of Problem 5

X	Approximated Solution	Analytic Solution	Relative Error
0.2	3.54×10^{-5}	3.35×10^{-5}	5.46×10^{-2}
0.4	9.93×10^{-4}	9.90×10^{-4}	2.88×10^{-3}
0.6	4.58×10^{-3}	4.57×10^{-3}	9.29×10^{-4}
0.8	7.48×10^{-3}	7.47×10^{-3}	1.28×10^{-3}

4. CONCLUSION

A new iterative scheme for the numerical approximation of nonlinear fractional order BVPs involving Caputo's derivative is proposed and hence, successfully applied in this paper. We used implicit compact finite difference scheme of order four for first and second-order derivatives and trapezoidal rule for numerical computation of integrals. Numerical experiments are performed on a collection of five nonlinear fractional orders BVPs. For the five test problems considered in this paper, convergence of the proposed iterative scheme till reaching optimal accuracy is achieved after no more than 20 iterations. We believe that the optimal accuracy can further be improved by using higher order finite difference schemes for the derivatives involved and using other numerical integration techniques for numerical computation of the integrals.

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Research Article

Some Properties of Harmonic Univalent Functions in a Conic Domain

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Abstract: We investigate a new subclass $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,k,t)$of harmonic functions satisfying condition:

$$\Re\left\{\alpha z\mathfrak{L}''(z)+\frac{\mathcal{K}(z)}{z}\right\}>k\left|\alpha z\mathfrak{L}''(z)+\frac{\mathcal{K}(z)}{z}-1\right|+1-|\gamma|(z\in \mathrm{E}),$$

Where $\mathfrak{L}(z)=z^t-\sum_{j=2}^{\infty}|a_j|z^{j+t-1}$ and $\mathcal{K}(z)=\sum_{j=1}^{\infty}|b_j|z^{j+t-1}$, $|b_1|<1$ .We also determine the coefficients inequalities, growth and distortion bounds, radius of star likeness for the analytic part of the harmonic functions $\mathcal{F}(z)=\mathfrak{L}(z)+\mathcal{K}(z)$. For specific values of parameters involved, our findings may be related to the previously known results.

Keywords: Harmonic and univalent functions, Coefficients inequalities, Radius of starlikeness

1. INTRODUCTION

Harmonic functions are important because of their applications in minimal surfaces and these functions play a vital role in applied mathematics for example, [3, 5, and 6]. Harmonic functions have close connections with conformal mappings are not analytic in general and hence the Cauchy-Riemann equations do not hold. These functions were first studied by differential geometers and then complex analysts involved in their study which was initiated by Clunie and Sheil-Small [4] in 1984.

A continuous function $f(z)=u(x,y)+iv(x,y)$ is harmonic, if both u and v are harmonic. A harmonic function f takes the canonical form: $f(z)=l(z)+k(z)$, where l and k are analytic in $\mathbb{U}$. The characterization of f for local univalence and sense-preserving is just $|l'(z)|>|k'(z)|$ in $\mathbb{U}$. For detail, we refer [4,7]. Let $\mathcal{H}$ be the class of functions $f(z)=l(z)+k(z)$ univalent and sense-preserving in $\mathbb{U}$ such that $f(0)=f_z(0)-1=0$. A function $f\in\mathcal{H}$, is expressed as:

$$f(z)=z+\sum_{j=2}^{\infty}a_jz^j+\sum_{j=1}^{\infty}\overline{b}_j\overline{z}^j,|b_1|<1 \qquad (z\in\mathbb{U}). \qquad (1.1)$$

This function f reduces to l for $k=0$. Jahangiri [10] introduced the class $\mathcal{T}_{\mathcal{H}}(\alpha)$ comprising of functions f such that

$$l(z)=z-\sum_{j=2}^{\infty}|a_j|z^j, \quad (z\in\mathbb{U}). \qquad (1.2)$$

and

$$k(z)=\sum_{j=1}^{\infty}|b_j|z^j, \quad (z\in\mathbb{U}). \qquad (1.3)$$

A function $f\in\mathcal{T}_{\mathcal{H}}(\alpha)$, if it satisfies the condition:

$$\frac{\partial}{\partial\theta}\left(arg\, f(re^{i\theta})\right)\geq\alpha,\ \ 0\leq\alpha<1. \qquad (1.4)$$

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Jahangiri proved that a function f satisfying (1.4) along with

$$\sum_{j=1}^{\infty}\frac{j-\alpha}{1-\alpha}|a_j|+\sum_{j=1}^{\infty}\frac{j+\alpha}{1-\alpha}|b_j|\leq 2, 0\leq\alpha<1, \tag{1.5}$$

where l and k are defined by (1.2) and (1.3) respectively, then f is sense-preserving and convex of order α in $\mathbb{U}$.

Frasin [9] defined the class $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma)$ consisting of functions $f: f(z)=l(z)+k(z)$ satisfy the condition:

$$\Re\left\{\alpha z l''(z)+\frac{k(z)}{z}\right\}>1-|\gamma|, \text{ where } \gamma\in\mathbb{C},\ \alpha\geq 0, z\in\mathbb{U},$$

where l and k are of the form (1.2) and (1.3) respectively. He also proved that if $f\in\mathcal{T}_{\mathcal{H}}(\alpha,\gamma)$, then

$$\sum_{j=2}^{\infty}\left[\alpha j(j-1)|a_j|-\frac{1-3\alpha}{j+\alpha}\right]\leq|\gamma|,$$

for $a_1=b_1=1, 0\leq\alpha<\frac{1}{3}$ and $\gamma\in\mathbb{C}$. Let $\mathcal{F}(z)=\mathfrak{L}(z)+\mathcal{K}(z)$, where

$$\mathfrak{L}(z)=z^t-\sum_{j=2}^{\infty}|a_j|z^{j+t-1}(z\in\mathbb{U}) \tag{1.6}$$

and

$$\mathcal{K}(z)=\sum_{j=1}^{\infty}|b_j|z^{j+t-1}(z\in\mathbb{U}). \tag{1.7}$$

Makinde and Afolabi [11] introduced the class $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,t)$ consisting of functions $\mathcal{F}(z)=\mathfrak{L}(z)+\mathcal{K}(z)$ such that $\mathfrak{L}$ and $\mathcal{K}$ are of the form (1.6) and (1.7) respectively and satisfying the condition:

$$\Re\left\{\alpha z\mathfrak{L}''(z)+\frac{\mathcal{K}(z)}{z}\right\}>1-|\gamma|(z\in\mathbb{U}),$$

where $\gamma\in\mathbb{C}$ and $\alpha\geq 0$. In this particular article, Makinde and Afolabi studied various properties of the class $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,t)$. In the following, we define a new class $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$.

Definition 1.1. *Let* $\mathcal{F}(z)=\mathfrak{L}(z)+\mathcal{K}(z)$ be a harmonic function defined in $\mathbb{U}$ such that $\mathfrak{L}$ and $\mathcal{K}$ have the series representations (1.6) and (1.7) respectively. Then $\mathcal{F}\in\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$, if $\mathfrak{L}$ and $\mathcal{K}$ satisfy the condition:

$$\Re\left(\alpha z\mathfrak{L}''(z)+\frac{\mathcal{K}(z)}{z}\right)\geq m\left|\alpha z\mathfrak{L}''(z)+\frac{\mathcal{K}(z)}{z}-1\right|+1-|\gamma|(z\in\mathbb{U}), \tag{1.8}$$

where $\alpha\geq 0, m\geq 0$ *and* $\gamma\in\mathbb{C}$

Functions in the class $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$ are related with the uniformly harmonic functions. The image domain of such functions is basically conic depending on the values of m. The condition described above in (1.8) is equivalent to:

$$\Re\left[\left(1+me^{i\theta}\right)\left(\alpha z\mathfrak{L}''(z)+\frac{\mathcal{K}(z)}{z}\right)-me^{i\theta}\right]\geq 1-|\gamma|, \qquad (\pi\leq\theta\leq\pi, z\in\mathbb{U}). \tag{1.9}$$

To avoid repetition of parameters in the Definition 1.1, we assume these parameters with the above specific restrictions. If we take $m=0$ in (1.8), we obtain the class $\mathcal{T}_{\mathcal{H}}(\alpha,\gamma,t)$, see [11] with references therein. For other related results, we also refer [1-2, 8, 11-18].

2. PRELIMINARY RESULTS

In this section, we include a useful lemma. This lemma deals with the conditions on the infinite series of coefficients involving in the representation of $\mathcal{F}$.

Lemma 2.1. *Let* $\mathcal{F}(z) = \mathfrak{L}(z) + \mathcal{K}(z)$ *be so that* $\mathfrak{L}$*and* $\mathcal{K}$ *are given by* (1.6)*and* (1.7) *respectively. If* $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha, t)$*, then*

$$\sum_{j=1}^{\infty} \frac{j+t-1-\alpha}{1-\alpha}\left(|a_j| + |b_j|\right) \le 2,$$

where $a_1 = b_1 = 1, 0 < t \le 1,$ *and* $0 \le \alpha < 1$.

3. RESULTS

In the following, we find estimates on the coefficients bounds. These coefficients bounds further lead to the estimates of growth and distortions related to the functions $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha, \gamma, m, t)$.

Theorem 3.1. Let $\mathcal{F}(z) = \mathfrak{L}(z) + \mathcal{K}(z)$, where $\mathfrak{L}$ and $\mathcal{K}$ satisfy (1.6) and (1.7) respectively. If $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha, \gamma, m, t)$, then

$$\sum_{j=2}^{\infty} \left\{ \alpha(j+t-1)(j+t-2)|a_j| - \frac{2-t-3\alpha}{j+t-1+\alpha} \right\} \le \frac{|\gamma|}{1+m}, \tag{3.1}$$

where $a_1 = b_1 = 1$, $0 < t \le 1$, $\frac{1}{3} \le \alpha < \frac{2}{3}$, *and* $\gamma \in \mathbb{C}$.

Proof. Let $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha, \gamma, m, t)$. In view of (1.6), (1.7), (1.9) and for $t_j = j + t - 1$, we have

$$Re\left[(1+me^{i\theta})z^{t-1}\left(\alpha t(t-1) + |b_1| - \sum_{j=2}^{\infty}\{\alpha t_j(t_j-1)|a_j| - |b_j|\}z^{j-1}\right) - me^{i\theta}\right] \ge 1 - |\gamma|.$$

Choosing z to be real and letting$z \to 1^-$ in the above inequality and simplifying, we obtain

$$(1+m)\left[\alpha t(t-1) + |b_1| - \textstyle\sum_{j=2}^{\infty}\{\alpha(j+t-1)(j+t-2)|a_j| - |b_j|\}\right] \ge 1 - |\gamma| + m.$$

For $0 < \alpha t(t-1) + |b_1| \le 1$, we write

$$\sum_{j=2}^{\infty}\{\alpha(j+t-1)(j+t-2)|a_j| - |b_j|\} \le \frac{|\gamma|}{1+m}. \tag{3.2}$$

From Lemma 2.1, we have

$$\sum_{j=1}^{\infty} \frac{j+t-1+\alpha}{1-\alpha}|b_j| \le \sum_{j=1}^{\infty}\left\{\frac{j+t-1-\alpha}{1-\alpha}|a_j| + \frac{j+t-1+\alpha}{1-\alpha}|b_j|\right\} \le 2\,,$$

where $0 \le \alpha \le 1, a_1 = 1$. This implies that

$$|b_j| \le \frac{2-t-3\alpha}{j+t-1+\alpha}, \qquad j \ge 2. \tag{3.3}$$

For $0 < t \le 1$, $\frac{1}{3} \le \alpha < \frac{2}{3}, \gamma \in \mathbb{C}$ and on substituting (3.2) into (3.3), we get

$$\sum_{j=2}^{\infty}\{\ \alpha(j+t-1)(j+t-2)|a_j| - \frac{2-t-3\alpha}{j+t-1+\ \alpha}\} \leq \frac{|\gamma|}{1+m}.$$

In the following, we deduce the conditions for the coefficients bounds for functions in the class $\mathcal{T}_{\mathcal{H}}\ (\alpha, \gamma, m, t)$.

Corollary 3.2. *Let* $\mathcal{F}\ (z) = \mathfrak{L}(z) + \mathcal{K}(z) \in \mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$, *where* $\mathfrak{L}$ *and* $\mathcal{K}$ *satisfy* (1.6) *and* (1.7) *respectively. Then*

$$|a_j| \leq \frac{(j+t-1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha(j+t-1)(j+t-2)(j+t-1+\alpha)(1+m)},$$

where $\gamma \in \mathbb{C}$ *and* $j \geq 2$.

The following theorems deal with the growth and distortions related problems of the function $\mathfrak{L}$ involved in the class $\mathcal{T}_{\mathcal{H}}\ (\alpha, \gamma, m, t)$.

Theorem 3.3. Let $\mathcal{F}(z) = \mathfrak{L}(z) + \mathcal{K}(z) \in \mathcal{T}_{\mathcal{H}}(\alpha, \gamma, m, t)$, where $\mathfrak{L}$ and $\mathcal{K}$ satisfy (1.6) and (1.7) respectively. Then

$$r^t - \theta(\alpha,\gamma,m,t)|r|^{t+1} \leq |\mathfrak{L}(z)| \leq r^t + \theta(\alpha,\gamma,m,t)|r|^{t+1}, \tag{3.4}$$

where

$$\theta(\alpha,\gamma,m,t) = \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha\ )}{\alpha t(t+1)(1+m)(t+1+\alpha)}.$$

Proof Let $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}\ (\alpha,\gamma,m,t)$. Then from (3.2), we have

$$\alpha t(t+1)\sum_{j=2}^{\infty}|a_j| - \sum_{j=2}^{\infty}|b_j| \leq \frac{|\gamma|}{1+m}. \tag{3.5}$$

From (3.3) and (3.5), we write

$$\sum_{j=2}^{\infty}|a_j| \leq \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha t(t+1)(1+m)(t+1+\alpha)}. \tag{3.6}$$

From (1.6), we have the following inequality:

$$|\mathfrak{L}(z)| \geq r^t - \sum_{j=2}^{\infty}|a_j||r|^{t+1}\ . \tag{3.7}$$

Combining (3.6) and (3.7), we thus obtain

$$|\mathfrak{L}(z)| \geq r^t - \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha t(t+1)(1+m)(t+1+\alpha)}|r|^{t+1}. \tag{3.8}$$

Adopting similar procedure, we write

$$|\mathfrak{L}(z)| \leq r^t + \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha\ )}{\alpha t(t+1)(1+m)(t+1+\alpha)}|r|^{t+1}. \tag{3.9}$$

On combining(3.8) and (3.9), we obtain (3.4).

Theorem 3.4. If $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha, \gamma, m, t)$, then for $|z| = r < 1$,we have

$$tr^{t-1} - \theta(\alpha,\gamma,m,t)r^t \le |\mathfrak{L}'(z)| \le tr^{t-1} + \theta(\alpha,\gamma,m,t)r^t, \qquad (3.10)$$

where

$$\theta(\alpha,\gamma,m,t) = \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha t(1+m)(t+1+\alpha)}.$$

Proof. For $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$, from (3.2), we write

$$\alpha t \sum_{j=2}^{\infty} (j+t-1)|a_j| \le \frac{|\gamma|}{1+m} + \sum_{j=2}^{\infty} |b_j|. \qquad (3.11)$$

From (3.3) and (3.11), we obtain

$$\sum_{j=2}^{\infty} (j+t-1)|a_j| \le \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha t(1+m)(t+1+\alpha)} \qquad (3.12)$$

Also for $|z| = r < 1$, we can write

$$|\mathfrak{L}'(z)| \ge tr^{t-1} - \textstyle\sum_{j=2}^{\infty}(j+t-1)|a_j| r^t. \qquad (3.13)$$

From (3.12) and (3.13), we have the following

$$|\mathfrak{L}'(z)| \ge tr^{t-1} - \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha t(1+m)(t+1+\alpha)} r^t. \qquad (3.14)$$

Also for $|z| = r < 1$, we obtain that

$$|\mathfrak{L}'(z)| \le tr^{t-1} + \frac{(t+1+\alpha)|\gamma| + (1+m)(2-t-3\alpha)}{\alpha t(1+m)(t+1+\alpha)} r^t. \qquad (3.15)$$

Combining equations(3.14) and (3.15), we have the desired result.
In the theorem, we calculate the radius of starlikeness for $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha, \gamma, m, t)$.

Theorem 3.5. *Let* $\mathcal{F}(z) = \mathfrak{L}(z) + \mathcal{K}(z) \in \mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$,
where $\mathfrak{L}$ and $\mathcal{K}$ are given by (1.6) and (1.7) respectively. Then $\mathfrak{L}$ is starlike of order ρ $(0 \le \rho < 1)$in $|z| < r_1$, where

$$r_1 = \operatorname*{Inf}_j \left[\frac{(2-\rho-t)(1-\rho)\{\alpha+t-1)|\gamma| + (1+m)(2-t+3\alpha)\}}{\alpha t(1+m)(1-\rho)(t+j-\rho-1)(t+j-1)(t+\alpha+1)}\right]^{\frac{1}{j+t-2}}.$$

Proof. To obtain the desired result, it is enough to show that:

$$\left|\frac{z\mathfrak{L}'(z)}{\mathfrak{L}(z)}-1\right| \leq 1-\rho \text{ for } |z|<r_1$$

From (1.6), we have the following representation:

$$z\mathfrak{L}'(z) = tz^t - \sum_{j=2}^{\infty}(j+t-1)|a_j|z^{j+t-1}.$$

Thus, on simplifications, we can write

$$\left|\frac{z\mathfrak{L}'(z)}{\mathfrak{L}(z)}-1\right| \leq \frac{(t-1)+\sum_{j=2}^{\infty}(j+t-2)|a_j||z|^{j+t-2}}{1-\sum_{j=2}^{\infty}|a_j||z|^{m+t-2}}, \quad (a_1=1).$$

The inequality $\left|\frac{z\mathfrak{L}'(z)}{\mathfrak{L}(z)}-1\right| \leq 1-\rho$ holds only if

$$\frac{t-1}{1-\rho}+\sum_{j=2}^{\infty}\left(\frac{j+t-1-\rho}{1-\rho}\right)|a_j||z|^{j+t-2} \leq 1. \tag{3.16}$$

The coefficients inequality (3.16) along with (3.12) yield the following bounds

$$|z|^{j+t-2} \leq \frac{(2-\rho-t)(1-\rho)\{\alpha+t-1)|\gamma|+(1+m)(2-t+3\alpha)\}}{\alpha t(1+m)(1-\rho)(t+j-\rho-1)(t+j-1)(t+\alpha+1)},$$

that is,

$$|z| \leq \left[\frac{(2-\rho-t)(1-\rho)\{\alpha+t-1)|\gamma|+(1+m)(2-t+3\alpha)\}}{\alpha t(1+m)(1-\rho)(t+j-\rho-1)(t+j-1)(t+\alpha+1)}\right]^{\frac{1}{j+t-2}}. \tag{3.17}$$

Hence, from (3.17) we deduce the radius of the starlikeness of the functions $\mathcal{F} \in \mathcal{T}\_\mathcal{H}\ (\alpha,\gamma,m,t)$, that is,

$$r_1 = \operatorname*{Inf}_{j}\left[\frac{(2-\rho-t)(1-\rho)\{\alpha+t-1)|\gamma|+(1+m)(2-t+3\alpha)\}}{\alpha t(1+m)(1-\rho)(t+j-\rho-1)(t+j-1)(t+\alpha+1)}\right]^{\frac{1}{j+t-2}}.$$

This completes the proof.

3.5. Remarks

We may also calculate radius of convexity of the functions $\mathcal{F} \in \mathcal{T}_{\mathcal{H}}(\alpha,\gamma,m,t)$.

4. CONCLUSIONS

In this research, we introduced a new class $\mathcal{T}_{\mathcal{H}}\ (\alpha,\gamma,m,t)$of harmonic functions. We obtained the coefficients inequalities, growth and distortion bounds, radius of starlikeness for the analytic part of the harmonic functions involved in this newly defined class. For specific values of parameters involved, our findings may be related to the previously known results.

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