

Research Article

Fuzzy Soft Relative Method and its Application in Decision Making Problem

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Abstract: In day to day problems, so many situations are faced which are full of dissatisfaction and uncertainties. Fuzzy soft set theory has evolved as an effective decision making tool to cope with problems with uncertainties. This work develops a new technique, Fuzzy Soft relative (FS-relative) method for solving such problems. Underlying concept is inspired from fuzzy soft set aggregate approach. We find maximal set and apply it to FS-set to get a relative set which contains relative fuzzy approximation functions values. Then FS-relative operator is generated and the values are applied with maximal set by FS-relative operator to get a single relative fuzzy set. The proposed FS-relative method has been effectively applied to find the optimum solution for selecting the best teacher in a High School according to the teacher specific characteristics.

Keywords: Soft Sets, Fuzzy Sets, Fuzzy Soft Sets, FS-Relative Method.

1. INTRODUCTION

Many real world problems related to engineering, basic sciences, industries, society, medical, environmental sciences etc. cannot be precisely stated without uncertainties. The conflictions and uncertainties are involved due to many reasons like fake and conflicting information and incomplete information about the problem. To overcome these uncertainties theories of fuzzy sets, rough sets, vague sets, intuitionistic fuzzy sets, sets ideas and covered values sets are employed to check out dissatisfaction conditions. Molodtsov [1] pioneered the concept of soft sets and successfully applied his work in many directions [2, 3, 4]. He also established the new ideas like soft number, soft derivative and soft integral.

Rough set theory was introduced by Pawlak [5]. Maji and Biwas [6] used rough sets to make decision making under soft set theory. Chen and Wang [7] presented soft set for reduction of parameterization. Kong and Wang [8, 9] introduced soft set for normal reduction of parameterization and generated a new algorithm for it and applied

it for decision making problem. Xiao and Yang [10, 11] pointed out synthetic evaluating method and provided soft information using soft set theory. Majumdar and Samanta [12] discussed on measure of soft set in a similar way. Ali and Shabir [13] extended the concept of soft set theory and generated some new ideas and techniques on soft set theory.

Zadeh [10] pioneered the idea of fuzzy soft sets. Maji et al. [7, 14, 15, 16] hybridized soft sets with fuzzy soft sets and applied resulting fuzzy soft set theory in decision making problems. Som [17] augmented the work of Maji [16] and established links among soft set theory, soft relation and fuzzy soft relation. Krishna and Chandra [19] used fuzzy soft sets in real life problems. Bhardwaj and Nayak [18] introduced fuzzy soft reduction method in decision making problem. Saeed et al. [20] initiated a simple technique between soft set and fuzzy soft set on Topsis. Studies in references [6, 11, 12] pointed out some deficiencies in the study of soft set. Cagman [21] promoted the concept of fuzzy soft set and found out new concept and ideas. Cagman [21, 8] introduced soft matrix theory

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in decision making problem and also used it to develop parameterized fuzzy soft set. Cagman et al. [10] introduced application of fuzzy soft set in decision making problems.

Alkhazaleh et al. [22] introduced the idea of fuzzy soft set to possibility fuzzy soft set. Kalaiselvi et al. [23] used the concept of Alkhazaleh et al. [9] to solve the selection of a hockey player problem by using possibility fuzzy soft set. Pal [23] used the fuzzy soft aggregate method of Cagman to take decision on selecting a bridal. Many other researchers have also used fuzzy TOPSIS to solve decision making problems [12, 24]. Saeed et al., used TOPSIS for Multi Criteria Decision Making in Octagonal Intuitionistic Fuzzy Environment by Using Accuracy Function [25]. Nowadays researchers are focused on the development of new theories to solve MCDM problems. Recently many researches are done in the field of fuzzy numbers [26-35].

In this work we use a new concept of fuzzy soft relative method in a teacher selection problem. The results of FS-relative method are found better than FS-aggregate method on the considered problem.

2. THE MAIN TEXT

In this section some basic definitions in soft set theory have been revisited. Let a non-empty set U represents the universal set and non-empty set J represents the set of parameters respectively throughout the study.

2.1 Definition (Soft Set)

Let P be a mapping from J into G(U) such that

$$P: J \to G(U)$$

Where G(U) is the power set of U. The ordered pair (P, J) is said to be a soft set and is denoted by (P, J).

2.2 Definition (Fuzzy Set)

A fuzzy set Y on U is a set having the form as:

$$Y = \{(b, \theta_Y(b)) : b \in U\}$$

Where the function $\theta_{Y}: U \to [0,1]$ is called the membership function and θ_{Y} (b) depicts the

degree of membership function of each element *b* ϵ *U*. Mathematically, membership function ($\theta_{Y}(b)$) can be written as:

$$\theta_{Y}(b) = \left\{ \frac{\theta_{Y}}{b} : b \in U, \theta_{Y} \in [0,1] \right\}$$

If $\theta_{Y}(b) = 1, \forall b \in U$, then Y becomes a crisp (or ordinary) set. We denote the class of all fuzzy sets on U by FS(U).

2.3 Example 1

Let U = { b_1, b_2, b_3, b_4 } be the set of four buses and J = { j_1, j_2, j_3 } be the set of parameters in which j_1, j_2 and j_3 have been characterized that buses are 'precious', 'charming' and 'low-priced' respectively. Let D = { $j_1, j j_3$ } \subset J then a soft set can be written as (P,J)={ P(j_1) = { b_2 }, P(j_3)={ b_3 }. This set makes the alternative choices of a person wants to buy a bus.

2.4 Definition (Fuzzy Soft Set)

Let U be an initial universal set and E be a set of parameters. Let G(U) represents the set of all fuzzy sets of U. Let $Y \subseteq E$. A pair $F_y = (\theta_y, E)$ is called a fuzzy soft set (FS-set) over U, if there exists a mapping θ_y as $\theta_y : E \rightarrow G(U)$ such that $\theta_y(b) = \varphi$, if $b \notin Y$ where φ is a null fuzzy set. Here, θ_y is called fuzzy approximate function of the FS-set F_y , and the value $\theta_y(b)$ is a set called b-element of the FS-set, $\forall b \in E$. Mathematically, FS-set (F_y) over U can be written as:

$$F_{Y} = \{ (b, \theta_{Y}(b)) : b \in E, \theta_{Y}(b) \in G(U) \}$$

Sets of parameter and approximation functions are attached to each other in the theory of soft sets. On the other hand, only sets of parameter are attached but approximation functions are independent. Here, approximation functions are represented by fuzzy subsets of U. Fuzzy soft sets are denoted by T_A, T_B $, T_C, T_D, \dots$ etc. and the fuzzy approximation functions are denoted by $t_A, t_B, t_C, t_D, \dots$ etc.

2.5 Example 2

Consider $U = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ be any non-empty universal set and corresponding set of parameters is $J = \{j_1, j_2, j_3, j_4, j_5, j_6\}$ If $D = \{b_2, b_3, b_4\} \subset U$, the approximation functions are:

$$t_D(b_2) = \left\{ \frac{0.7}{j_2}, \frac{0.4}{j_6} \right\}, \ t_D(b_3) = \left\{ \frac{0.9}{j_1}, \frac{0.3}{j_4}, \frac{0.1}{j_6} \right\}$$
$$t_D(b_4) = \phi$$

Fuzzy soft set T_D is written as:

$$T_D = \left\{ (b_2, \left\{ \frac{0.7}{j_2}, \frac{0.4}{j_6} \right\}), (b_3, \left\{ \frac{0.9}{j_1}, \frac{0.3}{j_4}, \frac{0.1}{j_6} \right\}) \right\}$$

Fuzzy soft set T_D can be written in form of table presented as Table 1.

2.6 Definition

If be any element of fuzzy soft sets over U

1- If $t_D(j) = \phi$, $\forall j \in J$

 T_D is called an empty FS-set and it is denoted by T_a .

2- If
$$t_D(j) = U, \forall j \in J$$

 T_D is called A-universal FS-set and it is denoted by $T_{\tilde{D}}$ 3- If D = J

 $T_{\rm D}$ is called A-universal FS-set becomes as $T_{\tilde{i}}$

2.7 Example 3

Suppose that $U=\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ be a universal set and $J=\{j_1, j_2, j_3, j_4, j_5, j_6\}$ is a set of all parameters. If $D=\{b_1, b_3, b_4, b_6\}$, fuzzy approximation functions are:

$$t_D(b_1) = \left\{ \frac{0.2}{j_1}, \frac{0.7}{j_4}, \frac{0.3}{j_7} \right\}, \ t_D(b_3) = U,$$
$$t_D(b_4) = \left\{ \frac{0.9}{j_3}, \frac{0.4}{j_6} \right\}, \ t_D(b_6) = \phi$$

The fuzzy soft set $T_{\rm D}$ becomes

$$T_D = \left\{ (b_1, \left\{ \frac{0.2}{j_1}, \frac{0.7}{j_4}, \frac{0.3}{j_7} \right\}), (b_3, U), (b_4, \left\{ \frac{0.9}{j_3}, \frac{0.4}{j_6} \right\}) \right\}$$

Table 1. Fuzzy soft set of Example 2

T_D	j_2	j_3
b_1	-	0.9
b_2	0.7	-
b_3	-	-
b_4	-	0.3
b_5	-	-
b_6	0.4	0.1

1-If $H = \{j_1, j_4\}, t_H(j_1) = \phi, t_H(j_4) = \phi$

FS-set T_H is an empty FS-set. It means $T_H = T_{\phi}$

2- If
$$Q = \{j_3, j_6\}, t_Q(j_3) = U, t_Q(j_6) = U$$

FS-set T_{Q} is a Q-universe FS-set. It means $T_{Q} = T_{\tilde{Q}}$ 3- If R = J and $t_{R}(j)=U, \forall j \in J$, FS-set T_{R} is a universe FS-set.

Where each $\theta_{tD}(b)$ in Table 2 is called the membership function of T_D .

It means $T_R = T_{\widetilde{J}}$.

3. FS-RELATIVE ALGORITHM

Formatting FS-set and approximation functions of all the FS-set must be nature wise fuzzy. Further getting its maximal set and applied this maximal set to FS-set and getting a relative set. This set contains relative fuzzy approximation functions values. Now FS-relative operator is generated. These values are applied with maximal set by FSrelative operator and getting a single relative fuzzy set. Everybody can easily find the best single crisp from this alternative value set after getting a single relative fuzzy set.

Step 1: Generating fuzzy soft set (T_D) , where $T_D \epsilon G(U)$.

Suppose that
$$U = \{b_1, b_2, ..., b_{\lambda}\}$$
,
 $J = \{j_1, j_2, ..., j_{\mu}\}$ and $D \subset J$

$G_{\scriptscriptstyle A}$	$\dot{J_1}$	j_2		j_{μ}
b_1	$\theta_{t_D}(b_1)(j_1)$	$\theta_{t_D}(b_1)(j_2)$		$\theta_{t_D}(b_1)(j_{\mu})$
b_2	$\theta_{t_D}(b_2)(j_1)$	$\theta_{t_D}(b_2)(j_2)$		$ heta_{t_D}(b_2)(j_\mu)$
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
b_{λ}	$\theta_{t_D}(b_{\lambda})(j_1)$	$\theta_{t_D}(b_{\lambda})(j_2)$		$ heta_{\scriptscriptstyle t_D}(b_\lambda)(j_\mu)$

Table 2. Fuzzy soft set TD and its membership functions

Where each $\theta_{D}(b)$ in Table 2 is called the membership function of T_{D} .

If $[d_{lm}] = \theta_{lD}(b_l)(J_m)$; where $l = 1, 2, ..., \lambda$ and $\prod_{m=1,2,...,\mu} (d_m) (d_m)$.

 T_{D} can be uniquely written in matrix form as below.

This is known as λ by μ matrix of the FS-set T_D .

Step 2: Find the Maximal Set mT_D of T_D .

Let $T_D \epsilon G(U)$ then the maximal set T_D is denoted by mT_D and is defined as under:

$$mT_D = \left\{ \theta_{\frac{mT_D(j)}{j}} : j \in J \right\}$$

is a fuzzy set over J.

Thus membership function θ_{mT_D} of mT_D is defined as $\theta_{mT_D}: J \to [0,1]$ such that:

$$\theta_{mT_D}(j) = \max \{ |\theta_{t_D}(b_l)(j)| \},\$$

where $l = 1, 2, ..., \lambda$.

Then the maximal set mT_D is uniquely expressed in row matrix formed as below.

$$[v_{lm}]_{l\times\mu} = [v_{11}, v_{12}, \dots, v_{1\mu}]$$

This is known as maximal matrix of the maximal set mT_p over J.

Step 3: Find the relative set rT_D of T_D , where rT_D means relative fuzzy soft set over U.

For finding rT_D the following Table 3 is needed to be constructed.

; where 1,2,...., and 1,2,..., .

Then rT_D can be uniquely written in matrix form as:

This is known as λ by μ relative matrix of the FSset T_{D} over U.

Step 4: Find the relative fuzzy set T * D of T_D .

Let $rT_D \in G(U)$ and mT_D denotes maximal set of T_D , then FS-relative is denoted by G_{rel} refined as:

$$G_{rel}: rG(U) \times mG(U) \rightarrow G(U)$$

Table 3. Membership functions of rT_D

rT_D	v_1	v_2		${v}_{\mu}$
b_1	$\theta_{t_D}(b_1)(j_1v_1)$	$\theta_{t_D}(b_1)(j_2v_2)$		$\theta_{t_D}(b_1)(j_\mu v_\mu)$
b_2	$\theta_{t_D}(b_2)(j_1v_1)$	$\theta_{t_D}(b_2)(j_2v_2)$		$ heta_{t_D}(b_2)(j_\mu v_\mu)$
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
b_{λ}	$\theta_{t_D}(b_{\lambda})(j_1v_1)$	$\theta_{t_D}(b_{\lambda})(j_2v_2)$		$\theta_{t_D}(b_{\lambda})(j_{\mu}v_{\mu})$

 $[w_{lm}] = \theta_{\iota_D}(b_{\lambda})(j_{\mu}v_{\mu})$; where $l = 1, 2, \dots, \lambda$ and $m = 1, 2, \dots, \mu$.

such that;

$$G_{rel}(rT_D, mT_D) = T * D$$

Where,

$$T * D = \left\{ \theta_T * \frac{D(b)}{b}, b \in U \right\}$$
 is a fuzzy set of U.

Here T * D is a relative fuzzy set of T_D , and its membership function is $\theta_T * D$ and is defined as:

$$\theta_T * D : U \rightarrow [0,1]$$

such that

$$\theta_T * D(b) = \frac{\sum_{t_D} (b)(jv) \max \left\{ \theta_{t_D}(b)(j) \right\}}{|J|}$$

Where, |J| denotes the total number of elements in the set of parameters.

If possible, let, $U = \{b_1, b_2, ..., b_{\lambda}\}$, then FSrelative operator i.e. T * D can be operated as in Table 4.

Table 4. FS-rel	lative operator
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T_D	$\theta_{_{T}}*D$
b_1	$\theta_T * D(b_1)$
b_2	$\theta_T * D(b_2)$
-	-
-	-
-	-
b_λ	$\theta_{_T} * D(b_{_\lambda})$

Thus the FS-relative matrix can be uniquely written in column matrix form as

$$[w_{lm}]_{\lambda \times 1} = \begin{bmatrix} w_{11} \\ w_{21} \\ - \\ - \\ - \\ w_{\lambda 1} \end{bmatrix}$$

This is known as the relative matrix of over U.

Step 5: Find out one of the higher membership value in $\theta_T * D(b)$. The best optimum alternate solution in relative fuzzy set T * D is that which has the higher membership value into T * D(b).

4. APPLICATION OF FS-RELATIVE METHOD TO TEACHER SELECTION PROBLEM

Let us suppose that there are seven high school teachers in District Lahore of Pakistan. The aim is to select the best of the seven teachers. These seven teachers make the universal set as $U=\{b_1,b_2,b_3,b_4,b_5,b_6,b_7\}$ constructed on the basis of characteristics of a high school teacher that are good personality, good sense of humor, kindness, class room management skills, subject command, students friendly and determination. These characteristics are presented as a set of parameters respectively as $J=\{j_1,j_2,j_3,j_4,j_5,j_6,j_7\}$

Pal [23] used FS-aggregate method to solve such uncertainty problems. In this paper a new technique, FS-relative method is proposed. In FS-relative method, every weighted value is individually characterized and more refined. So, this technique gives a better decision to select specific goals than FS-aggregate method.

4.1 Characteristics Influencing Teacher Selection Decision

J_1 - Good Personality

Every student observes the personality of the teacher. Someone consider the teacher as their leader and role model. A good teacher has in good outer word as well as inner word personality. Outer word personality or external word personality means that he is good in attitude, habits, nature and environment. Inner word personality or internal word personality means that he wears good dress, good in style, look, smile, gesture and habits.

J₂ - Good Sense of Humor

Teaching the students at their mind level is an important aspect. A good teacher creates some fun in the class. Pauses is also very important during study.

J₃ - Kindness

A teacher is sympathetic and kind hearted and settle their behavior according to student's habits and ages.

J_{a} - Class Room Management Skills

A good teacher maintain discipline, regularity, punctuality and ensure student effective study and work habits.

J₅ - Subject Command

A good teacher is full of knowledge in subject matter and complete his/her knowledge in their subject.

J_6 - Friendliness and Congeniality

There is no alternative of a friend. A great teacher is

like a friend of the students.

J_7 - Determination

A great teacher is determined in solving their student's problems and keen interesting all matters of the students.

4.2 Solution of a Teacher Selection Problem

Step 1: Representing the FS-set T_D in the form of following Table 5.

It can be written in FS-matrix $[d_{lm}]_{\lambda xu}$ form as:

	[1	0.7	0.9	0.8	0.5	0.9	0.6
	0.7	0.9	0.8	0.9	0.6	0.7	0.8
	0.5	1	0.7	1	0.7	0.8	0.9
$\begin{bmatrix} d \end{bmatrix}_{\lambda \times \mu} =$	0.9	0.3	0.8	0.5	1	0.9	0.9
	0.8	0.9	0.8	1	0.7	0.4	0.5
	0.3	0.7	0.9	0.5	0.9	0.7	1
	0.8	0.8	0.7	0.8	1	0.9	0.8

Step 2: Finding maximal set mT_D of T_D is as follows:

$$\mu_{mT_{D}}(j_{1}) = \max\{1, 0.7, 0.5, 0.9, 0.8, 0.3, 0.8\} = 1$$

$$\mu_{mT_{D}}(j_{2}) = \max\{0.7, 0.9, 1, 0.3, 0.9, 0.7, 0.8\} = 1$$

$$\mu_{mT_{D}}(j_{3}) = \max\{0.9, 0.8, 0.7, 0.8, 0.8, 0.9, 0.7\} = 0.9$$

$$\mu_{mT_{D}}(j_{4}) = \max\{0.8, 0.9, 1, 0.5, 1, 0.5, 0.8\} = 1$$

$$\mu_{mT_{D}}(j_{5}) = \max\{0.5, 0.6, 0.7, 1, 0.7, 0.9, 1\} = 1$$

$$\mu_{mT_{D}}(j_{6}) = \max\{0.9, 0.7, 0.8, 0.9, 0.4, 0.7, 0.9\} = 0.9$$

$$\mu_{mT_{D}}(j_{7}) = \max\{0.6, 0.8, 0.9, 0.9, 0.5, 1, 0.8\} = 1$$

Step 3: Representing the relative fuzzy soft set rT_D by the following Table 6 as under.

It can be written rFS-matrix $[w_{lm}]_{\lambda x \mu}$ form as

	[1	0.7	0.81	0.8	0.5	0.81	0.6
	0.7	0.9	0.72	0.9	0.6	0.63	0.8
	0.5	1	0.63	1	0.7	0.72	0.9
$[w_{lm}]_{\lambda \times \mu} =$	0.9	0.3	0.72	0.5	1	0.81	0.9
	0.8	0.9	0.72	1	0.7	0.36	0.5
	0.3	0.7	0.81	0.5	0.9	0.63	1
	0.8	0.8	0.63	0.8	1	0.81	0.8

T_D	j_1	j_2	j_3	j_4	j_5	j_6	j_7
b_1	1	0.7	0.9	0.8	0.5	0.9	0.6
b_2	0.7	0.9	0.8	0.9	0.6	0.7	0.8
b_3	0.5	1	0.7	1	0.7	0.8	0.9
b_4	0.9	0.3	0.8	0.5	1	0.9	0.9
b_5	0.8	0.9	0.8	1	0.7	0.4	0.5
b_6	0.3	0.7	0.9	0.5	0.9	0.7	1
b_7	0.8	0.8	0.7	0.8	1	0.9	0.8

Table 5. FS-set related to the best teacher selection problem

Table 6. Membership functions of rT_D

rT_D	v_1	<i>v</i> ₂	v ₃	v_4	<i>v</i> ₅	v_6	<i>v</i> ₇
b_1	1	0.7	0.81	0.8	0.5	0.81	0.6
b_2	0.7	0.9	0.72	0.9	0.6	0.63	0.8
b_3	0.5	1	0.63	1	0.7	0.72	0.9
b_4	0.9	0.3	0.72	0.5	1	0.81	0.9
b_5	0.8	0.9	0.72	1	0.7	0.36	0.5
b_6	0.3	0.7	0.81	0.5	0.9	0.63	1
b_7	0.8	0.8	0.63	0.8	1	0.81	0.8

Step 4: The relative fuzzy set $\theta_T * D$ can be found out as

	[1	0.7	0.81	0.8	0.5	0.81	0.6	[1]	
	0.7	0.9	0.72	0.9	0.6	0.63	0.8	1	Í
	0.5	5 1	0.63	1	0.7	0.72	0.9	0.9	
$\theta_T * D = \frac{1}{7}$	0.9	0.3	0.72	0.5	1	0.81	0.9	1	
/	0.8	8 0.9	0.72	1	0.7	0.36	0.5	1	
	0.3	3 0.7	0.81	0.5	0.9	0.63	1	0.9	
	0.8	3 0.8	0.63	0.8	1	0.81	0.8	1	
5.058] [0.723							
5.115		0.731							
5.315		0.759							

 $= \frac{1}{7} \begin{vmatrix} 4.977 \\ 4.872 \\ 4.696 \\ 5.496 \end{vmatrix} = \begin{vmatrix} 0.759 \\ 0.711 \\ 0.696 \\ 0.671 \\ 0.785 \end{vmatrix}$

So,

$$T * D = \left\{ \frac{0.723}{b_1}, \frac{0.731}{b_2}, \frac{0.759}{b_3}, \frac{0.711}{b_4}, \frac{0.696}{b_5}, \frac{0.671}{b_6}, \frac{0.785}{b_7} \right\}$$

Step 5: The largest membership value in the relative fuzzy set is 0.785. This valve is represented in the universal set by the seven teachers. Therefore b_7 is the best teacher among the rest of seven teachers.

5. CONCLUSIONS

In this study an innovative idea in shape of FSrelative method has been developed for finding the optimum alternative solution. The developed FS-relative technique is more accurate and refined method than FS-aggregate method. The comparison of FS-relative method to the methods in [9] and [11] has revealed that FS-relative results are in good agreement according to the weights. The proposed approach is evidently capable of finding solution of the problems with uncertainties. Moreover, FSrelative method can be viewed as a generalized form of methods in [9] and [11]. At the end, we discuss the following findings into the consequences of conclusion as:

- We conclude that a teacher is recommended as the best teacher from rest of other teachers whom has more quality on subject command and friendly behavior with their students. These professional qualities made a teacher the best.
- The novel method (FS-relative method) has designed for the solution of uncertainties of real life problems in which provided values are not in pattern such as some values are so small and remaining values are so much large.
- FS-relative method provides the best result which is related to the situations because each value is individually characterized.
- Furthermore, FS-relative method also provides accurate solution because each value is refining with the help of maximal set. This process reduces the truncation error and enables the technique for the best results.

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