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Review Article

Potential Investigation on Multiphase Flow of Loaded Dispersion for the Production of Metallized Paper

Saadat Ullah Khan Suri, and Mohammad Siddique

Department of Chemical Engineering Balochistan University of Information Technology, Engineering and Management Sciences (BUITEMS), Quetta, Pakistan

Abstract: The current review research's main objective is to develop dispersion models in the multilayer curtain coating with the production of metallized paper. To achieve this, the curtain coating on the paper substrate is employed with respect to multilayer coating of polymers. The first layer of polymer is applied to the paper and then it is subjected to vacuum metallization with aluminum deposition. After it, another second layer of polymer is subjected on it to prevent it from oxidation. These coated polymers are different in nature. The metallized paper will be produced which has high strength will be formulated in this application of curtain coating. The instability of curtain and air entrainment will be minimized from high Weber number, low Reynolds number, Optimum web speed and Coat weights. The above demonstrated process simulation will be modelled in Ansys-CFD. The dispersion of solids in the curtain flow through substrate moving on the web will be evaluated from different numerical methods. Each method has its own characteristics to study the nature of solids dispersion. The high loaded solids dispersion will be investigated from numerical methods including Langrangian Point Particle, Coarse grained molecular dynamics, Stokesian Dynamics, Brownian Dynamics, Point Particle Method Reynolds Averaged Navier Stokes equation, Eulerian Method, Langrangian-Eulerian Point Particle, Large Eddy Simulation point particle, Combined discrete element and Large Eddy Simulation and Discrete Element Methods.

Keywords: Deposition, Impingement, Curtain, Algorithms.

1. INTRODUCTION

The invention of metallized paper has secured importance with its multi-dimensional uses till the recent past. To acquire the polished surface on a metallized paper, it is essential to pre-enamel the surface of the paper substrate. This can only be achieved by refining the paper and making a feasible technology that can work at an optimum rate. Therefore, there comes surprising finding through the motivation of pre-enameling surface with organic solvents and the high gloss on paper surface. The state of art of present research study on metallized paper with different numerical methods is to improve the methodology for an optimum effectiveness to gain desire quality [1-3].

This solution of methodology encompasses with upgraded metallized paper to scrutinize the nature of solid dispersion and to encounter this problem with suitable paths significantly. It is essential to have definite values of applied layers on the paper substrate. Hereafter, we can obtain a composition, which has certain rheological and static surface tension characteristics making it acceptable for metallized paper preparation [3-6].

The metallized paper is manufactured from direct high vacuum metallization. In this technology, the substrate that needs to be metallized is coated with different layers of liquid and metal films. It is settled to accumulate smooth surface on paper substrate. The coated substrate is subjected to a vacuum chamber. The aluminum metal is applied on the surface of the substrate there. There are coated another layer of film to protect the metal layer from oxidation and corrosion. Its composition comprises of mineral and synthetic pigments, latex

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*Corresponding Author: Saadat Ullah Khan Suri <saadatullah.khan@buitms.edu.pk>

binder and conventional additive. The synthetic polymer pigment should be from Styrene, Acrylic acid, Acrylonitrile, Homo-polyolefin, Co-polymer or Urea-formaldehyde resin of diameter around 0.1-30 um. The added plasticizer amount should be kept up to 25 % with a synthetic polymer agent. Its cast coating layer value is 18-26 g/m2. The Acrylic, Styrenic derivatives, Epoxy resins and Nitrocellulose are used for these purposes. These chemicals have the tendency to form a film when a solvent evaporates.

After the material coating on a substrate, the coating materials pH should be greater than 7. The moisture content is adjusted to 5 to 15 % after the dying operation. There should be mineral pigment to 25-75% and synthetic polymer at 5-25% in coats. These layers have thicknesses from 1 to 3 g/m. It is obligatory to accomplish glass metallized property on metallized paper. We have to reduce the solid dispersion to attain it. There are various metallic papers in usage and their varieties depend on surface smoothness and high adhesion. On that account, there is an advantage for increasing the production rate to 2,000 m/min [7-15].

In the multilayer curtain coating process, the liquid curtain falls from an upward direction on a moving substrate under the gravitational force at a constant height that deposits films, as depicted in Figure 1. The present invention provides a method to manufacture a metallized paper by elaborating different numerical methods for solid dispersion. There are mainly three stages of this process. (i)-Coating of an aqueous layer film on the moving substrate at web. (ii)-Deposition of a metal layer on the substrate. (iii)-Applying the second aqueous film layer on metallized substrate to prevent it from oxidation. So, the correct formulation of this novel process becomes with three zones. (i)-Curtain formation. (ii)- Curtian spreading. (iii)-Impingment. respectively [14].

This technology has two advantages over other coating technologies. This technology has the ability to coat different surfaces with thin liquid films. The second advantage is that the operation has high speed, which causes wetting failure through hydrodynamic pressure produces from liquid near the contact line. On the other hand, there are some challenges in multilayer curtain coating technology that need to address with a serious attention [15].

1.1 Special Issues

Figure 2 (a) represents the space domain of the curtain coating process by demonstration of two dimensionless numbers; (i) Weber number, it is a ratio between inertial and capillary forces based. It is also based on web speed and curtain velocity. (ii) Reynolds number, it is the ratio of inertial force to viscous force. Figure 2 (b) shows that a thin liquid film can be deposited on the solid substrate of limited area under the operating parameters range. It is addressing different issues which are (i)- Air entrainment, (ii)- Bead pulling, (iii)- Heel

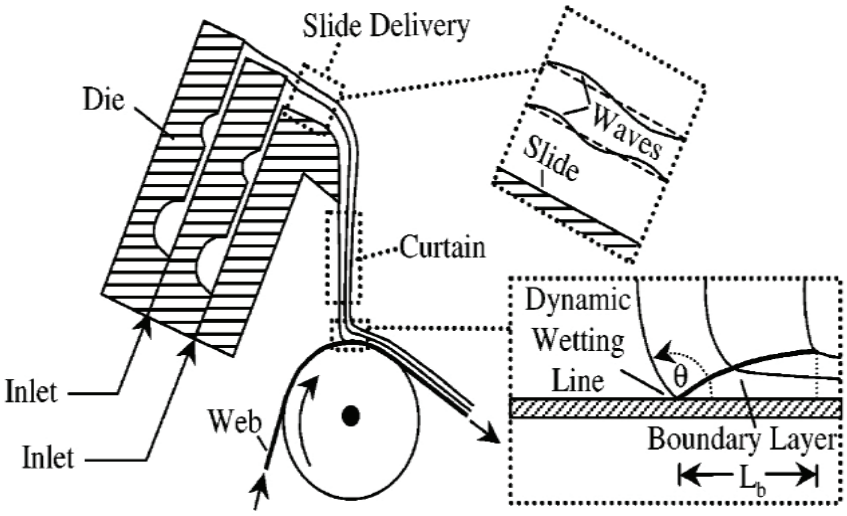

Fig. 1. Schematic diagram of multilayer curtain coating process with moving substrate on web beneath slide deliver [15].

formation and (iv)- Curtain breakup. This limited space is referred as the coating window.

At first, these four special issues during the curtain coating process cause the liquid curtain breakup and raise instability. Further, the stability of the liquid curtain solely dependent on the Weber number. The deep analysis tells us that the curtain meets stability at Weber number >1.5. The impingement zone has some physical operating conditions which elaborate the dynamics of film layer formation and its operation through multilayer curtain coater above the web and substrate. These conditions stand out for the process feasibility. Firstly, there is formed (Heel-Heel) when there does not arise movement in curtain to the web direction. Secondly, there occurs (Strands-Strands) at low flow rate which produces shear in the curtain resulting a break-up. In consequence, a pulled layer forms; it materialized due to coating layer that does not impact on substrate beneath the slot in moving web direction [16].

The stability in curtain has unique role for an effective coating. The curtain instability appears from solid dispersion at coats. Curtain unsteady performance causes by the air boundary layer. There is a need to encounter the curtain instability with relative troubleshooting. The curtain coat weights vary at high speed. These instabilities are recognized from sinusoidal frequency. While, the stability is reached at constant amplitude and variable frequency range.

The other curtain instabilities are termed as splashing and burps. The induced viscosity and surface tension result in these changes. The low values of curtain viscosity and Weber number coat result in splashing and intermittent burps.

The Weber number is varied along x-direction at the surface. The effectiveness of surfactant in multilayer curtain coating process leads towards low Weber number low. The Weber number provides helpful work about uniform liquid curtain. This can control adding surfactant and there comes stability in liquid curtain. The surface tension can also become low with the addition of a suitable surfactant. Hereafter, it becomes constant from the variable. As a result, surfactant has a vital tendency to decrease the surface tension and increase the stability of multilayer curtains effectively. A low liquid flow is found in a curtain due to the break of liquid films, as demonstrated in Figure 3. This transition accounts of forming a liquid film along curtain layers map out.

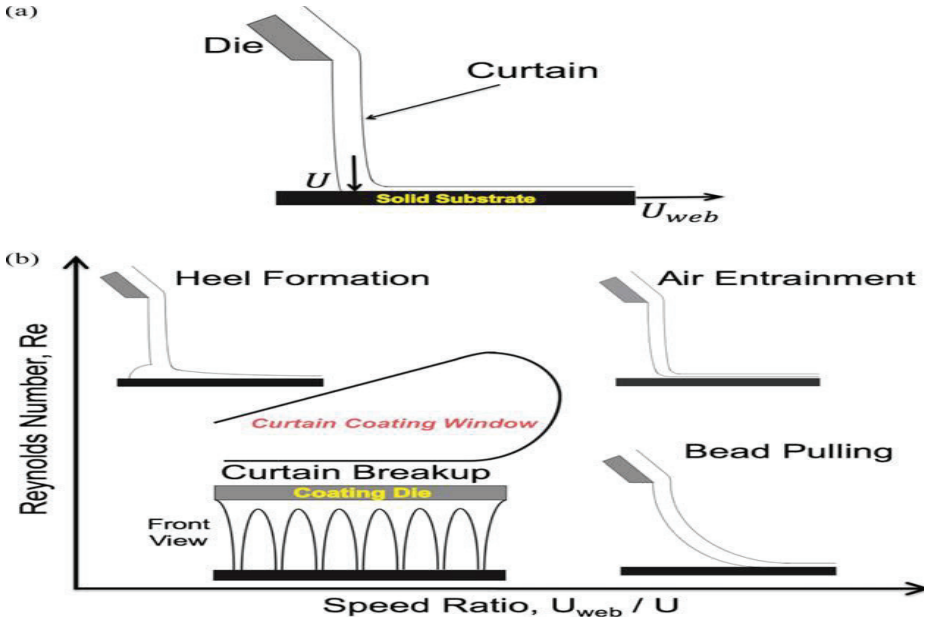

Fig. 2. (a) Illustration on Specific operation from dimensionless numbers, (b) Illustration on liquid film deposition at substrate of curtain coating process [16].

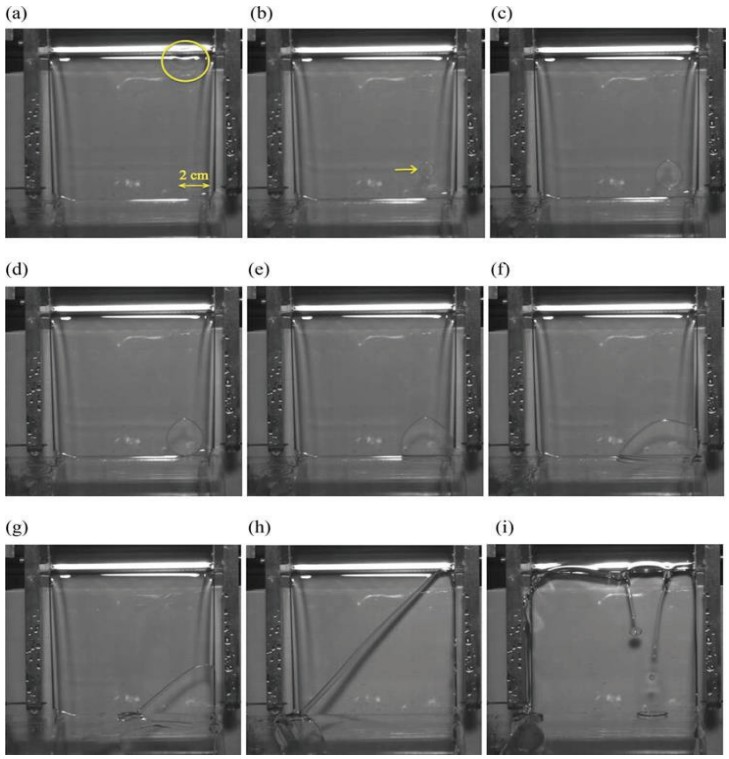

Fig. 3. Illustration on breakage of liquid curtain at 70 wt. % of glycerol/ distilled water with μ = 19.1 mPa.s and variable time intervals at (a) t= 26.5 ms. (b) t= 182.5 ms. (c) t= 186 ms. (d) t=188.5 ms. (e) t=192.5 ms. (f) t= 207 ms. (g) t= 359.5 ms. (h) t=689.5 ms. (i) t =967 ms [17].

In addition, there is an observation from results in Figure 3 that sudden break of liquid curtain can also occur due to low flows of liquid layers. There are formed topological structures when liquid layers left the die slot with equally spaced arrays. The purpose of the investigation is to compare curtain layers dynamics with theoretical and experimental observations of formation and steadiness in the inscribed multilayer curtain coating process. There are taken several observations with different liquid properties. The paramount certitude is break-up of the liquid curtain due to the formation of holes that take place before disintegrating liquid film.

(Heel-Heel) sets up when the curtain is not completely moved in the direction of the web, but flows slightly towards opposite direction. (Strands-Strands) founds due to insufficient flow, which causes well establish curtain to break-up and contract into strands. A pulled film forms when the coating fails to impact on substrate surface right below the slot and it is pulled forward together with the web [17].

The particles dispersion in the industrial processes has gained importance from reduced turbulence with upgrading it. These dispersed particles laden flows occur in different processes including pneumatic conveying, particle separation, multilayer curtain coating and fluidized beds. It is necessary to optimize these processes and their corresponding equipment for minimizing any error. This fact accounts of upgrading the process selectivity. There are several numerical simulation methods that can demonstrate the complex fluid flows and relative solid characteristics. The Computational fluid dynamics (CFD) techniques show the wide ranges of fluid flows from it salient tools. These CFD models are showing the particle-fluid and particle-particle interaction with quite precision. The present research paper is focused to compare the investigation on different CFD models to review comprehensively the nature of solid dispersion for the production of metallized paper using multilayer curtain coating technology. The identification of the correct optimum multilayer curtain coating model with physical parameters is

a future need which is intensively described at the end of paper.

In general, there occurs coupling between dispersed and continuous phase. It is evaluated from the volume fraction of the solids. It is based on degree of particle scale time $\tau P = \rho PdP$ and Kolmogorov time scale $\tau p = (\nu/\varepsilon)1/2$ simultaneously. This carrier fluid flow affects the particle dynamics, although these solids do not affect the flow turbulence.

1.2 Economic Value

Till the recent past, the curtain coating has become an emerging technology from its high speed. It has attained commercial success as the pre-metered and non-impact coating process. This technology has attained potential impact due to its ability to imply low coat weights on moving substrate. This process follows the non-impact and non-contact type operation. Multilayer curtain coating is referred to as a defect free coating surface process because there does not occur split patterns as the lamination comes above the base sheet. This process possesses uniqueness from various ranges of coating and coat weights through a single coating head [18].

There is a paramount need to address the multilayer curtain coating mechanism and identify the specific reasons why the process fails? The designed experimental tests have a key role as the testing will show its viability with any theory or hypothesis. The performance can only be enhanced by elucidating the errors which grow inaccuracy. While the CFD analysis is costly but the hydrodynamic dispersion mechanism is vital to upgrade the multilayer coating process. This process design is critical for increasing product reliability. The phase visualization with its brief mechanism is inaccessible to scientists working on this process. It is due to geometric features and non-transparent liquids. The evaluation of the configurations can be done with experimental tests. To save the cost and time with the permission of exploring the parameters have a key for real process effectiveness.

The CFD can simulate the phase interaction accurately. It offers a wide range to upgrade the full-scale model with reliable configurations. Scientists have gained interest in estimating the nature of solids with other phase interactions for optimization. Further, validation comes from the data. A different mathematical model can approach the nature of solid dispersion (Particle-particle, Gas-particle and Liquid-particle). In dilute systems, the Eulerian-Eulerian mathematical model has the capacity to trace the particles in the continuous phase with its low computational cost. The CFD has proved its utility to simulate the solid-particle interaction in the curtain at isothermal conditions too [19].

The present research study is focused on seeing the validity factor of this process. In the beginning, the performed mathematical models with their recommended features can be modified for testing, or their differences can be reduced with any combination, if required. There are available different simulation softwares such as (Ansys-CFD, Ansys-CFX, Star-CCM and Open-Foam). There is a serious need to compare the accuracy of these mathematical models through the nature of dispersion with relative troubleshooting. The critical evaluation of various numerical models will describe the process parameters optimization. It will keep a focus on the nature of flow with interphases dispersion. We will be able to obtain the most reliable mathematical model which can express multilayer curtain coating optimization in contrast to other simulation models [20-22].

In Langrangian-Particle mathematical model, it solely describes the diffusion and transportation of solid tracers in a medium. It combines with trajectories of large number of particles. The major advantage of this model is that there is no numerical diffusion takes place.

On the other side, in other models; a solid released from its point source instantaneously. While, the Langrangian-mathematical model is independent of the computational grid. In general, it has an infinitesimally small resolution. This model has a wide range. So, it can simulate the meso-scale transport, diffusion, dry and wet deposition.

Besides other models, the Eulerian-Langarangian mathematical approach can evaluate the interphase dispersion. This combined modeling approach has the ability to capture a wide range of

particles in various forms, including concentrated and dilute flows, linear and non-linear and multiphases interaction in non-equilibrium state. To achieve it, the Reynolds Averaged Navier Stokes equation is used at continuous phase which is called carrier.

The solid dispersion is shown from tracked particles in gas flows. There is momentum exchange which causes an interaction. The large particles are idealized as point particles. In this mathematical modeling approach, the particle boundary, flow region and meso-scale particle range can not be approached. These features can be simulated from functional correlations which have empirical parameters. The Point-Particle model can be evaluated through a continuous phase. It is achieved by using the different mathematical models including Langrangian Point Particle, Coarse-grained molecular dynamics, Stokesian Dynamics, Brownian Dynamics, Point Particle Method Reynolds Averaged Navier Stokes equation, Eulerian Method, Langrangian-Eulerian Point Particle, Large Eddy Simulation point particle, Combined Discrete Element-Large Eddy Simulation and Discrete Element Methods.

Consequently, the present research study aims to stabilize the curtain coater at an optimum speed which will develop a novel investigation of the multilayer curtain coating process from theoretical numerical methods. There is a comprehensive research plan to evaluate the various features, which comprise substrate, curtain stability, runnability and quality of metallic film intensively.

1.3 Applications

Curtain coating technology has tremendous applications in different industrial sectors, which include Optical films, Metallized papers, Steel sheets for home appliances, Utensils, and Specialty film formation [23].

2. Literature Review

2.1 Assessment of Particles Tracking Model

The Eulerian-Langrangian point particle method is used to explore the solid dispersion in a turbulent flow. There are used CFD and Open-foam softwares for the direct simulation. The precise investigation on solid dispersion in multilayer coating for metallized paper can be performed in the future. It will be really helpful for scientists who are undergoing any troubleshooting on multilayer coating of metallized paper. The dispersion model is evaluated from particle motion. In order to get simulation on the steady and unsteady state modes; These effects will rise to investigate the coupling between continuous and dispersed phase. The combined modeling effect of Eulerian and Langrangian models will give a conclusion that the dispersed phases is in agreement with simulation runs and experimental tests. The Ansys-Fluent software does not provide exact measurements on the dispersed phase. On the other hand, the Open-foam software overestimates the dispersion. These differences are due to low values of mass and solid volume fraction and mass [24].

The precise dispersed flow is being obtained with Langrangian-Eulerian combine network related to conservation equations. The prevailed conditions at Newtonian fluid are isothermal and incompressible. The Reynolds Averaged Naviers Stokes equation is able to demonstrate the mass and momentum transport. The simulation of Reynolds force is performed with the eddy-viscosity approach. For this purpose, two turbulence models are used, which are ((k – ε) and (k − ω-SST)). These are based on eddy viscosity approach. Their coding has been performed in the Ansys-Fluent and Open-foam. The solid particle's motion gets traced from the differential equation. The paramount significance of these approaches is that it can locate the solid particles independent of their velocity and shape [25].

2.1.1 Langrangian Point Particle Method

The Langrangian model is able to find the precise trajectories of solid particles with small air samples. It enables to find the transport and diffusion of small solid particles. The basic advantage of the current method beside Eulerian method; this method has key to demonstrate the numerical diffusion. Moreover, the Langrangian method is independent from computational grid and has infinite small resolution. The Flexpart tool is able to simulate the small sized solid particles in meso-scale through transport, diffusion, dry and wet deposition and also

radioactive decay of solid particles from a single source with Langrangian method. It can also find the solid particle in domain filling mode from where the entire region is presented by solid particles of equal mass. It enables the simulation of the solid dispersion from their releasing source in backward time to show the valuable contribution from their receptors. The weather prediction model of the European Center Forecast uses Flexpart software for numerical weather prediction [26-28].

2.1.2 Demonstration for Dispersed and Continuous Phases:

There is a need to define the coupling of dispersed and continuous phases for simulating the solid dispersion. The major work function of coupled phases is based on the volume fraction. The solids in diluted flows are affected by aerodynamic forces acting on them. When the solid particle's interaction and its notable effect on dispersed phase with fluid dynamics increase. [29].

2.1.3 Dispersed Phase Mathematical Simulation

The validation of the particle laden dispersed phase comes out from profiles of solid particles. The solid particle profile is plotted against the continuous phase. These profiles are generated according to experiments. The solid particles are traced in the form of slices bar of thickness around 0.15 H. Its center is lying at the measurement points exactly. Figure 4 depicts the date sets of experimental and numerical runs in order of one and two-way coupling. There is an injection at a time of around 0.07 s. There is also an observation that particle's velocity lies in x-direction and particle's clouds in the y-direction. The results are demonstrated in Figure 4 with two-dimensional meshes A and B and C explicitly.

The gathered data sets from the experimental tests reveal that the particles follow the fluid dynamics. Their shape and magnitude is identical to the fluid dynamics profiles. The simulation results indicate that there is a difference in the experimental reading by a spatial dimension. The problem of data sets is kept in Ansys-Fluent with a mesh A. It depicts the velocities with profile distribution which is non-realistic. While, the experimental tests show that particle cloud do not follow the span-wise path. There does not occur the dispersion of the solid particles in the recirculation zone.

Furthermore, there are high magnitudes of velocities in all these profiles. The two-way coupling detects that there are no particles at x/H = 2 at a specified time. The most suitable results come through simulation on mesh C using Ansys-Fluent. It has cell layers in a z-direction span wise. The results match with the experimental tests. The magnitude and shape of the solid particle velocities are in good agreement. The simulated solid particles are higher at the first four positions and then these profiles become block-shaped. At the position x/H = 12, these velocities curves in mesh C do overlapping. The two-way simulation coupling from Ansys-Fluent undermines the expansion of solid particle clouds at the position x/H = 2, 5, 7. The one-way coupling enables the particles to spread on the second measuring value [30-32].

To conclude, the computational domain of Ansys-Fluent should be three dimensional to facilitate the solid particle's turbulent dynamics.

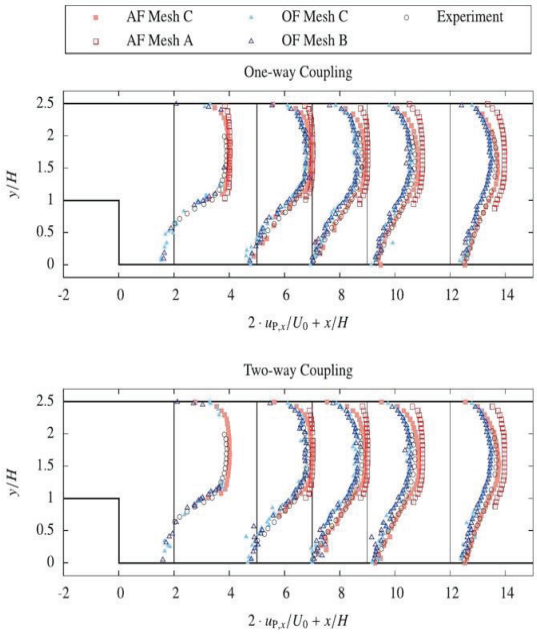

Fig. 4. Illustration on velocity profiles of the dispersed solid phase with main direction uP,x(y) behind from simulations in Ansys-Fluent on meshes A, B and C and experimental tests [32].

2.2 Dimensionless Numbers for Solid Dispersion

It is convenient to write the differential equations and their boundary conditions in terms of dimensionless groups. Dimensionless groups may have a smaller numerical values which certainly facilitates a numerical solution. These analyses of the system are described in a more convenient manner. The investigation on liquid curtain can be critically monitored at low value of dimensionless Weber number. The Weber number is able to distinguish the liquid curtain flow on a moving substrate meticulously. There are surface waves that are generated due to pressure gradient from meniscus on the substrate. The effects of gravity and pressure have a paramount role for the stability of liquid curtain to achieve the optimum multilayer coating; described in Figure 5 [33-35]. In multilayer curtain coating, there are developed dispersion models for the production of metallized paper. To achieve this, the curtain coating on the paper substrate is employed with respect to the layered coating of polymers. The main gain of the present research is to study the highly loaded dispersion of coating polymers on the first coating on a paper substrate moving on a web. Secondly, aqueous film deposits on said metallized paper. This would be characterized on the basis of shear viscosity and static surface tension. The first curtain polymer will be chosen from Acrylic group, an Acrylic-styrene polymer and colloidal dispersion polymer is formed from it simultaneously.

The first curtain polymer will be chosen from the Acrylic group, an Acrylic-styrene polymer and colloidal dispersion polymer is formed from it simultaneously. This is also called a modified Acrylic polymer. The second polymer is colloidal dispersed and its size varies. It is prepared conventionally and it contains Acrylic polymer with an Amine hydroxyl group. These two curtain layer polymers quantities are selected. There are added Pigments, Surfactant, Polyurethane and Acrylic-acrylamide thickener for upgrading metallized paper quality in this multilayer curtain coating application [36-38]. Henceforth, the solid particles visualization has become a crucial part of providing strength on said metallized paper. These die particles velocities are being monitored by a high-speed camera. Their velocities are calculated with a rate of change of position with time. These velocities are varied from K= (0.8-1) m/s shown in Figure 6. The measured thickness is varied from 0.07-0.3 mm.

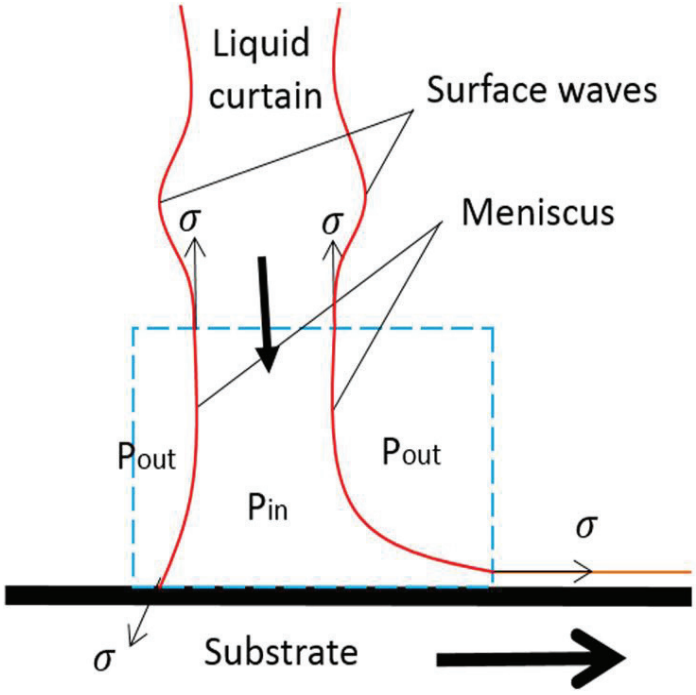

Fig. 5. Illustration on pressure gradient and surface tension at meniscus [33].

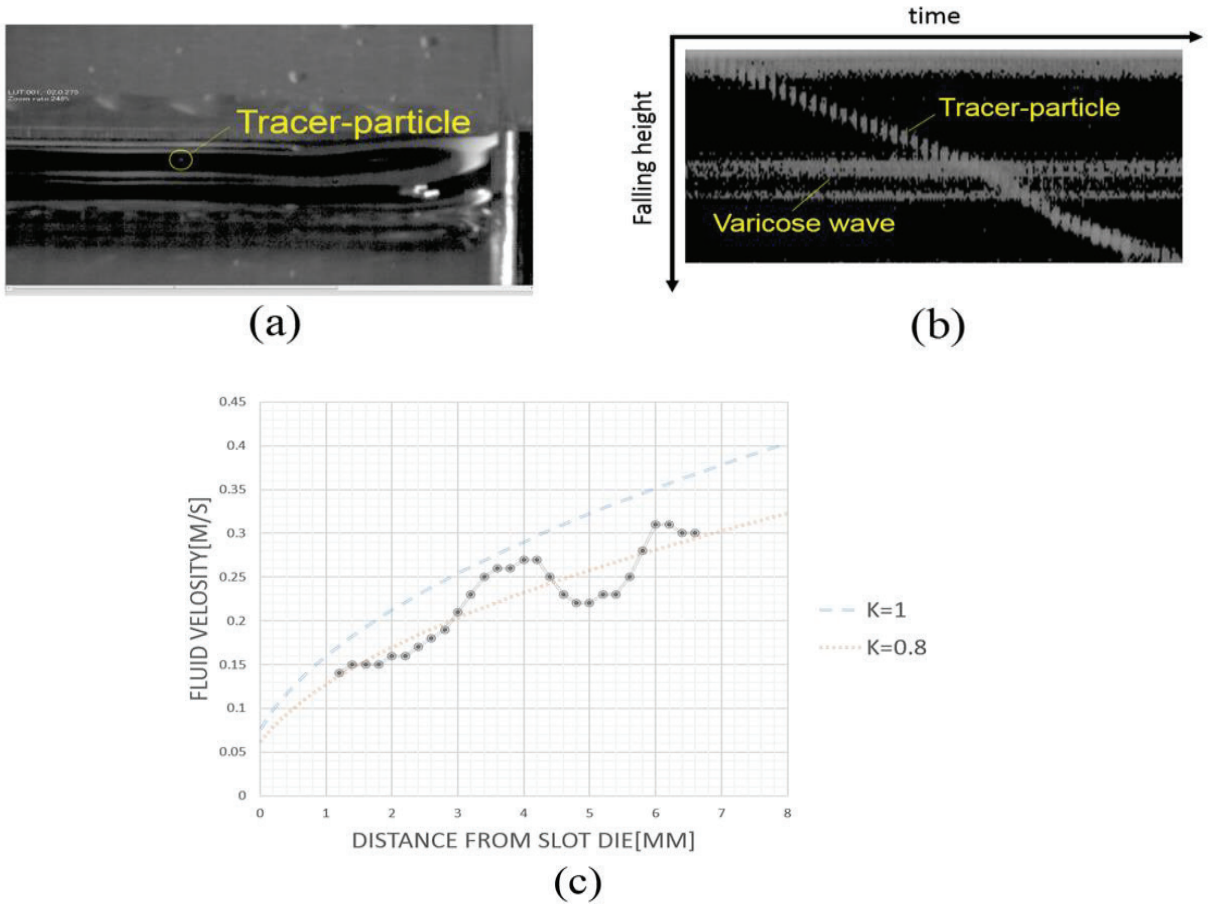

Fig. 6. Illustration on liquid velocities a) Tracer particles in liquid curtain with particle diameter 15 µm b) Solid tracer in liquid [38].

The wave number is dependent on dimensionless Reynolds and Weber numbers. On the other side, amplitude decay gets affected by capillary numbers. These numbers are changeable from the ambient condition, coat thickness and liquid curtain falling height.

There accumulate variable solutions of the curtains dynamics profiles from its curvature at low boundary value. The instability in liquid curtain increases with the effect of surface waves which have staggered and peak valley patterns. These waves are solely dependent on Reynolds and Weber numbers. There do not appear surface waves at low Weber number of 1, low substrate value and low viscosity of liquid curtain die [39,40].

2.2.1 Growth Dynamics of the Hole within the Liquid Curtain

There is breakup in the liquid curtain due to a hole. Firstly, this hole has a circular shape. The capillary force causes to make hole larger. There comes evaluation about curtain flows in a downward direction with an elliptic shape. It was measured using different liquids curtain dynamics. Figure 7 depicts the liquid solutions with a variable viscosity range from (19.1, 54, 92.8, and 177) mPa.s. At (177 mPa.s) liquid viscosity solution value, there is high resistance on hole expansion. The experimental observation on liquid curtain dynamics presents conservation of momentum with effect from solids dispersion. The 70 % solution of Glycerol is fed in liquid curtain and there generates variable effects of hole expansions in both x and y direction at different viscosities. It raises hole expansions, as shown in Figure 7 [40].

The solid particle in liquid curtain die accounts of increase in liquid film curtain viscosity. Furthermore, formation of holes near the edges causes this increase. The high-speed image analysis has proved this hypothesis a truth. The drawbacks of hole are that it slows downs the multilayer

coating process due to un-stabilization and it keeps lowering the curtain speed. The other experimental tests have shown that converging edge guides decrease the applied curtain width. Hence, a need to stabilize the curtain with multiple features has achieved importance in multilayer curtain coating process with solids dispersion investigation modes. [41,42].

2.3 Multi-Particle Collision Dynamics

The meso-scale particles simulation can notice the extent of their dispersion. The multi-particle collision dynamics simulation method has progressed as an emerging tool. The coarse-grained molecular dynamics (CGMD) are introduced between this method. Its characteristics can be procured from the multi-particle collision dynamics method coincides with to a stochastic process. The stochastic process comprises of point-particle interaction. From Figure 9, there is appeared that the algorithm is based on two steps; (i)-Streaming and (ii) Collision. In the streaming process, there is an updated particle position with the ballistic movement accompanied by their position. It has proved correct from the written mathematical correlation in Equation 1;

ri(t+h) = ri(t)+hvi(t) →Eq. 1

whereas; ri and vi(t) are the position and velocity of particle i, simultaneously. The collision time is denoted by h. The collision step consists of sorting the particles into the cubic cells. These particles are rotated with an angle α along the axis.

vi(t+h) = vcm(t)+R(α)[vi(t)−vcm(t) →Eq. 2

vcm is mean velocity at cell and α is the angle of rotation. The numbers of particle in cell are designated with NC. In addition, the mass, momentum and energy values are conserved. It facilitates the current hydrodynamics interaction in the current solid dispersion system.

2.4 Hydrodynamics Interactions

2.4.1 Brownian Dynamics

The solid particle motion in liquid curtain seeds the variation in their velocities. This interaction is known as inter-particle hydrodynamics interaction. Additionally, the hydrodynamics interaction is upraised from inducing diffusion. In addition, it is essential to simulate particle dynamics. There are stochastic differential equations with Brownian dynamics and their hydrodynamic interaction is nj(t) is a random vector satisfying [43].

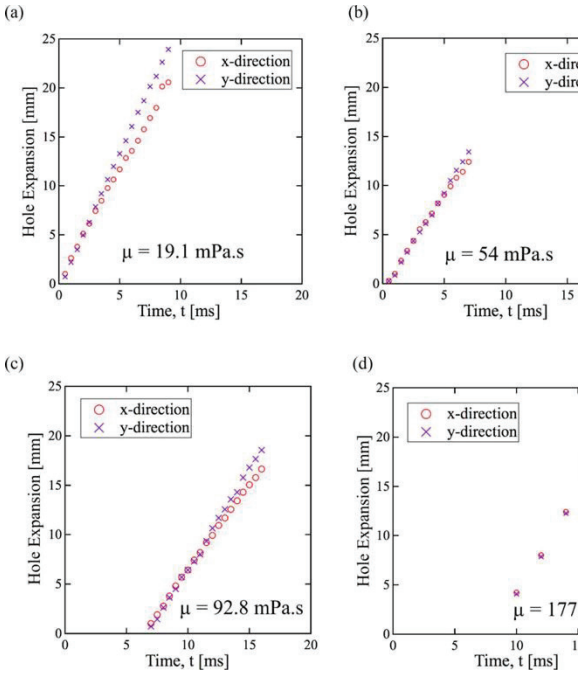

Fig. 7. Illustration on The hole evolution within the liquid curtains for (a) 70 wt % Glycerol solution (l519:1 mPa.s), (b) 79.5 wt % Glycerol solution (l554 mPa.s), (c) 85 wt % Glycerol solution (l592:8 mPa.s and (d) 90 wt % Glycerol [40].

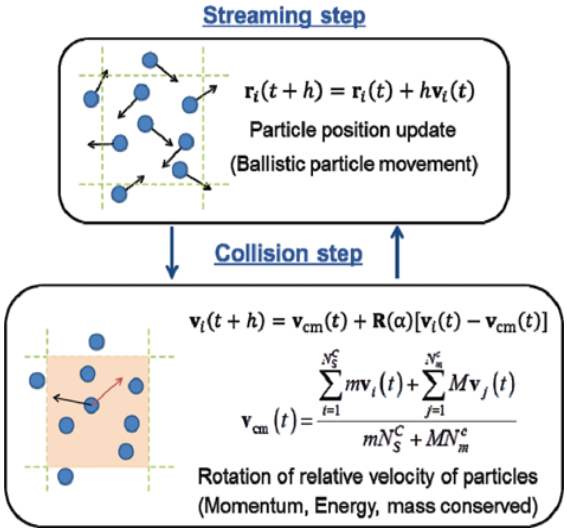

Fig. 8. Illustration on the solid particle hydrodynamic with theoretical approach [43].

There are used Brownian dynamics to expand the physical features of meso-macro molecules in a liquid curtain. The calibrated study on inter-particle hydrodynamic interaction is simply approximated to Stokesian friction as a term. It is inferred that Brownian dynamics cannot find an accurate hydrodynamics interaction. The coarse-grained molecular dynamics have been introduced to overcome these issues. The lubrication force is insufficient near the hydrodynamics particles interaction. However, despite this fact, the Brownian dynamics have a strong role in the investigation of solid dispersion, e.g. colloidal gel, colloidal glass and multilayer coating systems. In Brownian dynamics, inter-particle hydrodynamic interaction is less. In the case of multilayer curtain coating system, the solid particle movement is influenced from non-hydrodynamics inter-particle interaction. It is appeared from Van der Waals and depletion attractions. Consequently, the motion of solid particles is distinguished from volume effects. The theoretical calculations on inter-particle hydrodynamics have proved whole multilayer curtain coating process with non-uniform solids dispersion can be simulated by Brownian dynamics effectively [43].

Finally, the start-up of shear force shows non-rheological characters from overshooting. The Brownian dynamics can work for inquiring of microstructure change on the substrate. It has a deciding role to reveal the microstructural change on a substrate in a non-equilibrium state. It has also reproduced exact information about the microstructural change that denotes the source of stress fluctuation.

To conclude, the analysis on Brownian dynamics optimizes the solid dispersion in multiphase flow of loaded dispersion with the production of metallized paper explicitly [44].

2.4.2 Stokesian Dynamics

The Stokesian dynamics do not have a clear solution of the solid dispersion. Though, indicted mathematical correlation in Figure 8 authorizes to find the inter-particle hydrodynamic interaction rigorously. The Inter-particle and resistance tensors root into torque and stochastic displacement.

While this disturbance causes the change of drag force on solid particles to mention as inter-particle hydrodynamic interaction. The most accurate value of inter-particle hydrodynamics can be obtained from Stokesian dynamics due to the absence of diffusion tensor. It has multi-body far-field interaction and lubrication force which is assembled from resistance tensor R and mobility tensor M. Figure 8 mainly reveals the forces acting on solids dispersion [45].

2.5 Point Particle Reynold Averaged Navier Stokes Equation method

The Reynolds Averaged Navier Stokes equation is time dependent. The foremost theme of the equation is used to inspect fluid dynamics with relative time. The instantaneous quantity is converted by averaged time and fluctuating fluid dynamics. The fluid dynamics can be explored from it absolutely. The time dependent solution can be acquired which is based on the physical properties of fluid.

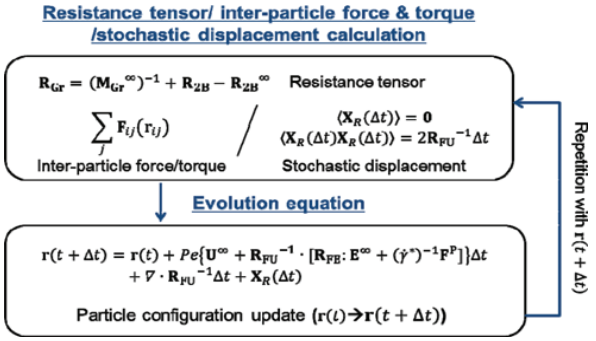

Fig. 9. Illustration on particle configuration update with respect to resistance tensor, inter-particle force torque and stochastic displacement calculations [45].

It can also resolve the dispersion in incompressible turbulent fluid dynamics which is a workhorse of CFD. There is a machine learning framework that enlarges the prognosis about steady-state turbulence. The eddy viscosity is interpolated with parametric studies of pressure and velocity. Hereafter, the exploration on dispersed particles in multilayer curtain coating is executed.

This evolution has been proved functional with this methodology by speed and accuracy properties. These closure optimizing strategies have progressed with upgrading model selection to get high selectivity and efficiency [47].

2.6 Eulerian Method

The Eulerian based methods have different phases which are based on interpenetrating continuous. The single phase does not come over on other one. There is being established a volume of fraction after implementing it. The Eulerian based methods vary continuously with time and space. There are conservation equation of mass, energy and momentum for each phase. These equations are similar to the constitutive relation of empirical investigation.

The Eulerian-Eulerian method consists of three models that are Volume of fluid (VOF), Mixture and the Eulerian [48].

2.6.1 Optimization of Solid Dispersion

The Eulerian method has a different mode for optimization besides other methods. The methods have a focus on shape optimization. For instance, shape node does not move. The assigned function is proportional to state function with respective grid coordinates.

The one side of shape once assigns a positive value at the grid. It comes to inside of the shape after zero is assigned at the boundary. When there is assigned a negative value, it comes on outside of shape. Whenever the shapes evolve, their function is updated from the state change with every node.
The Eulerian method is identical to the Finite element method. Its operating marks coincide into the fundamental features of Level set method. They can be used for process optimization of solids dispersion in multilayer coating process to produce metallized paper of high quality [49,50].

2.7 Langrangian-Eulerian Point Particle Method

The Lagrangian-Eulerian point-particle method uses both Langrangian and Eulerian frameworks. This is also called a Hybrid method. When the boundary conditions become different, the Chimera mesh is a suitable tool for analyzing large solid dispersion. The two sets of meshes are known as Langrangian and Eulerian ones. The moving boundary is surrounded by Lagrangian mesh. The solid dispersion in the region is characterized by the Euler method. When the two meshes do overlapping, a transition region is formulated. The meshes that are in Eulerian mesh are computed as flow variables. Chimera mesh is able to interchange the solid dispersion value using an overlapping zone. It is cost effective in comparison to the mesh regeneration method. While, it is capable of measuring high solids dispersion. A comparison between the experimental and theoretical indicates that they are well expressed. The slight defect comes to rise from errors generated from the coupling of two meshes. There generates some difficulty with digging a hole on Eulerian mesh when the geometry structure is moving [51].

2.8 Large Eddy Simulation Point Particle Approach

The Large-eddy simulation can be performed for the inspection of solid particle interaction in the continuous phase of multi-layer curtain coating process. There is a finding that fluid dynamics (Solid dispersion) is observed from a theoretical correlation. This correlation is derived from filtering the Reynolds Averaged Navier Stokes and continuity equations. When this Large-eddy simulation method is applied on the continuous phase to know the rate and nature of solid dispersion, the empirical correlation of solid dynamics needs substantial parameters. For example, pressure gradient, phase's velocities and mass [53-55].

2.8.1 Combined Discrete Element Method and Large Eddy Simulation Method

This combination of the Large eddy and Discrete

element method can deploy the intense exploration of solid particles dispersion. In this mathematical technique, the Hertz-Mindlin approach related to Johnson-Kendall-Roberts cohesion takes on mathematical simulation. The effects of various particle-particle energy interactions cause turbulence. Consequently, it advances the analysis on solid particles interaction in close channels and regions of high concentration [56].

2.9 Discrete Element Method

A Discrete element method is a numerical tool which is in a position to integrate the dispersion processes most accurately. It is also called the distinct element method. There are granular particles that move in various industrial processes. There is a specific need to store and transport them. The CFD coupled with the Discrete element method demonstrates these complex solid dispersion processes effectively beside other mathematical correlations and models. CFD-DEM coupling designates the strength of the open simulation source. It also guides the installation and setup processes. These software tools highlight the solid dispersion in multilayer curtain coating through its functional framework.
The method can expedite the molecular dynamics from rigid elements. Local deformation permits the operating condition requirements. Laterally, distinct numbers of models are formulated for solid materials in discrete element method [57].

The discrete element algorithm is based on conceptual straightforward. It also remains equally with the computational sequence of DEM. This mathematical model boundary gets updated with contact forces act on.

2.9.1 Operating Principle

The high value of dispersion is found in multilayer curtain coating process. It can be reduced from Weber number, Reynolds number and Web speed. The nature of solid dispersion can be inquired into series of calculation of traced particles. In this numerical method, the interaction of the solid particles monitored from the equation of linear motion closely. Their rotational motions are observed from forces drawn on it. The basic methodology stands up with an assumption that disturbances do not breed from one solid particle to other. These solid particles are not in direct contact at a single time frame. These solid particles are simulated from the governing equations of Newton's second law of motion and their dynamics with time-based algorithms coded in DEM [58].

This feasible calculation aims to provide acute study on particulate processes; (i)- Solid-particles. (ii)-Particle-fluid interaction. Its significance escalated in powder technology. This technology has a key role in mixing, blending, the powder flows and dies etc. Contrarily, the population balance models are not effective in contrast to DEM. Hence, the numerical method has a detailed representation of complex solid dispersion for increasing the yield of metallized paper from multilayer curtain coating process [59,60].

3. SUMMARY

The current review study has identified a study for the solid dispersion study in the multilayer curtain coating process for metallized paper production. The pressure gradient is responsible for surface waves at liquid curtain. It can be minimized from Weber number, Reynolds number and Capillary number. The instability in the production of metallized paper from curtain coating only arises from surface waves, liquids boundary condition, falling heights, substrate speed, meniscus angle and curtain curvature surface. A simple and accurate technique to study the irregularity in solid dispersion for this process is providing an obvious guideline from Langrangian Point Particle, Brownian dynamics (BD), Stokesian dynamics (SD), multi-particle collision dynamics (MPCD) and Self-consistent particle dynamics (SC), Eulerian, Langrangian-Eulerian Point Particle, Large Eddy Simulation Point Particle, Combined Discrete Element-Large Eddy Simulation and Discrete Element Methods.

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5. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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Research Article

Agile Software Development Techniques: A Survey

Marriam Nawaz[1], Tahira Nazir[1], Seema Islam[2], Momina Masood[1], Awais Mehmood[1], and Samira Kanwal

[1]Department of Computer Science, University of Engineering and Technology, Taxila, Pakistan
[2]Department of Computer Science, Comsats University Islamabad, Taxila Campus, Pakistan

Abstract: In this IT era, where there is a race of software development, it is necessary to introduce such types of software development techniques which will help the practitioners to deliver fast solutions. In the past, various traditional approaches were used for this purpose, but now agile techniques are getting more popular because conventional software development approaches are not efficient in managing the changing requirements. The agile software development process is one of the most emerging lightweight software development methodologies, which uses iterative and prototype development approaches to accommodate changes in software requirements. Final software products are delivered to the end-users in short iterations. One of the most noticeable drawbacks of agile methods is their limited courtesy to the structured and architectural design of the system. Hence this development approach will restrict small to medium design decisions only. In this paper, we have performed the analysis of different agile techniques, which will help the readers to understand their positive and negative points and select the most appropriate technique suited to their projects.

Keywords: Agile Techniques, Fast Software Development, Survey, Traditional Development Approaches.

1. INTRODUCTION

It is an organization that thrives on delivering products faster, better, and cheaper ways. Many studies and suggestions have been conducted for the improvement of the software development process. Recently a new software development method has been introduced called Agile Software Development. Agile software development methods are introduced to tackle fast changes in organizational and business needs. Agile methods aid in generating quicker, faster, and efficient solutions. There is a huge difference between ASD and traditional approaches as ASD has more emphasis on a mechanism for change management during project development. In contrast, the traditional approaches have more focus on up-front and strict plan-based control.

The agile software development model is one of the major models of software development that is used extensively by industries, and huge research work is conducted on its methodologies nowadays [1-4]. This approach is used as an alternative to conventional methods of software development as they are document-driven and heavyweight software development processes [5, 6].

Traditional approaches used for software development consist of several phases where for each phase, there is a predefined outcome and target [7]. But this caused a lot of problems like failure of software projects, unable to respond to changing requirement [5, 8] and also piles of documents gathered at the end of project development, But as requirements get changed many times throughout the project, so most of the time we do not require these documents as they are useless, So to cope with these problems Agile Software development model was introduced [9].

The agile software development methodology is based on the idea that software requirements are changing during the whole development lifecycle

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*Corresponding Author: Tahira Nazir <tahira.nazir77@gmail.com>

[10, 11]. This approach provides a consistent way to deal with this dynamic behaviour of requirements as Process ability to iterate itself, having active interaction and communication among clients and development team, flexibility in project management and active involvement of customer during the whole development cycle are the main characteristics of agile software development [12, 13]. Another basic characteristic of the agile method is its provision of communication both among the development team and customers. The word "communication" has a very strong impact in the field of software development as it depicts that people who are working on the same projects will be agreed to the same standards, definition and will share their knowledge, provide information to others and have good coordination in their activities [14, 15]. So this will help to achieve its goal and result in customer satisfaction [16, 17]. Some examples of these practices are collaboration activities like scrum meetings which are held on a daily basis, pair programming, and having face-to-face discussions instead of using formal documentation methods [18, 19]. So as communication is the central property of agile methods and makes it distinct from other traditional approaches [20, 21].

Agile software development techniques are preferred to use in such an environment where there is a chance of sudden change or have to generate a quick reaction to changing requirements by delivering small increments or through continuous incorporation of customers [22, 23]. Several principles of the agile method exist, of which some are based on behavioural and some are based on managerial improvement for software development [24, 25]. Agile software development methodologies are mainly concerned with code development rather than documents driven [26, 27].

There exist several agile software development methods that promote development work, collaboration among team members, and increase the flexibility of processes to make them more adaptable throughout the development lifecycle [28]. These methods include XP (Beck, 1999), [29], FDD (Feature-Driven Development) (Palmer & Felting, 2002) [30], Scrum methodology (K. Schwaber & Beedle, 2002), [31], ASP (Adaptive Software Development (Highsmith, 2000)[32], and DSD (Dynamic Systems Development Method) (Stapleton, 1997) [33]. XP and Scrum are considered as the best agile software development methods[34, 35]. The main focus of Scrum is on software project management to increase their probability of success while XP is more concerned with project-level activities of software development [36]. All agile software development approaches (Scrum, XP, DSD, ASP, FDD, RUP) are iterative and Incremental and have focused on different parts of the software development lifecycle. Among them, some approaches have focused on different practices used for development like XP, Agile Modelling, and pragmatic programming while other concerns with software project management like Scrum approach [37-39].

This study is focused on a comparative analysis of agile software development techniques and their current practices in the industry. These approaches will be examined from the angle of their applicability, strengths, weaknesses, product delivery, standards used for coding, design standards, roles description, and complexity of design and workflow technique. This will lead the reader to find benefits, limitations, and difficulties in the transition from traditional to agile software development. Moreover, this paper explains the worth of employing agile techniques in software development by examining its various methods. The presented research work demonstrates that agile approaches have significant benefits as compared to the existing traditional methods. However, all benefits do not apply to all software projects and situations.

The rest of the paper is divided into the following sections: section 2 explains the traditional approach that is waterfall method for software development, section 3 contains a description of agile software method and its comparison with waterfall method. Further, this section gives a comparative analysis of agile software development methods and section 4 comprises of conclusion.

2. WATERFALL MODEL

The waterfall model is the first traditional model to be introduced. It is a static technique that linearly performs the software development. This approach is very simple to understand and completes one activity before starting another. The waterfall model divides the projects based on process activities like

planning, design, implementation, etc [35]. There is a predefined goal for each development phase. The first phase must be completed before going into the next phase and there is no way to go to the previous one [40]. Testing can only be performed when the whole project is completed [39].

This approach applies to those systems that are more structured and where there exists a small chance of modification after development [40, 41]. It is difficult to reuse and upgrade the software systems developed by using this technique because there exists a coupling between data and code[41]. So if data is changed then code must be modified according to it and this causes to increase the overall cost of a project because the whole process needs to be modified [42, 43].

Figure 1 shows the workflow of the waterfall model:

3. AGILE SOFTWARE DEVELOPMENT

The word Agile states 'moving fast' or 'quickly accepts changes'. It is a lightweight and practice-based technique used widely for software development nowadays. It understands and accepts the idea that handling each project varies from each other, so a more dynamic approach for modeling is required that can be tailored according to the needs of different projects [42, 43].

Instead of following a single long process for the development of projects Agile methodology divides the development cycle into small chunks called increments [44]. After completion of each increment, it is delivered to the users for their verification. It follows the iterative approach and the final product contains all required features of users [45]. Figure 2 presents a graphical representation of the Agile Methodology. Table.1 shows the basic principles of the agile technique[47]. Table.2 shows the comparison between the traditional approach and agile methodology.

3.1 Extreme Programming (XP)

XP is one of the first agile methodologies which are proposed to improve the quality of software. It is a lightweight technique that provides a quick response to the evolving requirements of users and supports a more iterative and well-planned method of software development. It contains a small team of developers and provides an intense level of interaction between the development team and client organization in the whole development

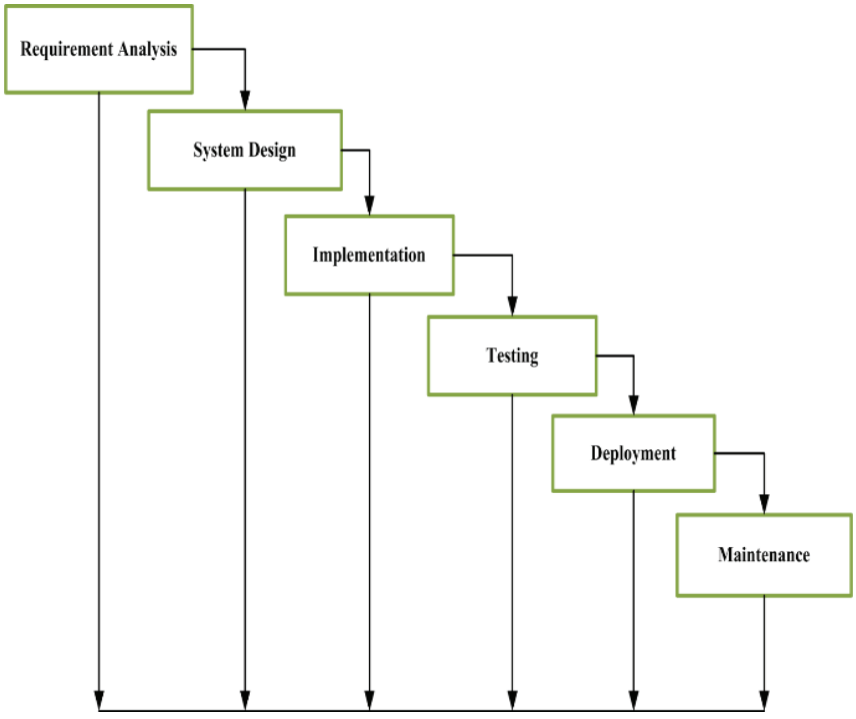

Fig. 1. Waterfall Model Development Lifecycle [41].

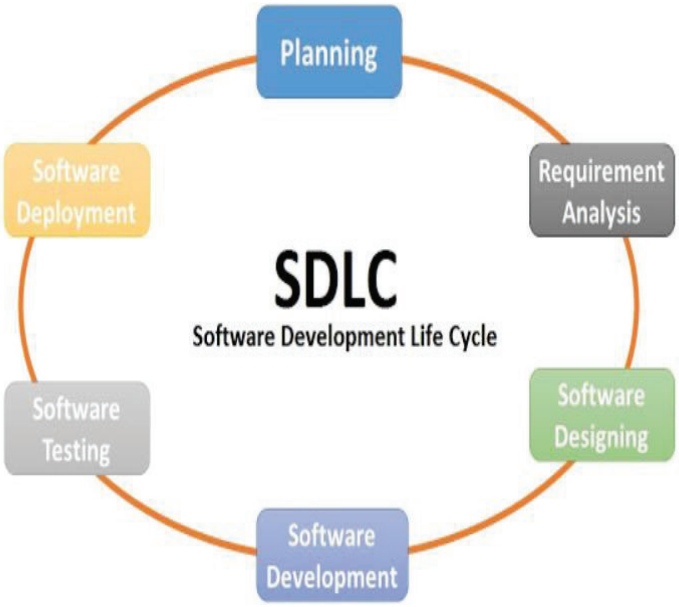

Fig. 2. Agile software development lifecycle [46].

lifecycle.

Being a type of agile methodology it provides fast "releases" in small cycles of development which not only results in increasing the production rate but also provides the points where evolving requirements of customers can be facilitated [48]. The name for this methodology comes from the idea of taking the different elements of traditional approaches to "extreme level". It addresses the different phases of the software development lifecycle like analysis, design, implementation, and testing phases with novel techniques that will cause to raise the quality of end product [49].

The basic principle of XP is to organize the people in such a way to improve the quality of end products and reduce the cost of accommodating the varying requirements of users by following multiple small phases of development [50]. Figure.3 explains the core practices which are used in XP:

Distinguishes features of XP [51] are as follows:

- Story Cards: Users define requirements as story-type scenarios, which are then presented in the form of story cards. Each story card is then further divided by developers to break them into smaller tasks. These smaller tasks are then prioritized with the help of customers for implementation.
- Simplicity: XP works with designing the simplest product to meet the basic needs of users. It is based on the principle to only develop what is demanded in the given requirement. Further functionalities are added to the product according to users' needs.
- Feedback: At the end of each release, proper feedback is obtained from the customers, and the next level of iteration is based on this feedback. In XP, for efficient feedback, small loops of design and implementation are built with the help of a pair programming technique and a test-oriented development method.
- Test-Driven Development: Extreme programming uses a test-oriented development technique in which test cases are pre-written before actual code implementation. Testing is used throughout the process of XP.
- Refactoring: It always encourages finding the

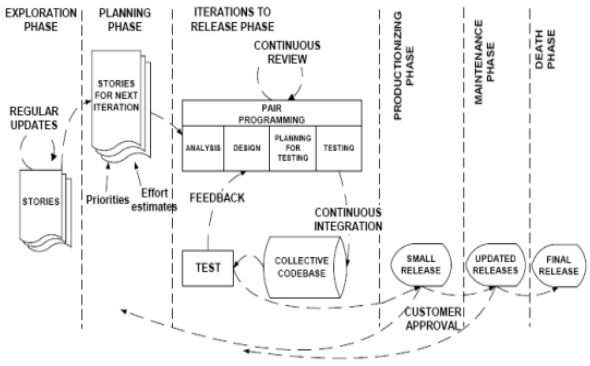

Fig. 3. XP workflow [45].

Table 1. Basic principles of agile methods

1	The main objective is the satisfaction of customers through fast and early delivery of valuable features.
2	No matter which phase of development you are, it must be able to accept and accommodate the changes.
3	Increment should be delivered quickly within weeks or months.
4	Strong communication between the development team and the customer organization.
5	Must be able to provide sustainable development to every stakeholder, whether he is a developer, customer, or sponsor, so that he has a constant pace.
6	The whole team should participate in identifying the ways of becoming more effective and then model their behavior according to this.
7	Involve trusted and motivated individuals in projects.
8	Continuous attention to technical excellence and good design.
9	Simplicity—the art of maximizing the amount of work not done—is essential.
10	The basic measure used for progress checking is working software.
11	The team should be self-organizing to select the best technique for requirement gathering, design, and architectures.
12	Both the development team and customer organization should work closely throughout the development lifecycle.

Table.2 Shows the comparison between the traditional approach and agile methodology

Factors	**Traditional Development**	**Agile Development**
Development Process	Linear	Iterative
Development style	Analytical	Adaptive
Development Orientation	Process-Oriented	People-Oriented
Requirements	Complete understanding of requirements and should be documented and stable.	Discover with the progress of the project. Emergent and rapidly changed.
Project Type	Suitable for large project size.	Suitable for small or medium project size.
Planning Scale	long-standing	short-standing
Style of management	More controlled and command-oriented.	More collaborative and leadership-oriented.
Documentation produced	High	Small
Response to change	Resistive	Accepted and adaptive
Client interaction	Low	High
Team Organization	Structured	Self-organized
Success measure	Plan conformance	Delivering business value

best practices for both design and problem solutions and using them to modify the existing solutions. This will cause to improve the quality of the product.

- Pair Programming: Pair programming is the distinguishing feature of XP, where a pair of programmers works dynamically. This results in immense savage of time and reduces the working load.

The main benefit of using this technique is that it is speeding up the process of development as this approach gives the right to the developer to fix a fault in code when it is detected. Standards related to development and designs are defined globally so that the whole team follows the same conventions. This technique is suitable for small size applications that do not need proper planning and specification efforts [52]. It results in cost reduction because it does not include useless documentation and help

the developers to concentrate on their basic task and performs better risk management. As simplicity is an important feature of XP so it creates more high-quality and faster products and contributes a lot in increasing the robustness of products. At the same time, the main limitation of this technique is that it does not take into count planning or measuring Quality Assurance of design and coding [53]. As it involves pair programming, so there is a huge chance of duplication of data. And it is a code-centric technique and can be irritated in large projects.

3.2 Feature-Driven Development

Software features are the basic focus of this approach because these features are the main driver of the whole development lifecycle [54]. This method is different from other techniques of agile development because the planning of the whole project and upfront design is its basic concerns. It has a basic five stages [55].

3.2.1 Develop an Overall Model

FDD approach is different from XP and Scrum because it demands team effort at the beginning of the project for completely understanding the main structure of the problem under consideration by developing its object model. The basic reason for building this model is to get a good idea and a shared understanding of the project. It captures the following things:

- Requirements of users
- Assumptions of users

3.2.2 Build a Feature List

Based on the first activity, a list of features is defined in this phase. Functional requirements are divided into smaller activities where each activity will deliver some business value to users.

3.2.3 Plan by Feature

A complete formal team is involved in this phase which consists of a project manager, head of the development team, and chief programmer. A complete plan is prepared here to determine the order in which features will be developed. The plan is prepared based on the priorities of the customer, dependencies between modules, risk, and complexities. Completion dates are also finalized here.

3.2.4 Design by Feature

All design packages like sequential diagrams class diagrams are defined here by the chief programmer. The sequential diagrams are developed by a group of people, but class diagrams and object models are defined and developed by owners of the class. Feature requirements are modified here with the help of domain experts.

3.2.5 Build by Feature

Here all classes and methods which are outlined and designed in the design phase are practically developed by developers and are checked and inspected for defects by using unit testing. Figure 4 shows the lifecycle of FDD.

The implementation work of all features is performed in parallel and each team has its owner which makes it distinct from XP. This approach is well suited to the projects of large size and five stages of the process allow you to perform the work in a better and disciplined manner [57]. It uses a predefined standard for implementation of the project, so it makes work easier for developers [58]. This technique does not perform well for small team sizes and the success of the project is dependent on chief programmers [59]. No documentation is available in written form in this methodology.

3.3 Scrum

Scrum is one of the iterative agile software

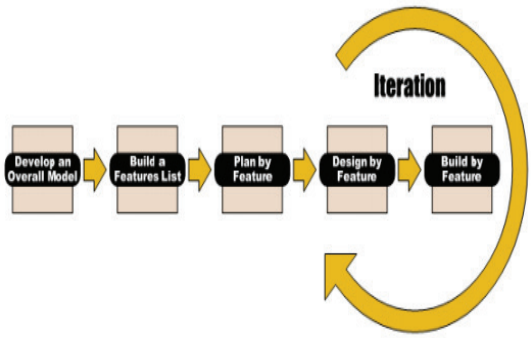

Fig. 4. FDD workflow [56].

development technique which is used for management of software product development [60, 61]. The main principle of this approach is to enable the development team to work as a unit and achieve common goals of the organization, enable the development team to self-organize and work at physical co-location where discipline and face to face communication of all team members are involved [62]. It is the responsibility of the scrum team to define organizational goals and then give their best to meet them.

3.3.1 Documents and Artifacts

Scrum team generally produced three main documents and artifacts these are Sprint Burndown chart, the Sprint Backlog, and the Product Backlog.

3.3.2 Sprint Burndown Chart

Burndown chart is one of the most common mechanisms for sprint tracking used by the scrum team. Burndown Chart is a graphical representation of time versus work left to do, time is often at a horizontal axis and work remaining on the vertical axis.

3.3.3 Sprint Backlog

A sprint is a list of all possible business and technology attributes and a list of all errors and defects that have to be managed and scheduled for the iteration on which we are currently working. The spreadsheet is used for defining Sprint Backlog. In which requirements are represented as tasks. The spreadsheet consists of a short task description region for each task. On basic daily spring, the backlog is updated by a daily tracker that keeps the latest estimate of work complete vs work remaining to complete.

3.3.4 Product Backlog

It is the prioritized queue of all technical functionalities that need to be developed by the development team and evaluate all the defects that need to be fixed. A unique identifier or ID is assigned for each requirement in the product backlog. Product Backlog is also kept in a spreadsheet. An overview of the whole process is explained in Figure 5.

The main power of this technique is that it conducts the meetings on daily purposes to keep the team focused and save both time and money Regular communication and interaction between SCRUM team members helps in attaining efficient completion [63]. In the SCRUM process, frequent testing is conducted which ensures that development work is going well. Regular feedback means changes can easily be tackled before the project grows too large [64]. This technique works well with small teams and can be inefficient due to slacking team members. Sometimes team member is not open to the flexibility it means that removal of one or two team members will cause disastrous damage to the whole team [65].

3.4 Dynamic Systems Development Method (DSDM)

The dynamic systems development approach is purely based on the development of such systems that focuses on the development of that business application whose purpose is to fulfill the needs of the business [67]. DSDM is an evolutionary development approach that uses the timebox and

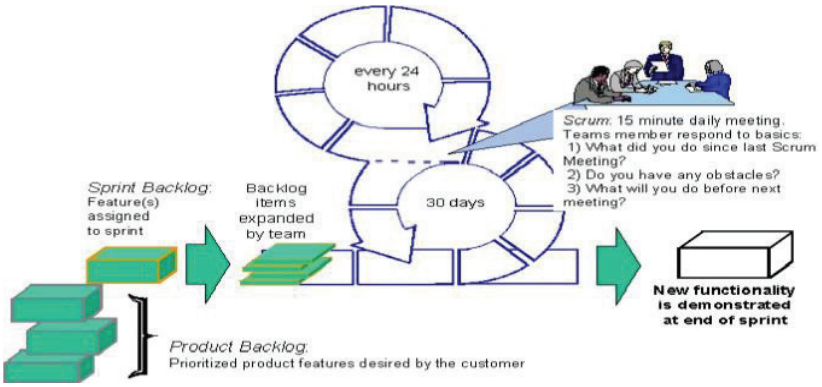

Fig. 5. Scrum workflow [66]

task prioritization approach. DSDM model has very strict standards and very inflexible deadlines for project completion. DSDM testing is an umbrella activity that occurs throughout the entire development life cycle. Feedback is gathered at each stage by the project team and project owner, who shared a physical or virtual workplace for efficient communication. DSDM works efficiently for large or medium-sized projects [68].
Implementation Build and design iteration, Functional Model iteration, Business study, and feasibility study are the few phases involved in DSD methodology.

3.4.1 Feasibility Study

In this stage feasibility report is generated, it is judged that either it will be suitable to develop a product with DSDM or not. Risk and other technical issues are also explored during this phase.

3.4.2 Business Study

System architecture and product outline are prepared at this phase. In this phase, primary business and technical information are studied, the process is defined according to business needs and requirements.

3.4.3 Functional Model Iteration

This is the iterative stage where the actual development starts, at the end of this phase, code prototype and analysis model are prepared.

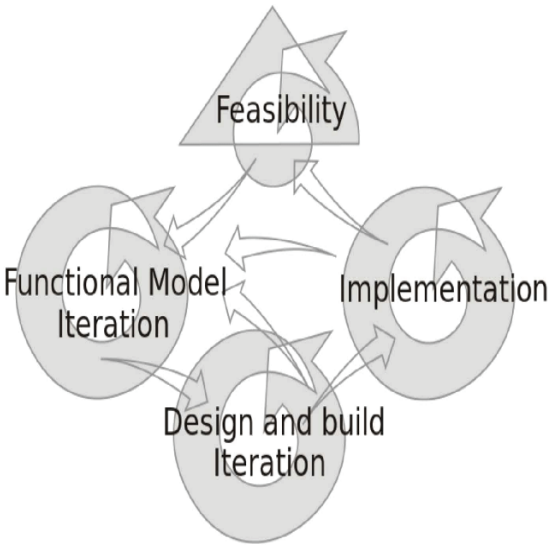

Fig. 6. Dynamic System Development Method Work-flow [69].

3.4.4 Build and Design Iteration

This is an iterative phase where customer requirements are evaluated, and direct communication is conducted with users to know that if end-users need further changes in development or not.

3.4.5 Implementation

This is also an iterative phase in which a completely implemented product is handover to the customers.

In this approach, Users get a stronghold of the software development process. As deadlines are un-flexible so quick delivery of functionality is possible [70]. But this technique is very costly to implement and for the small organization, this method is not suitable. If a user is not a domain expert, then the involvement of the user may be dangerous.

3.5 Crystal Methods Agile Software Development

Alistair Cockburn developed a crystal family (family of methodologies) [71]. Crystal methods are considered "lightweight software development methods" [72]. Cockburn [71] differentiates methodologies, techniques, and policies as follows:

Policies: Organizational standards or roles and regulations.
Techniques: Areas of expertise
Methodology: Practical tools

The Crystal family of methodologies assign a different color to different methods according to their "weight". Crystal yellow, crystal orange, or crystal-clear methods will be used if projects are small one. For safety-critical systems, crystal diamond or crystal sapphire will be used [37]. Crystal family has divided into the following colors: i) Crystal Clear ii) Crystal Yellow iii) Crystal Orange iv) Crystal Orange Web v) Crystal Red vi) Crystal Maroon vii) Crystal Diamond viii) Crystal Sapphire.

Some of the basic properties of the crystal family are as follows:

3.5.1 Frequent Delivery

Frequent delivery of software products by iterative development of the system. By releasing the product in iteration, end users can early identify the problems, and this then allows developers to tackle the problem earlier and ultimately will reduce the time and cost for re-development of a software system.

3.5.2 Reflective Improvement

In this approach, developers take a break from regular software development and explore new ways in which they can better develop software systems, feedback is taken at each iteration for further improvement.

3.5.3 Personal Safety

All people in the team should be allowed to speak freely about their ideas and suggestions, No one should be ridicule otherwise, they will be less likely to speak next time and overall team communication will be affected.

3.5.4 Easy Access to Expert Users

The developer will work with domain expert individuals, the greater the involvement of expert users the greater will be the chance of better product development.

Crystal family methods are suitable for small to very large projects. Face to face communication, consider talents, people, and community are the main aspect of these methods. But these approaches are not suited for medium-sized systems. Customer's unavailability can also degrade the performance of these methods.

3.6 Lean-Agile Software Development

Among all other agile software development techniques, the lean-agile methods are one of the most strategically focused methods. The main goal of this method is to develop the software system in one-third of the time with less budget and less amount of workflow.

Basic principles of LEAN agile software development are as follows [73]:

- Eliminate waste
- Respect people
- Optimize the whole
- Build quality
- Deliver fast
- Defer containment
- Create knowledge

By following this technique, the cost of the software development system will potentially reduce if elimination of overall efficiency is done earlier [74, 75]. It results in the early delivery of software systems and the efficient decision-making ability of the software development team [76]. The workflow of Lean agile software development is given in Figure.7.

3.7 Agile Modeling

This technique is used for documenting and modelling the software-based system by selecting an approach based on best practices. It consists of different values, practices, and principles which are used for documenting and modelling different software systems. This approach is more flexible and easy to practice as compared to traditional approaches [78]. The main objective of this technique is to document the systems by keeping its amount as low as, it is possible [79]. Different types of cultural issues exist, but they are resolved by encouraging and providing proper communication among team members [80]. This technique is used as an addition to other approaches of agile development like Scrum, XP, etc [81]. Figure 8 shows the lifecycle of agile modeling.

This approach helps to better maintain the significant documentation of the system. It provides a better resolution of cultural issues by providing good communication among team members [81, 83]. But it cannot provide a good result with poor modelling techniques and complex with large team size if proper tooling support is not available.

3.8 Adaptive Software Development

The adaptive software development (ASD) technique has emerged from the rapid application development approach. Different phases of this technique like speculate, collaborate, and learn are introduced to replace the traditional approaches used for software development [84]. These

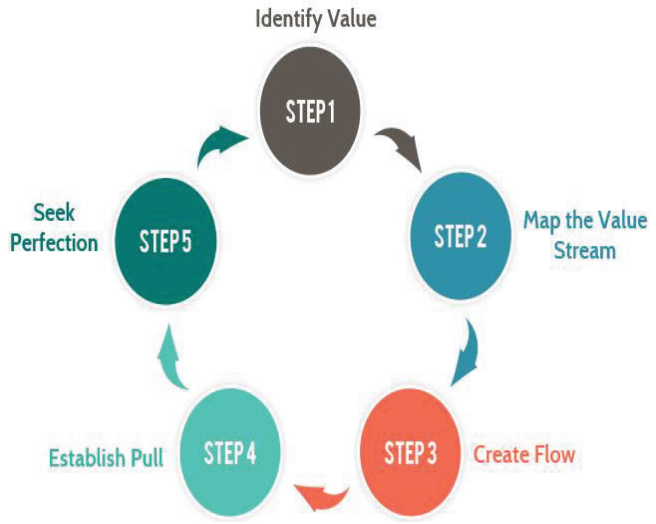

Fig. 7. Lean-Agile Software Development[77]

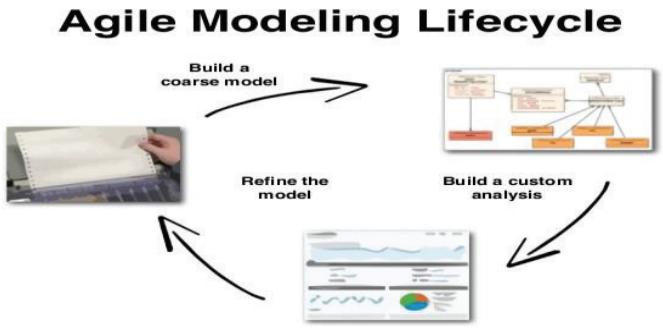

Fig. 8. Agile modelling lifecycle [82].

techniques are adaptable and can accommodate the changes easily in an unstable environment. Mission-focused, iterative in nature, provides tolerance to change, feature-based approach, and risk driven are basic characteristics of ASD [85, 86].

ASD consists of three phases:

3.8.1 Speculate

In this phase, the project is initiated, and all risk-driven plans are developed here. The basic motive of this phase is to completely understand the requirements of users so that the programmer can develop an understanding of the nature of the system under consideration. The success of this phase depends on bug identification and user reports for better guiding the project.

3.8.2 Collaboration

The parallel development of different components is performed in this phase. Proper customer and team collaboration are very important for the successful execution of this step which requires effective communication, creativity, and co-operated teamwork. For efficient requirements gathering JAD (joint application development) approach is preferred here. Instead of getting information about design details, code structure, or testing techniques 'collaboration' among developing team and client organization is the basic concern of this phase.

3.8.3 Learning

In this phase, all quality-related reviews are performed, and the newly created version of the project is made visible to users outside of the

Fig. 9. Adaptive software development Cycle [87].

development organization. Several bugs and user reports are produced here. Component's testing is performed thoroughly here. Figure 9 presents the flow of Adaptive software development.

All these phases show the dynamic and evolving nature of ASD which has replaced determinism with emergence [88]. This method is good to change adaption but as there is a fixed time of development so much pressure on the development team.

3.9 Kanban

This approach is gaining popularity in the field of software development. It provides a way to show and limit the progress of work during the development lifecycle. Its main focus is on doing proper scheduling of work so that product is timely delivered to customer organization [89, 90]. So the Kanban approach is responsible for the management of product development by ensuring its continual delivery to users without having to put a burden on the development team [91, 92]. Figure 10 shows the workflow of the Kanban methodology:

Distinguishes features of Kanban methodology are as follows:

3.9.1 Kanban Board

It is a tool used for visualizing the workflow of the project. It divides the work into different categories which are as follows:

- Backlog
- To-do
- In progress
- Done

3.9.2 Maximizes Productivity

By dividing the work into different groups this approach results in optimizing the workflow. It increases team productivity by minimizing idle time.

3.9.3 Continuous Delivery

This methodology is based on the continual releases of software increments rather than delivering the batches of functionalities.

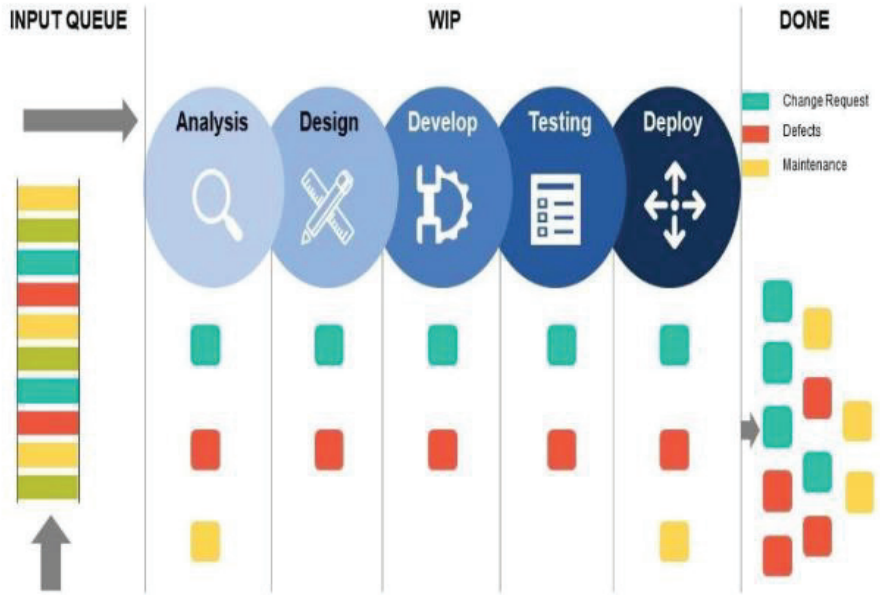

Fig. 10. Kanban workflow[93].

Table 3. Comparative analysis of agile methods in term of process

Factors	Scrum Methodology	Extreme Programming	Feature-Driven Development	Kanban Approach	Dynamic System Development System
Design Standards	Use complex design principles.	Use simple design and coding standards.	Use simple design approaches.	Guaranteed to reduce the waste by limiting the work in progress.	independent Framework for developing and implementation
Roles Description	Roles are predefined.	Roles are not predefined.	Roles are predefined.	Roles are predefined.	Roles are predefined.
Complexity of Design	Design complexity is high	Low design complexity.	Low design complexity.	Simple design.	Simple design
Workflow Technique	Work in iterations. Sprints are produced.	Does not work in iteration rather follow the task flow approach.	It is an incremental and iterative approach. A set of features is delivered.	Works in small iterations.	Iterative delivery of functionality.
Technique for Requirements Management	Product and sprint backlog is used for managing requirements in term of artifacts.	Story cards are used for requirement management.	Manage user requirements by building an object model of them	Kanban Boards are used for requirement management.	Timeboxing and Moscow principle is used.
Product Delivery Approach	Sprints are delivered on a defined time.	Continuous Delivery	Continuous Delivery	Continuous Delivery	Continuous Delivery
Standards used for coding	Coding standards are not defined.	Use defined coding standards.	Development practices are defined in advance.	Coding standards are not defined.	Coding standards are not defined.
Testing techniques	No formal techniques are defined for performing testing.	Use several testing techniques for product auditing like acceptance testing.	Use standard testing techniques.	At the end of each increment or work product testing is performed thoroughly.	Use standard testing techniques.
Changes acceptance	Changes are not acceptable in sprints.	It can accept changes at any phase of development.	It can accept the changing requirements of customers easily at all levels.	I can accept the changes at any time.	I can accept the changes at any time.
Process Owner	Scrum Master	Team Ownership.	Each class is owned by a class owner who works under a chief programmer.	Team ownership	Team ownership

Table 4. Comparative analysis of agile methods

Factors	Scrum Methodology	Extreme Programming	Feature-Driven Development	Kanban Approach	Dynamic System Development System
Customer Involvement	The presence of a customer on-site is not essential.	On-site customer presence and interaction are compulsory.	For the early two phases of FDD, customer involvement is mandatory.	Not essential for the on-site availability of customers.	It is a vendor or a customer independent approach.
Project director	Scrum Master	Extreme programming Coach	Project Manager	Teamwork	Teamwork
Collaboration among Team	Cross-functional teams	Self-organized teams	Teamwork.	The team consist of specific resources	Teamwork.

3.9.4 Waste Minimization

In this technique, tasks are only performed when they are needed. So, this approach results in avoiding over-production and wastage of time. Therefore, this approach is time-efficient.

3.9.5 Limits Work in Progress

Limiting the work in progress is the basic focus of this technique which optimizes the workflow of the system according to its capacity level.

4. COMPARATIVE ANALYSIS

In the presented work, we have discussed the detail of different agile development techniques. Moreover, the features and various phases of all approaches are also described.

In this section, we have presented a detailed comparison of agile techniques according to 3 p's (people, process, and product) of project management. Table 3 shows the comparison of all agile approaches concerning the process and Table.4 demonstrates the assessment of agile techniques from the perspective of people involved. After a detailed discussion of all agile techniques, we have selected different factors, which are used to perform the comparison of all approaches And for the third 'P' of project management, that is 'Product', it has been found that all agile methods apply to products of small and medium sizes.

5. RESULTS AND DISCUSSION

Software development approaches have been changing since the 1970s. To overcome the problems of traditional approaches, agile methodologies are introduced as these are lightweight in nature and help in accommodating the changes easily. In this paper, we present a comparison between traditional approaches and agile techniques used for software development. A comparison of agile methodologies is also performed in detail to highlight their various aspects. This study will help the readers to make a sense out of numerous agile techniques and can decide on which method is most suited to their problem. A major drawback of agile techniques is their inability to be used for big projects.

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Research Article

A Comparative Analysis of Mobile Application Development Approaches

Mohamed Abdal Mohsin Masaad Alsaid[1], Tarig Mohamed Ahmed[1,3], Sadeeq Jan[2*] , Fazal Qudus Khan[3], Mohammad[2], and Amjad Ullah Khattak[4]

[1]Department of Computer Science, University of Khartoum, Khartoum, Sudan
[2]Department of Computer Science & IT, University of Engineering and Technology, Peshawar, Pakistan
[3]Department of Information Technology, King Abdulaziz University, Jeddah, Saudi Arabia
[4]Department of Electrical Engineering, University of Engineering and Technology, Peshawar, Pakistan

Abstract: Over the last decade, there has been a significant increase in the development of mobile applications. The performance of the developed applications depends largely on the development approaches. There are two widely used approaches: (1) native, where the application is targeted and developed for a specific platform, (2) cross-platform, where the developed application runs on multiple platforms. This paper aims to address the question of which approach should be used in various scenarios. We have performed a detailed comparison of the two approaches by developing a mobile app using both approaches. Experiments are performed using Android and iOS, the two most well-known mobile Operating System. The criteria of deciding the best approach include performance, usability and support. Our results show that both approaches are viable depending on the requirements and type of the application to be developed, with native having an edge. Guidelines are presented at the end to help the developers in choosing the best approach. The fundamental differences and advantages of each approach are discussed.

Keywords: Mobile Applications, Native, Cross-platform, Development, Programming.

1. INTRODUCTION

In the age of technology, smartphones are the most widely used electronic device, used by billions of people. They have become a key part of our daily lives. But what makes a smart-phone "smart" is the functionality it provides beyond phone calls and texting, such as touch screens, GPS, camera, biometrics, and that they have fully capable operating systems that can run all sort of applications, like email, browsers, banking, health care, games and much more. There exist several smart-phone platforms, however, Android and IOS have been dominant, with their market share being 87.7% and 12.1%, respectively [1].

Android is an Open source mobile operating system that is developed and maintained by Google and is used by many smart-phones manufactures. IOS is a closed source mobile operating system developed by Apple company for their own devices. In the process of making a mobile application, one of the first decisions to make is choosing the target platforms and the technology stack to use for development. For most cases targeting Android and IOS is the way to go. The ideal situation would be that both platforms can run the same source code without compromising performance, support or usability.

With two major operating systems, it makes sense to target both platforms when developing an app that needs to target almost all users. There are two General approaches in mobile application development, native and cross-platform. Native application development involves using the

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*Corresponding Author: Sadeeq Jan <sadeeqjan@uetpeshawar.edu.pk>

supported languages and APIs directly, Native development in Android is done with Java/Kotlin and in ISO with Objective-C/Swift. Cross-platform solutions are made to achieve the goal of "write once run everywhere", the same source code runs in multiple platforms without the need to use platform-specific code (at least for most cases) and is usually done by implementing an abstraction layer upon wish the cross-platform code runs. Some notable cross-platform mobile solutions are React-native, Unity, Flutter, Apache Cordova, Xamarin and Kotlin-native.

The native application development approach adds to the performance of the mobile apps; however, it comes with additional technical and financial costs during the development maintenance phase. On the other hand, the cross-platform approach is beneficial for the users, however, it has several limitations like inferior performance and lack of support etc. Deciding between the two approaches is a challenging task. In this paper, we aim to facilitate the developers in choosing the right approach for developing their applications. We investigate the advantages and disadvantages of each approach in various scenarios and considers the parameters from different use cases. The main objectives of this research are:

- To assess and compare the two major mobile application development approaches, i.e., native and cross-platform development
- Checking performance, usability and support
- To implement an app using both native and cross-platform and then use various benchmarks to assess certain KPIs
- Recommend which approach to choose in general and depending on the tools and environment available.

This paper focuses on the two major mobile operating systems, Android and iOS. While some cross-platform solutions target more platforms, these two are the most prevalent. We conduct an analysis on native and cross platform, to evaluate certain parameters of interest. Mainly performance, and usability. Most of the focus will be on Flutter, from the cross-platform side and Android, from the native side.

Choosing the development approach or tool for developing Mobile applications has great significance in all aspects of the project, since the investment in a tool that ends up not meeting the desired outcome will waste plenty of resources. This research evaluates the two general approaches of app development, native and cross-platform, and the results can be used as guidelines when making the decision.

The rest of the manuscript is structured as follows: Section 2 describes the background of each development approach. In Section 3, a detailed comparison of the existing studies with our work is presented. Related work is explained in detail. Section 4 provides the research methodology adopted for conducting this study. Results are discussions are provided in Section 5, while Section 6 discusses the best approach based on our proposed evaluation criteria. Finally, the conclusion and future work is provided in Section 7.

1.1 BACKGROUND

1.1.1 Native Application Development

MA Native application is an application developed and targeted at a certain platform. Native apps directly use the platform's main language, tools, framework and APIs, to access and use the resources available for the system, in order to extend the device functionality and offer value. The tools used for developing native apps are often in abundance and have direct support from the platform they are targeting, since it's in the platform's best interest to make the developer's job as pain-free as possible. Tools here mean programming languages, IDEs (Integrated Development Environment), frameworks, libraries, documentation, courses, etc.

Native development also often has a big community, with open-source libraries and frameworks that lets the developer focus on their App's business logic rather than spend time doing something that has already been implemented and tested thoroughly. Given the scope of this paper, the platform means either Android or iOS.

1.1.2 Android Native Development

Android is a Linux-based operating system, meaning its kernel is written with C/C++. The framework layer is written in Java, as well as the APIs. Java is a high-level programming language

that is platform-independent. To achieve this, Java runs on top of what is called JVM (Java Virtual Machine). Java code is first compiled into ".class" files that contain byte code, the JVM then interprets the byte code in machine code that depends on the host platform the JVM is run on [2]. Even though Android uses Java, there are some differences. Given the resource limitations mobile devices generally have, some changes had to be made to the java environment in order to better suit mobile needs. Android has two first class supported languages, Java and Kotlin. Given the popularity and robustness of Java, Google made a choice to use it for the Android platform, and for a while, it was the only one with first-class support. However, in Google I/O 2017, the Android team announced first-class support for Kotlin [3]. Kotlin is a concise modern programming language that can run on the JVM, it also can be compiled to JavaScript code or LLVM native code.

1.1.3 iOS native development

Apple launched the App Store in 2008 with 522 apps. To make these apps, the language of choice for Apple was Objective-C, A C-like programming language with object-oriented features. Apple uses objective see for both of its major operating systems, OSX and iOS, with their APIs also written in it. As Objective-C aged apple decided it needs to find a replacement, so in 2014, during their Worldwide Developers Conference, Apple announced Swift, a new modern programming language for iOS and Mac OS applications. Swift was originally a side project for an apple employee, Chris Lattner. But after a while, it gained interest and attention within the company. Swift is a compiled language that uses the LLVM and Objective-C runtime, meaning it can leverage and interact with existing Objective-C code, which allows Swift to directly interact with the iOS framework APIs [4].

1.1.4 Cross-Platform Application Development

The cross-Platform purpose is to solve the problems caused by the fact that native applications are platform-dependent, by making an abstraction that works in more than one platform. To achieve this, there are different approaches categorized into web, hybrid, interpreted and generated apps [5].

1.1.5 Web Approach

Apps that are browser-based web apps take advantage of technologies like HTML and JavaScript to make platform-independent apps. Web apps depend on the browser they run on, which renders the HTML and interprets the JavaScript. This approach means any platform with a browser can run them. Web apps have limited access to the underlying structure of the platform, since they depend on what the browser exposes as capabilities. They also have to be downloaded each time they are used, since there is no installation process [5]. Figure 1 depicts the interaction of the mobile apps with the browser and backend.

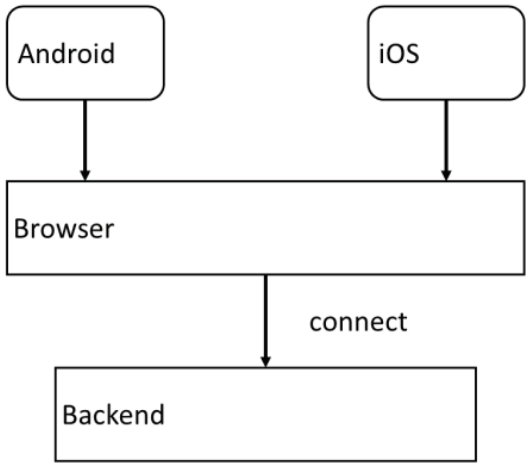

Fig. 1. Web approach

1.1.6 Hybrid Approach

As depicted in Figure 2, Hybrid apps are a mid-ground between Web Apps and Native Apps, they use the native browser, like UIWebView in iOS and WebView in Android, to run web pages. Their difference from Web Apps is that they are packaged and installed on device and have their content saved locally so they don't have to be downloaded each time. They also have access to the underlying capabilities of the platform they work on. An example for them is Cordova [6].

1.1.7 Interpreted Approach

Interpreted apps depend on the underlying tools to interpret the code to platform-specific native code, like some programming platforms as Java does. An example of a software environment that creates

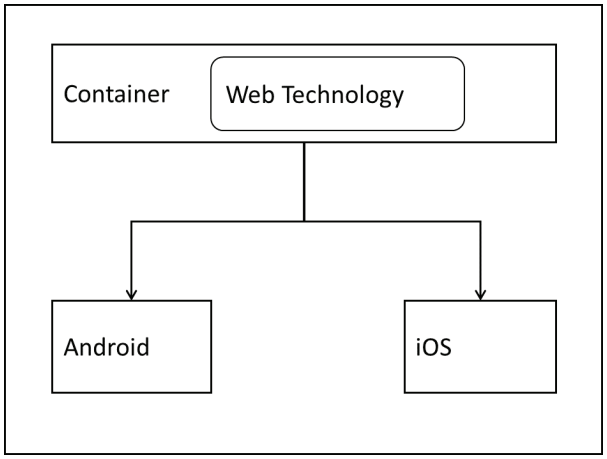

Fig. 2. Hybrid approach

Interpreted apps is Appcelerator Titanium [5].

This approach provides native user interfaces but has downsides like dependence on the tool, such as the case where a new user interface is available in the native platform but is not yet supported by the tool.

1.1.8 Generated Approach

Generated apps are compiled to platform-specific code depending on the target platform, so each platform will have different executable code. An example of a software environment that creates Interpreted apps is Applause [5].

1.1.9 MDA approach

MDA is a design approach that allows development using high-level constructs without having to deal with low-level details. MDA acts as a middleware that abstracts away operating systems, programming languages, etc., allowing focus on the business logic of the product [7]. MDA was defined by the Object Management Group (OMG) [8]. MDA consists of, well, models, as shown in Figure 3.

1.2 Related Work

In the paper "A Comparative Analysis of Cross-platform Development Approaches for Mobile Applications" by Spyros Xanthopoulos from the Aristotle University of Thessaloniki and Stelios Xinogalos from University of Macedonia [5], the authors suggested that the use of native application development technologies imposed "severe constraints", things like multiple development environments and increased maintenance cost were mentioned. The paper evaluates different cross-platform development types, which include, web, hybrid, interpreted and generated apps.

In the white paper "Analysis of Native and Cross-Platform Methods for Mobile Application Development" by Praveen Kumar S [9], the author conducts an analysis on the native and cross-platform by highlighting their respective features, advantages and limitation. The author suggests that in the future the choice of the development approach will become costlier as the process become more complex because of increased mobile device fragmentation.

In the paper "Evaluating Cross-Platform Development Approaches for Mobile Applications" by Henning Heitkötter, Sebastian Hanschke, and Tim A. Majchrzak [10], the authors evaluate cross-platform solutions for mobile, including the most prevalent at the time of the publishing, like PhoneGap and Titanium Mobile. Some of the criteria they used for the evaluation are Look and feel, ease of development Maintainability, scalability and application speed. The authors argue that cross-platform is mature enough that the native approach is not always needed.

In the paper "Cross-platform approach for mobile application development: a survey" by

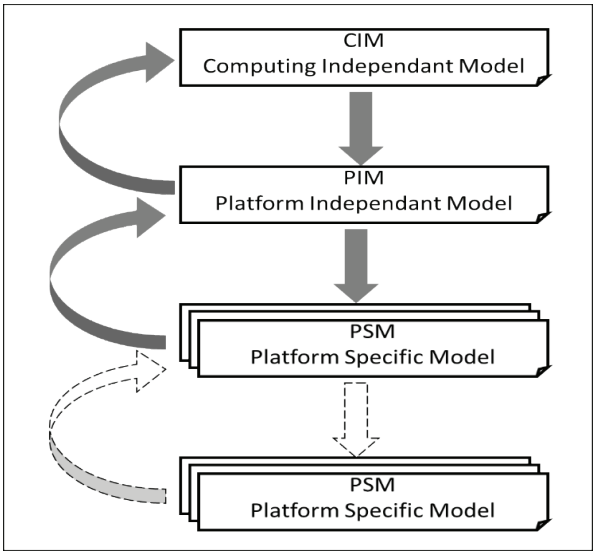

Fig. 3. Model Driven Architecture. Source (Sommer and Krusche, 2013)

LATIF, LAKHRISSI, NFAOUI and ES-SBAI [11], the authors conduct a survey on current cross-platform approaches while it puts emphasis on the MDA (Model Driven Architecture) approach, it also looks into We, Hybrid, Interpreted and cross-compiled approaches. The paper identifies desirable properties of any cross-platform solution, which includes Application Scalability and maintainability, Access to devices features, Security and Development Environment. The paper concludes that a cross-platform solution is favourable to native when time and cost constraints are present, and it recommends a solution using the MDA approach.

In the paper "Survey, Comparison and Evaluation of Cross-Platform Mobile Application Development Tools" [12], The authors discuss the decision criteria for choosing a suitable cross-platform tool. The authors first identify the desired requirements to be met in a cross-platform framework, then they discuss the general architecture of cross-platform development, and finally, they conclude with a survey of several cross-platform solutions (PhoneGap, Titanium, Sencha Touch). The paper concludes that the user experience in cross-platform applications is not as good as with native, but it still offers more potential to reach more users straightforward.

In the paper "Cross-platform mobile development approaches" [13], the authors present a comparison between several cross-platform approaches, including: Runtime, Sources Code Translators. Web-to-native wrapper, App factories and JavaScript frameworks. Some notable criteria present are: The type of the resulting App (Native, hybrid or web), the app size, performance hit (CPU or memory), supported platforms and access to underlying platform APIs. In conclusion the paper emphasizes the need to analyze the desired objective in order to choose a suitable cross-platform tool, and the paper present three factors that help make that choice, which are, programming habits, the importance of native look and feel, and the target OS.

In the paper "Baseline Requirements for Comparative Research on Cross-Platform Mobile Development: A Literature Survey" [14], the authors state how the technical implementations are used to test hypotheses in the computing field, and that research in the mobile field lacked a common baseline. The authors propose a baseline to be used for cross-platform mobile app development research. Their results include which tool to use for each cross-platform approach (like Xamarin. For cross-compiled / Generated), which devices to test on for each major mobile platform and the features to assess. The authors conclude that a signal baseline is not feasible, so they presented several baselines for different types of studies. They also conclude that the approaches and tools change and depreciate over time.

In the paper "Evaluation of cross-platform frameworks for mobile applications" [15], the authors conduct an evaluation of then current cross-platform tool against their native counterpart. The evaluation is done by assigning weights to certain desirable properties like functionality and developer support, and averaging for a final score. The native SDKs got the higher scores overall with the biggest gap appearing in the "Reliability & Performance" category. In conclusion, the authors argue that cross-platform solutions are of value if the performance hit is acceptable for the use case.

There also exist several other studies on native and cross platform development [16-23], however, none of them specifically targeted the comparison of the two approaches as we did in this paper.

2. MATERIALS AND METHODS

The procedure followed in this research include Goal Question Metric, criteria evaluation through investigation and Case Study to further measure different aspects of the competing approaches evaluation through investigation and Case Study to further measure different aspects of the competing approaches.

2.1 Goal Question Metric (GQM)

Goal Question Metric [24] is a top-down approach that works as a measurement mechanism that breaks down the study or project into Goals that need to be reached, Questions to be answered to reach the Goal and Measurements to evaluate said Questions. This paper uses the GQM method and uses the evaluation and case study to answer the

questions. Figure 4 depicts the association of the goal with the corresponding questions and metrics.

2.2 Evaluation

To evaluate the two development approaches for mobile, native and cross-platform, the paper identifies desirable characteristics that will be investigated in each approach. The paper uses some of the criteria identified by H. Heitkötter, S. Hanschke and T. A. Majchrzak [10], to test against selected candidates from each approach. These criteria include:

- License and Costs
- Access to platform-specific features
- Long-term feasibility
- Look and feel
- Application Speed
- Development environment
- Ease of development
- Scalability

2.3 Validation

To keep the validity of the thesis reasonably high, GQM is used to link the goal of the study, to the questions and metrics that help reach the goal, as listed in Table 1. Evaluation through investigation or case study is used where appropriate. While the case study provides actual real-world data, the investigation helps fill the gaps of criteria examination in the case study.

2.4 Case Study

The case study is a mobile app that displays a Timer that shows minutes, seconds and a two-digit fraction of a second. The App is a modified version of Alex Sullivan's timer, that is used for the same performance testing purpose [25].

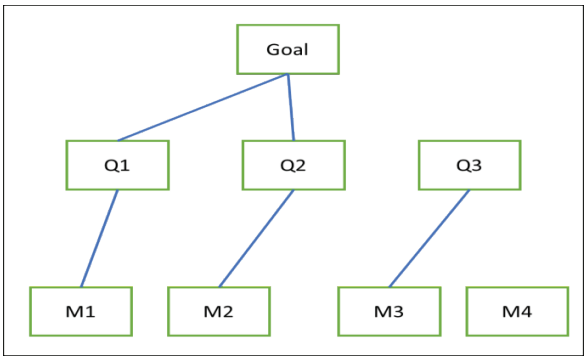

Fig.4. Goal Question Metric Hierarchy

2.5 Instrumentation

Following tools are used in our experiments:

- Android Studio
- Kotlin
- Flutter
- Dart
- Flutter plugins
- Benchmarking tools

2.6 Assumptions and Limitations

The paper won't evaluate all existing cross-platform technologies, and rather it will try to represent major technologies and approaches, choosing one candidate from native (native Android) and one candidate for cross-platform (Flutter). Flutter was chosen because it's a new tool that is gaining traction and because of the lack of studies using it. The performance data will also be affected by choice of and the implementation of the applications, the paper will try to minimize this effect by optimizing the applications to a reasonable degree. Lack of tools and hardware for iOS development limits the evaluation, in the case study, to the results present on android devices.

3. RESULTS AND DISCUSSION

When the App first starts, it saves the initial start time as the difference between the current time and boot time (for Native) or between the current time and time since the Unix epoch, this is called the start

Table 1 Questions and Metrics for Evaluation

Question 1	What approaches are available?
Question 2	What are the characteristic of said approaches?
Question 3	what criteria to assess to choose an approach?
Metric 1	survey available approaches and tools
Metric 2	investigate approaches and choose candidates
Metric 3	evaluate candidates
Metric 4	design set of questions to help choose an approach depending on the use case

time. After that, periodically, the time difference between the current time and start time is calculated and displayed in the timer in the proper format. The rapid, frequent update for the timer is good enough to display performance differences between the two approaches when looking at resource consumption.

The App also showcases the minimum requirements in terms of memory and storage. Since the app functionality is not memory intensive, and neither is it when it comes to storage, then most of the resources are needed for the framework to function. The tests were run on a Samsung Galaxy Note 5 Device, and the profiler bundled with Android studio was used. Figure 5 shows framed screenshots for the App in Android native and Flutter, respectively.

3.1 CPU Readings

CPU readings from the two apps, depicted in Figure 6, demonstrates the overhead that is present when using cross-platform solutions. The native App has a CPU usage of 3% while its flutter counterpart has a 5.5% usage.

3.2 Evaluation

Using the results and observations from the case study, as well as thorough investigation, the native Android and Flutter are evaluated using the criteria mentioned in the previous section.

3.3 License and Cost

Android Open Source Project (AOSP) prefers the usage of Apache 2.0 license [26], which meets their use case for openness and providing more options to manufacturers in means of how to use the platform.

The mentioned license is for Android itself, for development, however, using the Android SDK for native Android development incurs different terms and license [27], which grants a patent to developers to use the SDK, and the sources for the SDK are also available. Flutter is also open-source [28].

3.4 Supported Platforms

Native development, for Android or iOS, involves using the platforms SDK for developing applications targeted only to one platform. Hence native approaches, in the sense of native vs cross-platform, support only one platform.

Flutter builds cross-platform applications that target both Android and iOS [29], which is the main difference between native and cross-platform. The ability to target multiple platforms, with even the infrastructure to even support future platforms, is a very valuable trait.

3.5 Access to platform Specific Features

Native development for Android or iOS gives direct access to all available features of the platform. Flutter, out of the box, gives access to a part of the underlying platform's features in the form of read-made packages, such as, access to biometric technologies like a fingerprint for authentication [30] and storage access for storing simple key-value pairs [31], but it does not have cross-platform

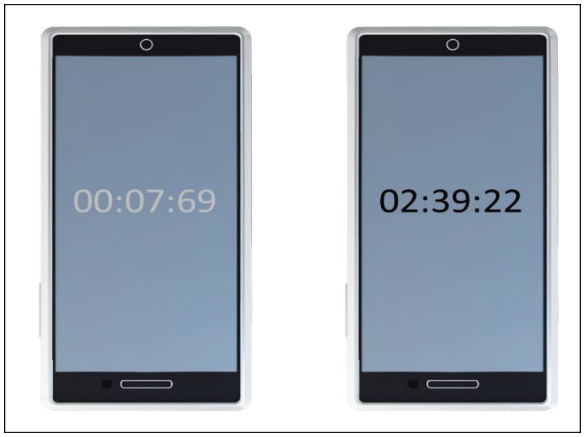

Fig. 5. Timer app in native android (left) &

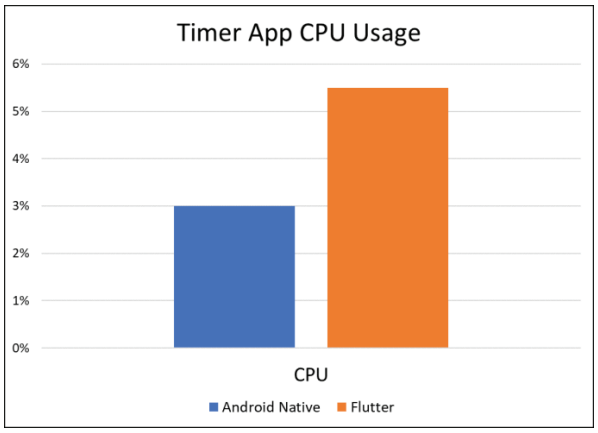

Fig. 6. Cpu comparison

packages for all platform-specific features and APIs, which is a decision explained to made to avoid the issue of "lowest common denominator" [29], which restricts the features of the cross-platform packages to the capabilities available in all platforms (Android and iOS).

For features not available as ready packages, Flutter allows developers to access platform features via message passing, allowing Flutter to message a part of the program that is written in the corresponding native language (Java or Kotlin for Android, Objective C or Swift for iOS), and get a response when the request is handled [32].

3.6 Long Term Feasibility

The native approach, for Android or iOS, is the official approach endorsed by the respective platform, and it's reasonable to assume that it will be supported as long as the platform is still relevant. Even still, some changes can happen in the ecosystem, such as the introduction of Kotlin development for Android [2], and Swift for iOS [33], but the underlying SDKs are still unchanged, and original programming languages can still be used. It can also be seen that each platform is striving to make native development a better experience for developers, which in this case is done by supporting new modern programming languages.

Flutter is a Google product, one of the largest tech giants with around 110 billion dollar earnings in the third quarter of 2018 [34], and also the company behind Android, which is a good indicator for flutters survivability, but it's likely will be ultimately decided with the degree of its success and adaptability, which is too early to tell given how it only recently hit its first stable release [35].

In Conclusion, the native approach is always better supported given that its critical for the platform in question, while the cross-platform approach is affected by the backing of its creators of or the community in case of open-source.

3.7 Look and Feel

Look and feel is platform specific, since users expect applications in a certain platform to have certain look and certain behavior, which is defined by how the native applications look and feel. For Android google use material design [36], while iOS use Human Interface Guidelines [37].

Flutter has widgets, layouts and themes that use Material design for Android and used Cupertino (iOS -like style) for iOS [29], which solves the issue of difference in the look and feel between the different platforms.

The conclusion is that Native has the most authentic look and feel to the platform, but cross-platform solutions like Flutter take a good approach to achieve the desired look and feel across platforms. Also, it can be seen that some applications take the approach of making their applications look and feel the same across platforms, with the intent of making the experience consistent and as a way to focus on the desired experience that the brand needs to convey to customers. In the Later approach, the cross platform can be more appealing in that regard.

3.8 Distribution

Whether it is native or cross-platform, applications can be distributed via the specific platforms app store, as long as they comply with the respective rules and policies.

3.9 Development Environment

Android native development is mostly done using Android studio, the IntelliJ based IDE (Integrated Development Environment) that is reached in features tools. Some of its offerings include things like Instant run, translation editor and APK analyzer [38].

Flutter can be integrated into an android studio; it also has a very straightforward way for SDK installation. Both contribute to a good experience, especially for developers making a move from the Native Android ecosystem.

In conclusion, both approaches give a good development environment, but with the native having generally more tooling available, by the fact it had quite a while to mature.

3.10 GUI Design

Native Android development using Android studio can make use of the build in WYSIWYG tool that facilitates building graphic user interfaces and see the result without the need for time consuming way building and running the application every time a change is done, this is achieved by interpreting the XML files which are used to describe the UI [38].

Flutter does not have a WYSIWYG, but it achieves the desired results in different ways. Flutter uses Dart programming language for both its logic and GUI portions and its uses hot reloading as a means to almost immediately reflect code changes into the running App, and it takes significantly less time to achieve so.

In conclusion, Flutter's hot reload is a very good quality of life feature that does not have a good enough counterpart in the Native approach, despite attempts to give a similar outcome using instant run.

3.11 Maintainability

Although the technologies used in the native affect its maintainability, the fact remains that a native application for both Android and iOS means that two separate code bases negatively affect maintainability. On the other hand, in Flutter and other cross-platform approaches, only one code base exists, which helps maintainability.

3.12 Speed and Cost of Development

Native development, for Android or iOS, requires specific knowledge about the framework, programming language and tools, resulting in doubling the work, increasing either or both of the cost and speed of development.

Flutter is a cross-platform framework, and one of the main points of cross-platform is to run the same code base on multiple platforms. But in reality, other factors can have an effect, and the lowest common denominator problem issue is solved in Flutter by allowing certain functionalities to be implemented separately for each platform. This means two separate implementations, which can become counter-intuitive depending on the type of the application. But nevertheless, the general rule is cross-platform is cheaper and faster. This is supported by the observation that the case study in the previous section took roughly the same time to develop in native and Flutter, while of course, flutter runs on two platforms.

4. BEST APPROACH

Using the outcomes of the evaluation, the following criteria can help decide which approach to choose for development:

4.1 Budget

Budget is the main form factor, the time and resources required vary depending on the nature of the application and the approach used for development. Native apps usually require a developer/team for each OS, and even if it's a single developer/team, there are still two separate code bases, on the other hand Cross-platform apps need one developer/team to get the job done which makes it cheaper. If the budget is not a problem, however, native apps offer uncompromised performance and user experience, making it more favourable in this situation.

4.2 Type of App

The type of the App is a significant factor, typical CRUD apps can be relatively straightforward to make using both approaches, but another kind of apps that require access hardware sensors or device, like a camera app, can become a nuisance to make using a cross-platform approach because the lowest common denominator problem. Native apps, however have direct access to all the APIs that expose device resources, making the approach more ideal for this type of App.

4.3 Developer Background

The developer's previous knowledge can determine how difficult to learn a new development framework, for example, web developers can find themselves at home using cross-platform frameworks that use web technologies or programming languages like react-native.

5. CONCLUSIONS

In this paper, we investigated the two widely used approaches (Native and cross-platform) by developing and evaluating a mobile app. We discussed our results from various perspectives highlighting the advantages and shortcoming of each approach, with native having the upper hand in criteria such as performance and access to platform specific features, and cross-platform showing an advantage in terms of cost and maintainability. We have used the well-known Goal Question Metric (GQM) as a measurement mechanism to breakdown our study into Goals and to answer the Questions for reaching our goals. The cross-platform approach has many established frameworks with different ideas to deliver on the write once promise and Flutter is a promising framework that builds upon the experience gained from the previous frameworks.

The decision of which development approach to use is a costly one, but the answer is not straightforward, it should be decided by the nature of the project, the team developing and the budget. However, based on our findings, native is still the safest approach for mobile application development. In future, the following options can be investigated:

- Address the limitations present in this paper, for example, by evaluation multiple cross-platform frameworks.
- Test different upcoming approaches e.g., Kotlin-native, that promises the best of both worlds, native and cross-platform.
- What if the number of dominant mobile operating systems increase? Is there a point at which native becomes impractical?
- With mobile devices becoming increasingly more performant in terms of hardware, will the performance advantage of the native approach become unnoticeable? Or will the applications become ever more demanding of resources?

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Research Article

Design and Stability Analysis of a proposed Microgrid for on Campus Diesel and Photovoltaic Power Sources

Muhammad Iftikhar Khan[1], and Muhammad Amir[1*]

[1]Electrical Engineering Department, University of Engineering and Technology, Peshawar, Pakistan

Abstract: Pakistan is currently facing an energy crisis that is detrimental to its growth. Due to persistent load shedding by the National Grid throughout the country, the latest trend is tilting towards generating power at localized level through a mix of renewable and conventional energy sources. Such a mixture is referred to as Distributed Energy Resource (DER). Part of such a DER (i.e. solar energy) is free as naturally and mostly available in Pakistan having no degradation problem while providing benefit to the locality. The idea of using a Microgrid for our local power needs morphed accordingly. In this paper, a Microgrid design for our local campus i.e. University of Engineering & Technology, Peshawar, KPK is proposed. Our campus generators are not interconnected. Consequently, even while servicing small load periods, all of our campus's diesel generators run at full capacity at the same time. Such a behavior is uneconomical, unacceptable and the generators run at low efficiency during islanding from the main utility grid. It is seen that behavior as such is endemic throughout Pakistan. Through design and analysis, it is proposed that if a Microgrid of such unconnected resources on campus is formed then during islanding, all generators and our Photovoltaic (PV) systems will economically and efficiently feed their common loads through load sharing. Synchronization, load flow analysis, short circuit analysis, harmonic analysis, transient stability, cost and reliability of our proposed Microgrid is analyzed using ETAP (Electrical Transient Analyzer Program) in this paper.

Keywords: Power, Microgrid, Diesel, Photovoltaic, Islanding.

1. INTRODUCTION

The demand of electricity generation to provide energy for today's modern living is on the rise [1-2]. The electricity demand in commercial side, residential side and industrial side is increasing due to two main reasons, first and foremost is the modern inventions/innovations in electrical appliances to facilitate modern living while the second one is due to increasing population [1]. The generation of electricity from conventional energy resources is too costly and the other major problem is that these energy resources are depleting day by day [2].

Currently, our old installed electricity supply system is not enough to fulfill the required electricity demands of modern populous. The rural and urban side is facing an unexpected problem of electricity shortage due to which they install isolated backup generators in order to accomplish their requirements which is costly and are less efficient [3-4].

The new concept of Microgrid is ascending to solve problems of modern life in which the locally generating resources are interconnected to fulfill electricity demand locally which is an efficient way of supplying electricity to local people [4-5]. The Microgrid works on both grid connected and islanding mode of operations [5]. In the generating resources of such an arrangement, renewable and some conventional energy resources are interconnected in order to form a Microgrid as shown in Figure 1. The Microgrid is mostly used in rural areas where supply of electricity is a major problem [6]. This research work proposes a Microgrid for our local campus i.e. the University of Engineering & Technology situated in Peshawar, KPK, Pakistan using ETAP version 12.6 simulation/ analysis software [7].

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*Corresponding Author: Muhammad Amir <amir@nwfpuet.edu.pk>

This paper shows the analysis of UET proposed Microgrid using ETAP in a single line diagram approach. The analysis includes load flow, harmonic, short circuit and transient stability case studies [8-9]. The harmonic study on one side includes analysis of distortion in voltage and current waveform [8]. On the other, the load flow study analyzes the steady state performance of the power system, analyzes the voltage profile at load buses, cable or transmission line under load, over load of transformer and under load, under excited and over excited generator [10-11]. While the short circuit study [9] is used to calculate all the short circuit current flowing in the power system, when a fault occurs and accordingly selects the protective devices ratings.

Moreover, the transient stability study [10] is carried out to analyze the system when subjected to sudden change in load, generation or a fault.

2. MATERIALS AND METHODS

2.1 Network Configuration & Modeling

The studied circuit in Figure 2 shows a Microgrid consisting of four diesel generators and one solar system. The rating of three generators is 400kVA and one of 60 kVA. The rating of solar system is 15 kW and having output DC voltage of 400 V.

As Microgrid operating in parallel with utility grid is not viable option here because it is an expensive choice due to high prices of diesel fuel. In future, the share of renewable energy resources production is expected to increase in UET, then grid connected mode will be under consideration. When the Microgrid is in islanding mode due to some failure of main grid or load shedding, the Microgrid will feed their common loads.

The Microgrid system frequency is 50Hz. The generation voltage of all four diesel generators and solar system is 400V, which is fed through cable to different departments of UET Peshawar. G1 is taken as swing generator and the remaining three generators are connected to the PV bus.

2.2 Analysis of Microgrid using ETAP

2.2.1 Load Flow Analysis

Load Flow Analysis is carried out to analyze the voltage profile at the buses, the power factor, the flow of active power and reactive power flow between different types of buses [12]. The load flow study is used to analyze the generator, transformer and cable/transmission line working during their load flow analysis of the entire power system.

In the load flow analysis [13], the generator

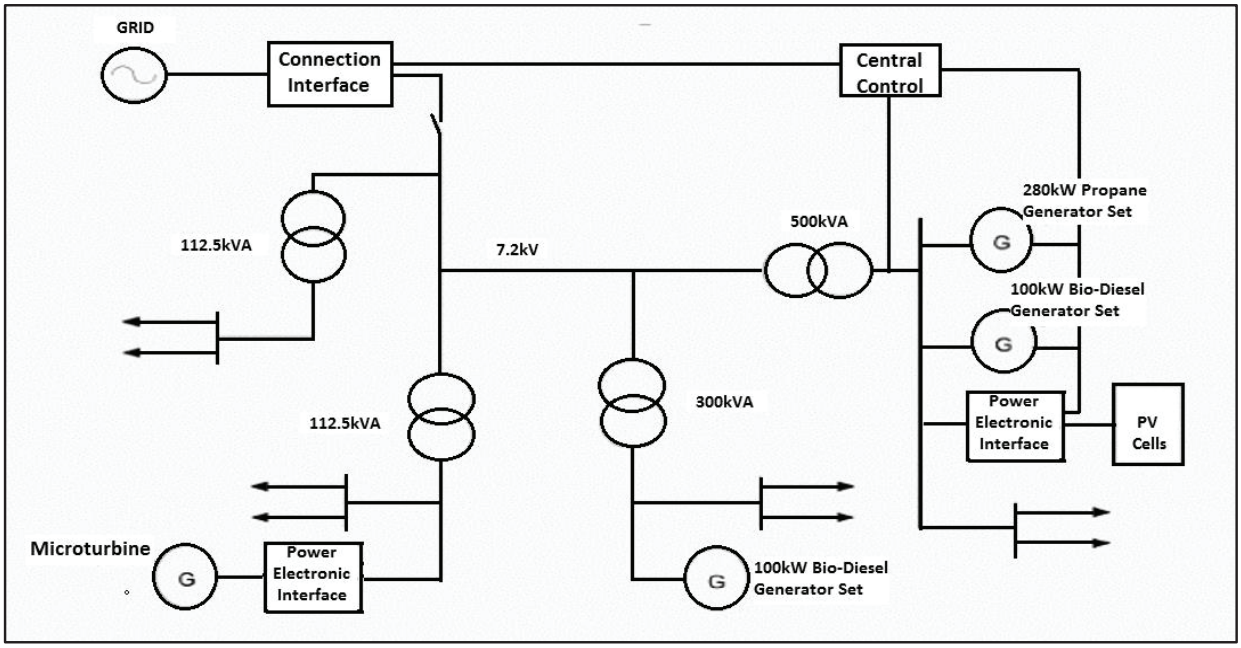

Fig. 1. Single line diagram of Microgrid

under load, over excited, under excited condition and the loading on cable is studied. The proper load sharing of generators is studied and the tolerance in voltage is taken up to the range of +5 to -5 percent.

The iterative method is used for load flow analysis which is Newton Raphson method [14] with 99 iterations of accuracy i.e. 0.0001. Figure 3 shows the load flow analysis results. Table 1 shows operating conditions of generators during load flow analysis. Table 2 on the other hand shows the loading conditions during load flow analysis. Table 3 shows information about each bus in load flow analysis while Table 4 shows all branches information during load flow analysis.

2.2.2 Short Circuit Analysis

Here, short circuit study is used to analyze short circuit current which is the combination of generator and motor current flowing in the overall power system when a fault situation occurs. There are mainly two types of faults i.e. symmetrical and unsymmetrical faults. The analyzed symmetrical fault is a three phase bolted fault, which is known as three phase balanced fault. The resume of a symmetrical fault includes line to ground, line to line and line to line/ground. Figure 4 above shows our short circuit analysis while Table 5 shows the short circuit reading after 3-phase fault occurs on Bus22.

2.2.3 Harmonics Analysis

The harmonic study analyzes the distortion in voltage and current waveform. In ETAP the total harmonic distortion in voltage is denoted by (%VTHD) and the total harmonic distortion in current is denoted by (%ITHD). The presence of harmonics in sinusoidal wave form voltage and current is due to presence of nonlinear devices. Due to this problem of harmonics present in voltage and current wave forms some area of the power system is greatly affected [15]. The frequency components present in the voltage and current wave form are usually integral multiple of the fundamental frequency which are known as harmonics. There are some problems arising due to presence of these harmonics in a power system, including equipment heating, reduction of system power factor and relay malfunction. Figure 5 here shows the harmonic analysis of UET Peshawar Microgrid while Fig. 6 shows the waveforms of voltage harmonics on different analyzed buses.

2.2.4 Transient stability Analysis

Transient stability study analyzes the stability of the proposed power system, when subjected to some sort of disturbance. The disturbance may be on source side i.e. some generators to stop working during some fault situation or the disturbance may be on a load side i.e. the load increases up to very high level or decreases up to very low level. The transient stability analysis of UET Peshawar Microgrid is done by a 3-phase fault which occurs at Bus21 as shown in Figure 4 and this fault is cleared at 0.02s. Figure 7 shows the transient stability of UET Peshawar Microgrid.

Figure 8(a) till 8(f) shows different transient stability graphs of four generators based on separate analyzed parameters while Figure 9(a) till 9(c) shows different transient stability graphs of lump loads. Moreover, Figure 10 shows the frequency stability of our proposed Microgrid under fault condition.

3. RESULTS - COST EFFECTIVITY ANALYSIS OF UET PESHAWAR MICROGRID

The main consideration in the Microgrid operation is economical and optimum generation that can fulfill the dynamic demand keeping intact the quality of the power system. The grid connected Microgrid analysis is not performed because of high prices of diesel oil and lack of high amount of renewable energy. The economic analysis of UET proposed Microgrid includes two operation modes i.e. (a) Without Microgrid and (b) With Microgrid which are discussed here.

3.1 Mode of operation (Without Microgrid)

In this mode, analysis is done without the implementation of Microgrid as shown in Fig. 11, in which all the generators and solar system are working independently when the main grid supply is off due to utility power failure or contingency on the power system. The percent generation of generators is given in Table 6. The percentage

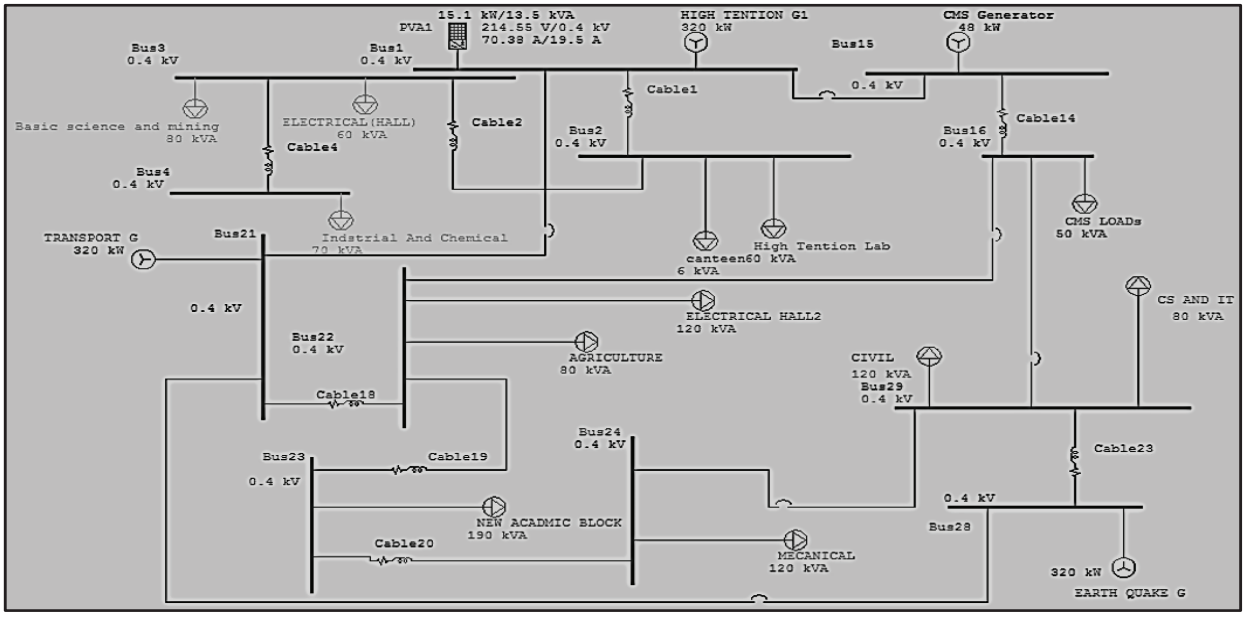

Fig. 2. Model of Microgrid network

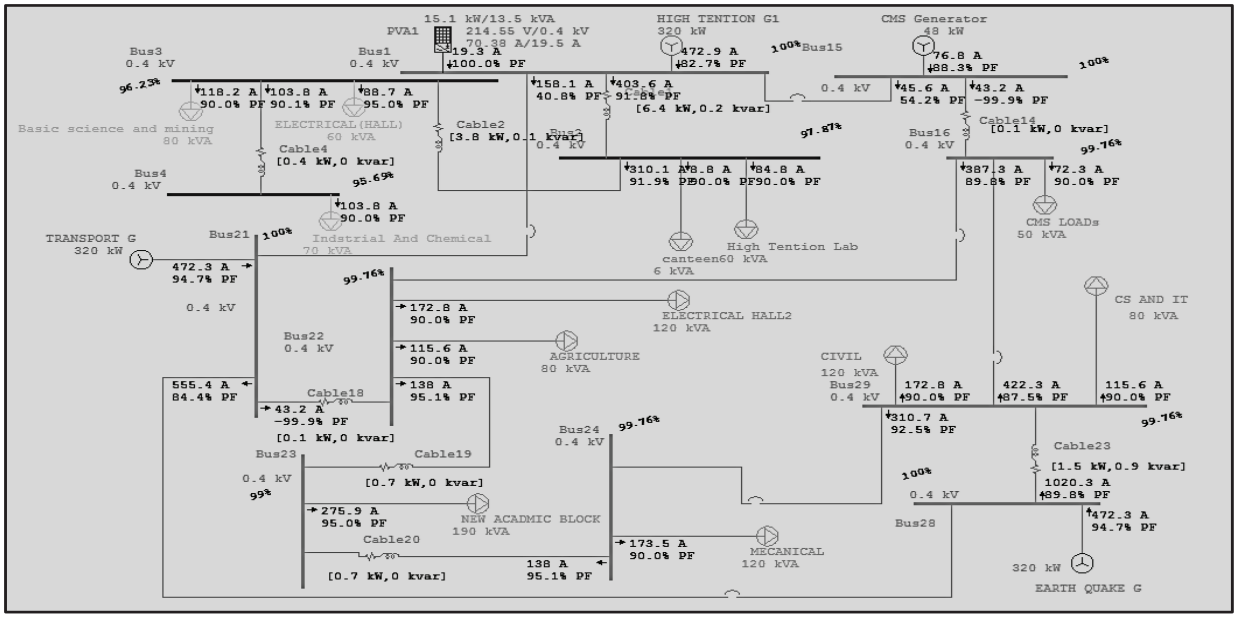

Fig. 3. Load flow analysis

Table 1. Generators operating in load flow

Generator ID	Rating (kW)	Rated (kV)	kW	kvar	Amp	%PF	% Generation
CMS Generator	48	0.4	47	25	76.84	88.29	97.9
Earth Quake G	320	0.4	310	105	472.3	94.74	96.9
High Tension G1	320	0.4	271	184	472.9	82.67	84.7
Transport G	320	0.4	310	105	472.3	94.74	96.9

Table 2. Loading condition during load flow analysis

Load ID	Rating (kVA)	Rated (kV)	kW	kvar	Amp	%PF	% Loading	% V(terminal)
Agriculture Department	80	0.4	71.93	34.837	115.6	90	100.1	99.76
Basic Sciences Department	80	0.4	70.935	34.356	118.2	90	102.4	96.23
Canteen	6	0.4	5.354	2.593	8.774	90	101.3	97.87
Civil Department	120	0.4	107	52.052	172.8	90	99.8	99.76
CMS Loads	50	0.4	44.956	21.773	72.27	90	100.1	99.76
CS&IT Department	80	0.4	71.93	34.837	115.6	90	100.1	99.76
Electrical Department Hall 1	120	0.4	107	52.052	172.8	90	99.8	99.76
Electrical Department Hall 2	60	0.4	56.157	18.458	88.66	95	102.4	96.23
High Tension Laboratory	60	0.4	51.725	25.051	84.76	90	97.9	97.87
Industrial & Chemical Departments	70	0.4	61.938	29.998	103.8	90	102.7	95.69
Mechanical Department	120	0.4	108	52.256	173.5	90	100.1	99.76
New Academic Block	190	0.4	180	59.092	275.9	95	100.6	99

Table 3. Information about each bus in load flow analysis

Bus ID	Nominal kV	Type	Voltage	kW Loading	kvar Loading	Amp Loading
Bus1	0.4	Swing	100	301	211	530.9
Bus2	0.4	Load	97.87	250	111	403.6
Bus3	0.4	Load	96.23	189	82.827	310.1
Bus4	0.4	Load	95.69	61.938	29.998	103.8
Bus15	0.4	Gen	100	47	26.531	77.9
Bus16	0.4	Load	99.76	285	141	460.5
Bus21	0.4	Gen	100	355	206	592.2
Bus22	0.4	Load	99.76	270	118	426.4
Bus23	0.4	Load	99	180	59.092	275.9
Bus24	0.4	Load	99.76	199	81.829	310.7
Bus28	0.4	Gen	100	635	311	1020
Bus29	0.4	Load	99.76	633	310	1020

Table 4. All branches information during load flow analysis

Cable ID	Type	kW Flow	Kvar Flow	Amp Flow	% PF	% Loading	% Voltage Drop	kW Losses	kvar Loss
Cable1	Cable	257	111	403.6	91.81	80.7	2.13	6.397	0.231
Cable2	Cable	193	82.963	310.1	91.89	77.5	1.64	3.777	0.137
Cable4	Cable	62.361	30.013	103.8	90.11	51.9	0.54	0.423	0.015
Cable14	Cable	29.868	-1.531	43.17	-99.87	21.6	0.24	0.073	0.003
Cable18	Cable	29.868	-1.531	43.17	-99.87	8.6	0.24	0.073	0.003
Cable19	Cable	90.64	29.573	138	95.07	34.5	0.75	0.747	0.027
Cable20	Cable	90.64	29.573	138	95.07	69	0.75	0.747	0.027
Cable23	Cable	635	311	1020	89.81	53.7	0.24	1.482	0.894

Table 5. Short Circuit reading for 3-phase fault occurs at Bus22

Short Circuit Report						
3-Phase fault at Bus: Bus22						
Pre-fault voltage = 0.400 = 100.00 % of nominal bus kV (0.400 kV) i.e. 100.00 %of base (0.400 kV)						
Contribution			1/2 Cycle			
From Bus ID	To Bus ID	% V from Bus	kA Real	kA Imaginary	Imaginary/Real	kA Symm Magnitude
Bus22	Total	0.00	2.335	-12.856	5.5	13.066
Bus21	Bus22	2.32	0.227	-0.341	1.5	0.409
Bus23	Bus22	2.83	0.205	-0.455	2.2	0.499
Agriculture Department	Bus22	100.00	0.165	-0.393	2.4	0.426
Bus15	Bus16	2.32	0.227	-0.341	1.5	0.409
CMS Loads	Bus16	100.00	0.103	-0.245	2.4	0.266
Bus28	Bus29	2.32	0.790	-9.645	12.2	9.677
CS&IT Department	Bus29	100.00	0.165	-0.393	2.4	0.426
Bus23	Bus24	2.83	0.205	-0.455	2.2	0.499
Mechanical Department	Bus24	100.00	0.247	-0.589	2.4	0.639
*Bus16	Bus22	0.00	1.738	-11.667	6.7	11.796
*Bus29	Bus16	0.00	1.408	-11.081	7.9	11.170
*Bus24	Bus29	0.00	0.453	-1.044	2.3	1.138
*Bus15	Bus1	2.32	-0.195	-0.107	0.6	0.223
*Bus1	Bus21	2.32	0.588	-4.013	6.8	4.056
*Bus21	Bus28	2.32	0.576	-6.658	11..6	6.683
No AC Decrement (NACD) Ratio = 0.13						

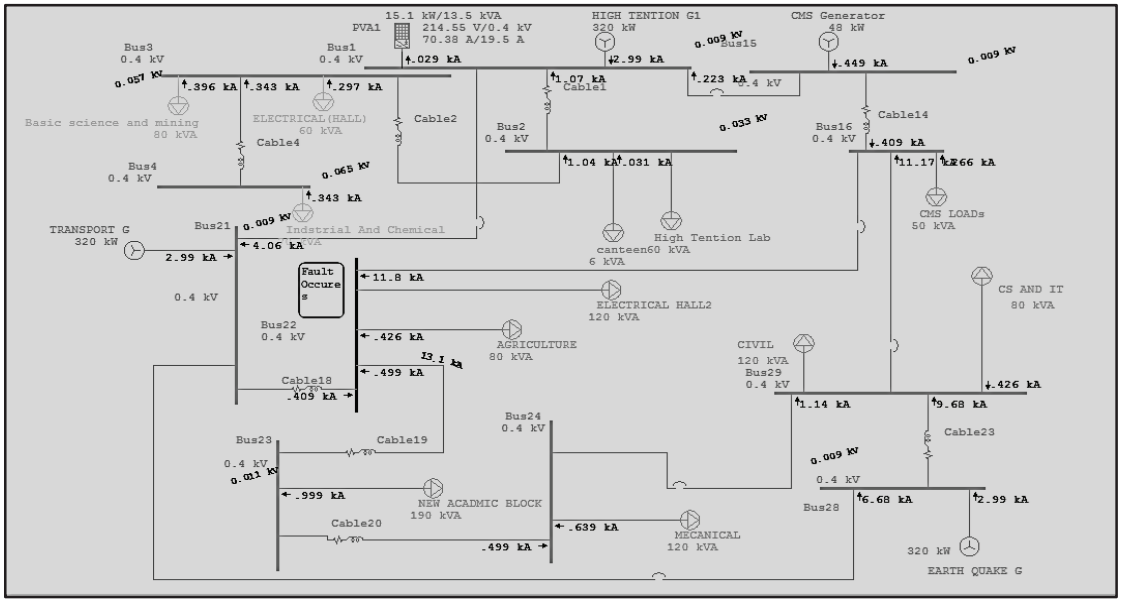

Fig. 4. Short circuit analysis

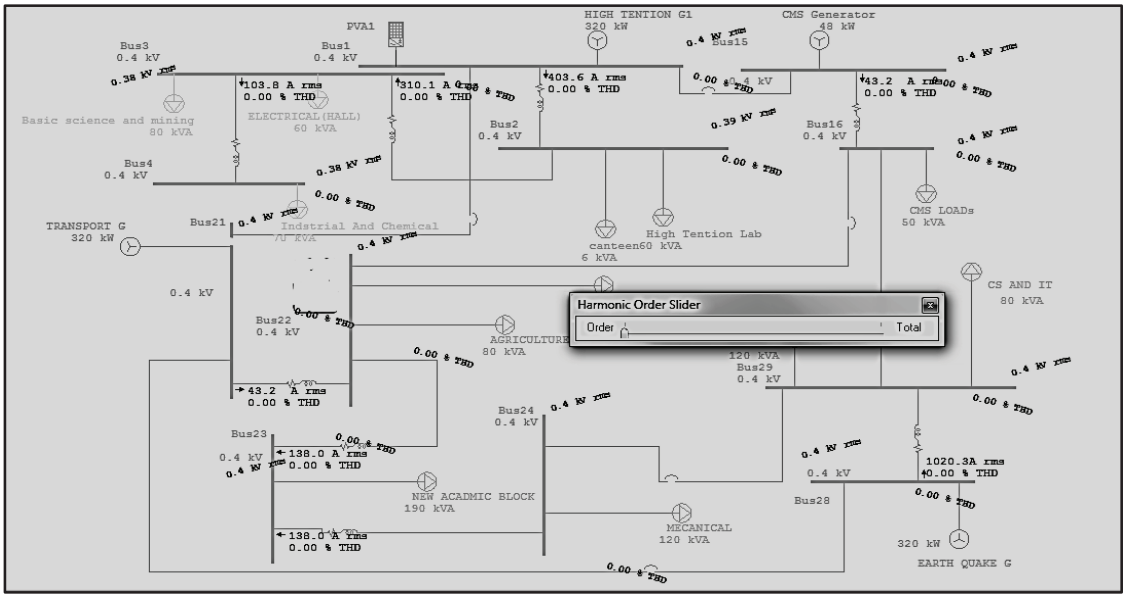

Fig. 5. Harmonic analysis of UET Microgrid

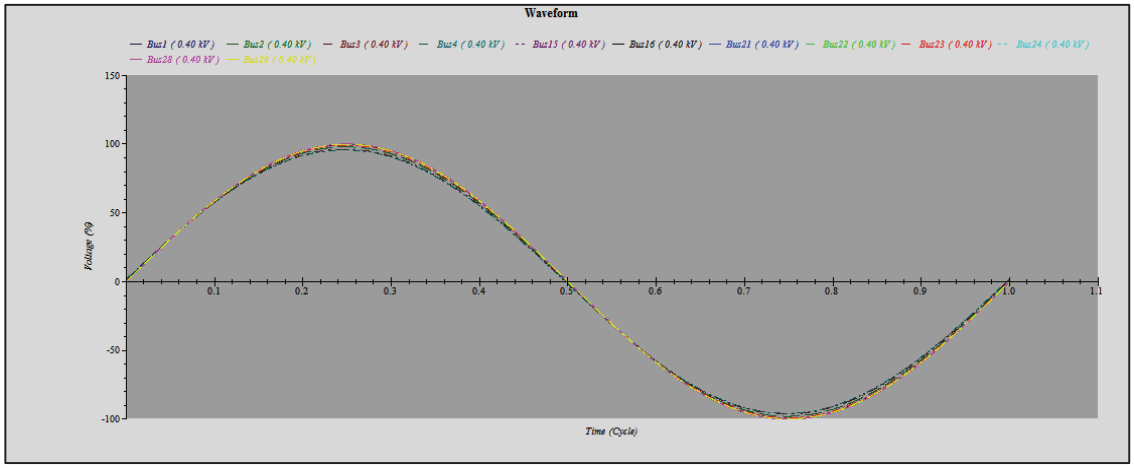

Fig. 6. Bus voltage harmonics waveform

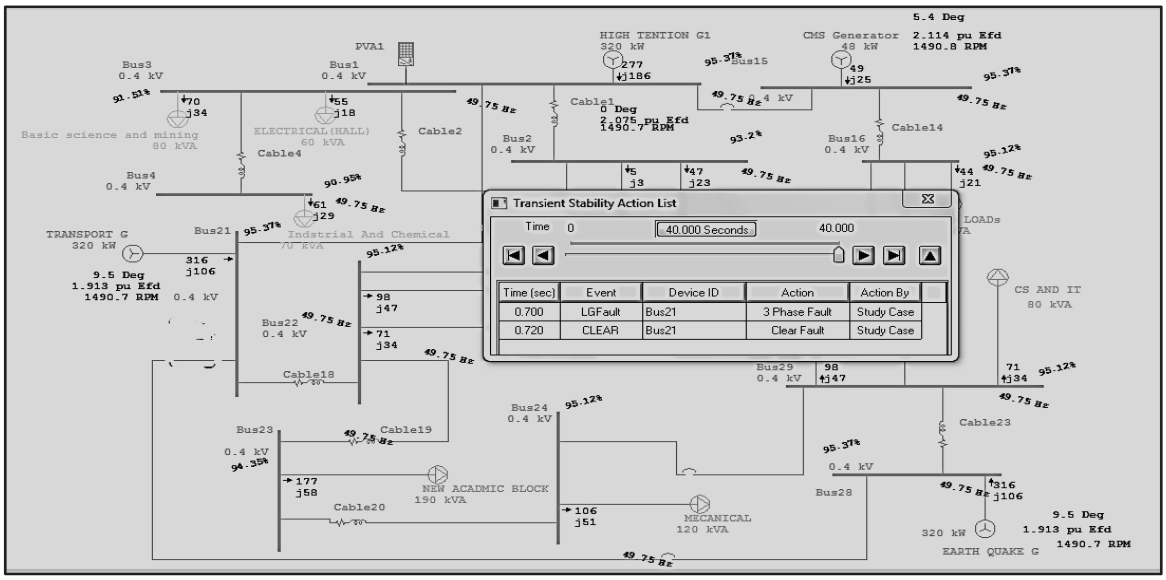

Fig. 7. Transient stability analysis of UET Microgrid

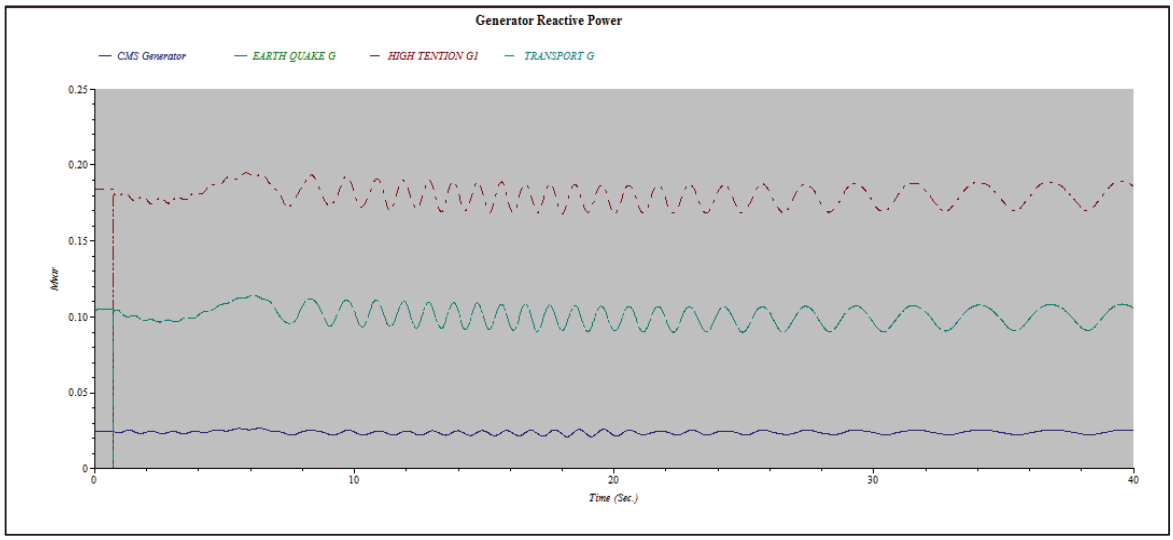

Fig. 8(a). Generator reactive power

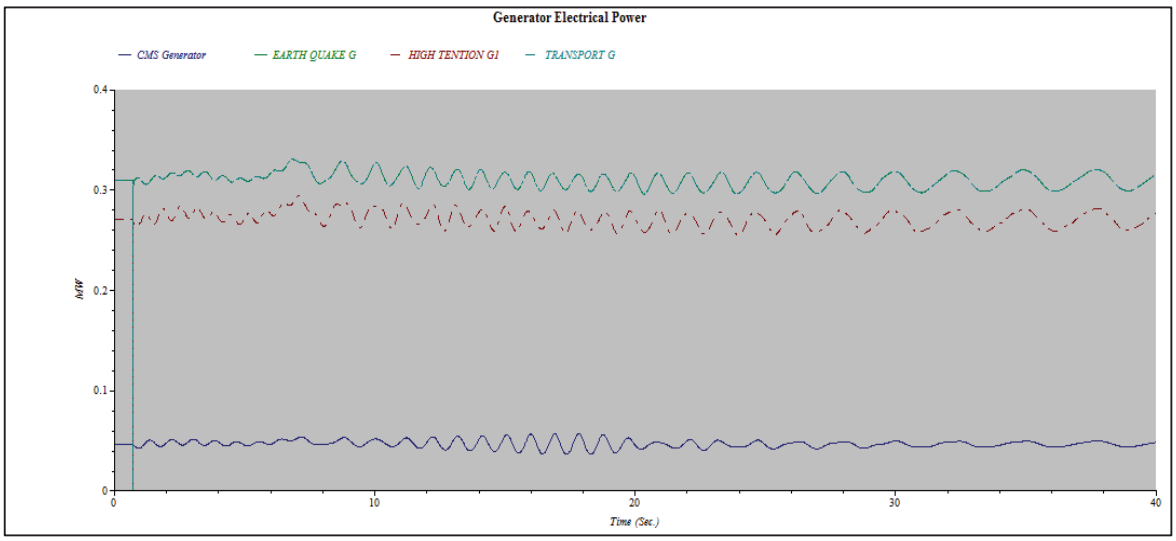

Fig. 8(b). Generator electrical power

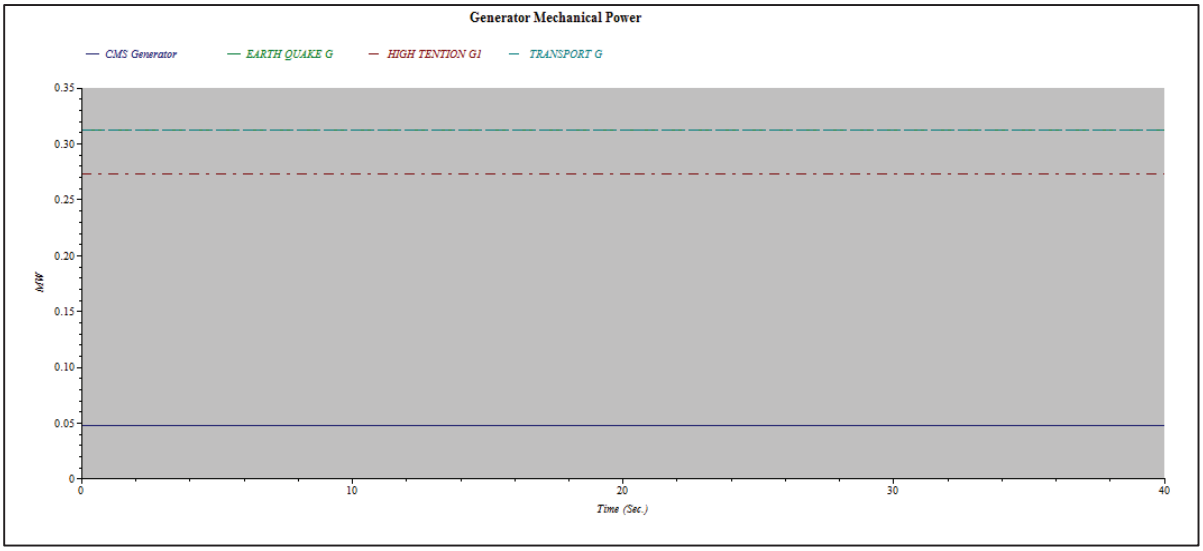

Fig. 8(c). Generator mechanical power

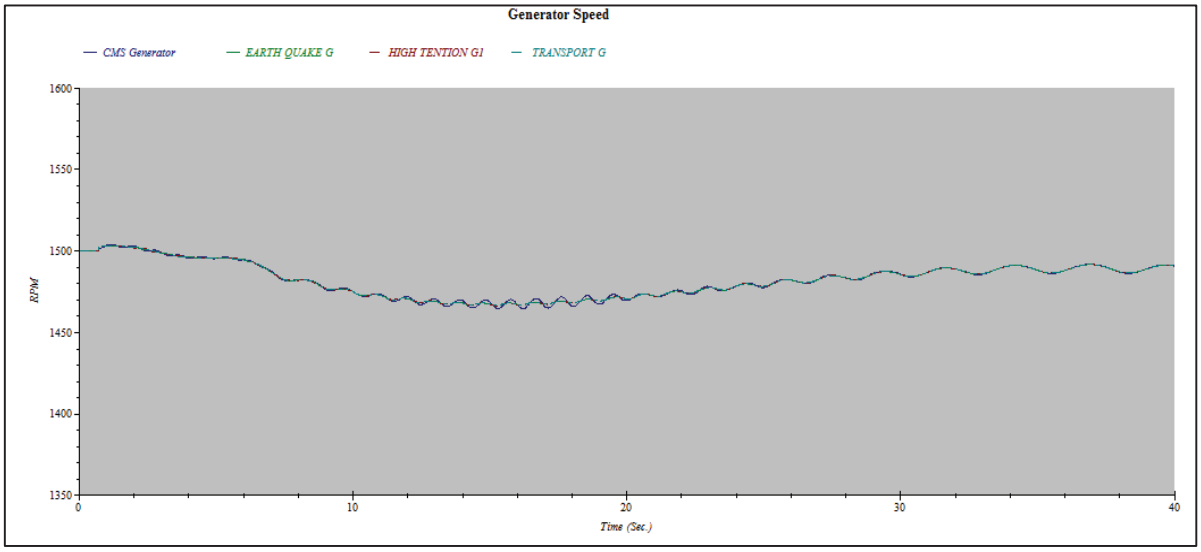

Fig. 8(d). Generator speed

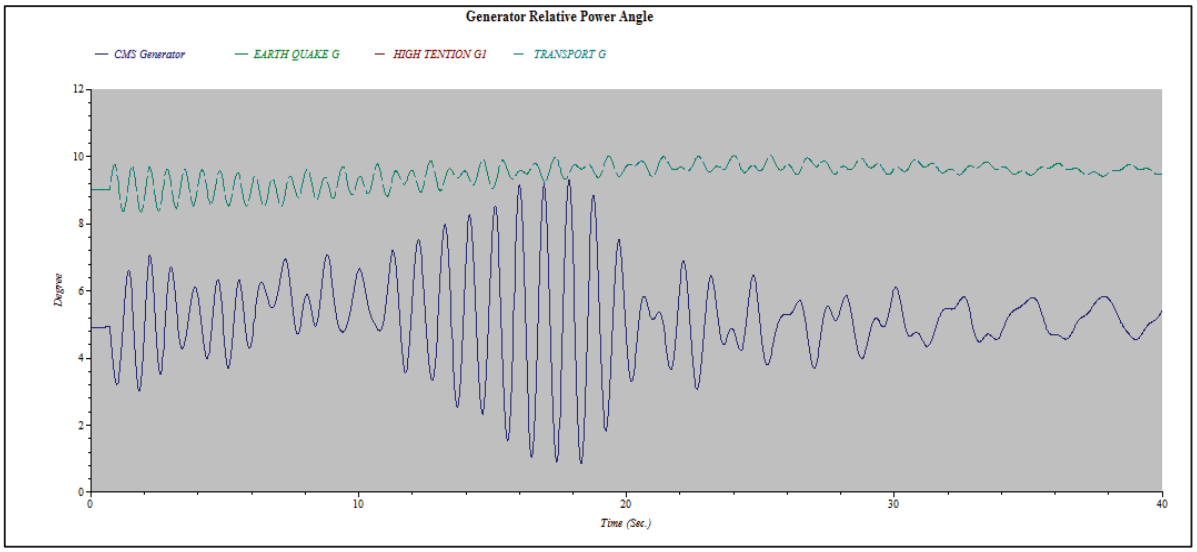

Fig. 8(e). Generator relative power angle

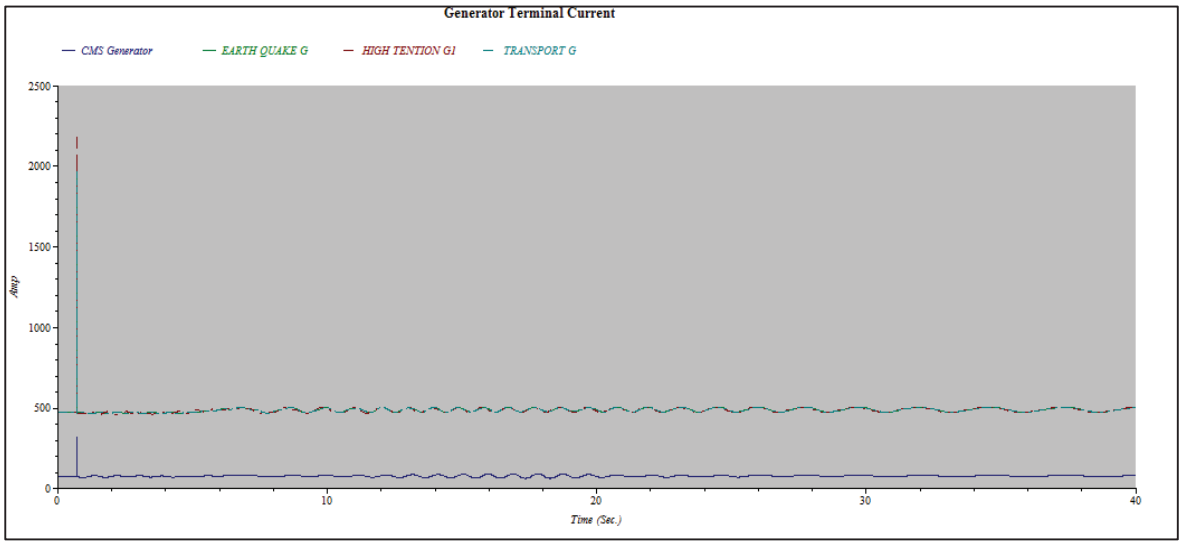

Fig. 8(f). Generator terminal current

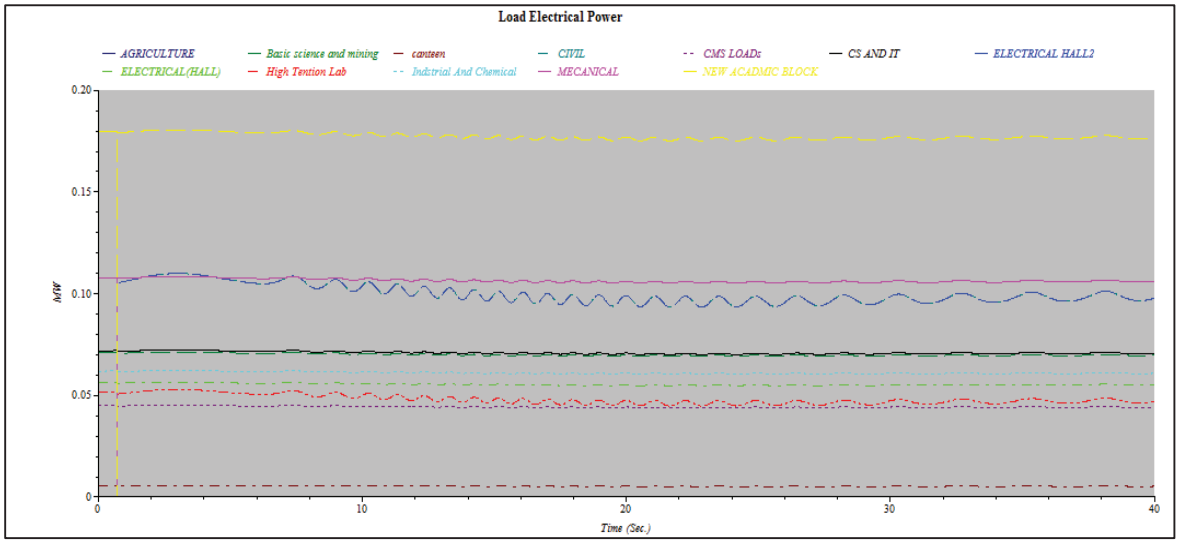

Fig. 9(a). Load electrical power

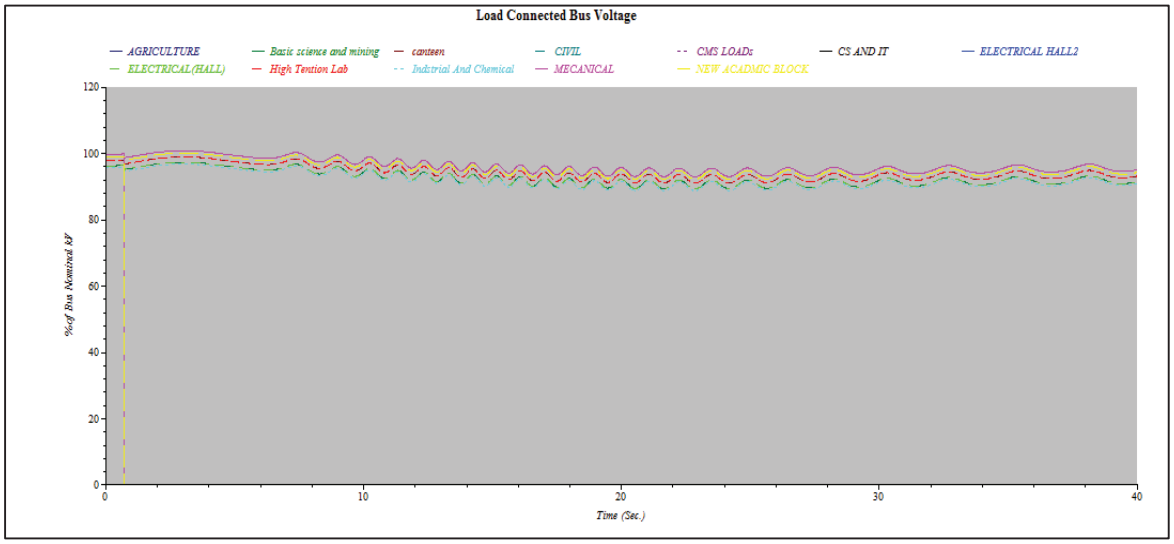

Fig. 9(b). Load connected bus voltage

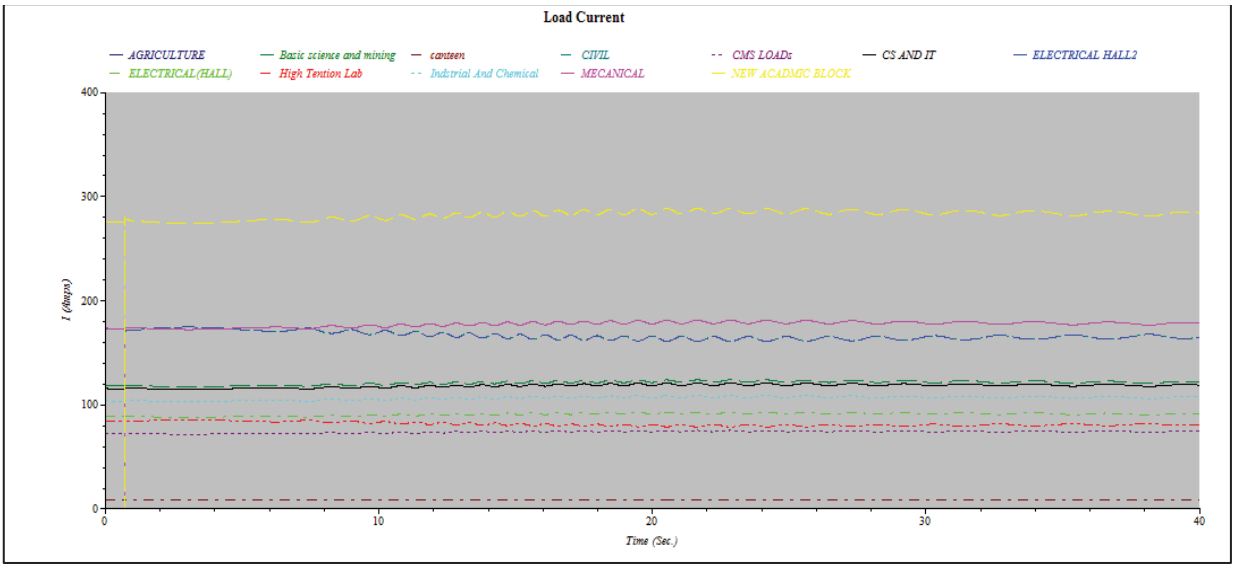

Fig. 9(c). Load current

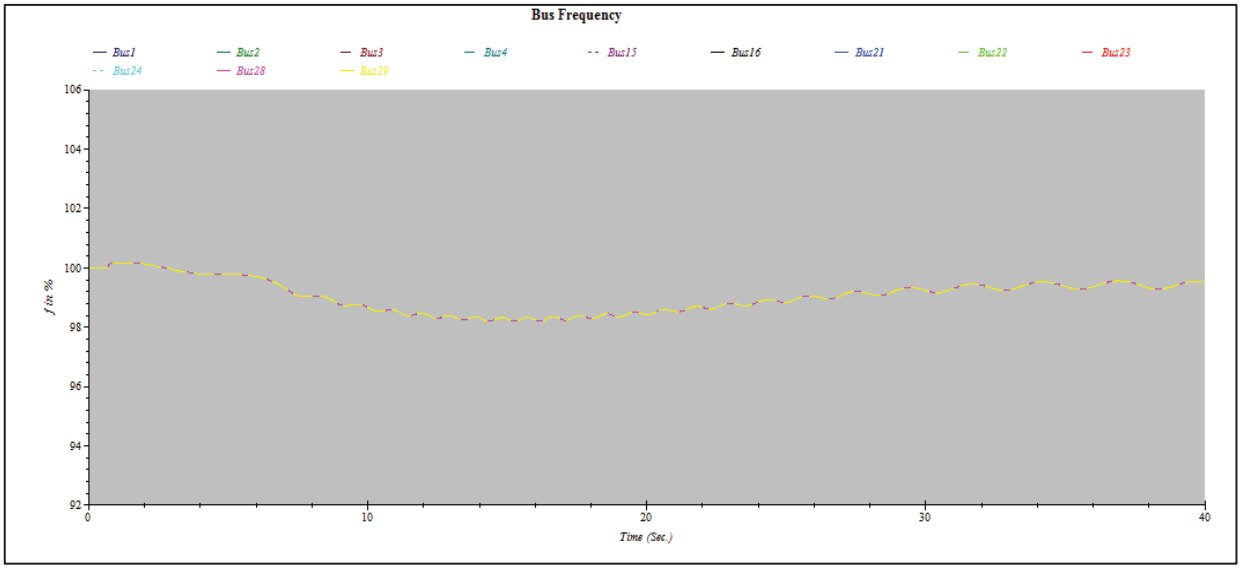

Fig. 10. Stability of the frequency of UET Microgrid

loading, percentage terminal voltage is shown in Table 7 while the total connected load and total generation is shown by Table 8.

3.2 Mode of operation (With Microgrid)

Here, the analysis of the proposed Microgrid is performed when supply from main grid is off. The Microgrid in this state feeds all loads of UET Peshawar campus. Cons equently, one small generator is switched off and other extra load is connected to the main generator while in islanded mode. Two operational modes of our Microgrid with economic benefits are described here.

3.2.1 One generator is switched off

The ability of the main Generator to meet all the demand forces the small generator of 60kVA to shut down. The load flow analysis of UET Peshawar Microgrid without the 60KVA generator is performed and depicted in Figure 12. The percentage generation of generators is shown in Table 9. The percentage loading, percentage terminal voltage is shown in Table 10 and the total connected load and the total generation are shown in Table 11. So in this way, cost of operation of extra generator is removed. The 60 kVA generator will be available for meeting the peak load or any

eventuality in the islanded mode. Diesel consume by 60KVA generator per hour when it is running at full load i.e. generating 48KWh is 13.5 litre/hr. So,

If one litre diesel price = Rs 100 Rupees
For 13.5 litre diesel price = Rs 1350 Rupees
One Unit or KWh price = Rs 28.125 Rupees
If the generator is running for three hours in a day, then total cost saving per day = Rs 4050 Rupees
Total cost saving for one month = Rs 121500 Rupees
Total cost saving in one year = Rs 1458000 Rupees

3.2.2 Addition of load to the Microgrid

As the High Tension lab generator shown in Figure 10 is generating at 48% so it's mean that additional load can be connected to the Microgrid. In this way the extra load of 160 kVA can be feed by the UET Peshawar Microgrid. The load flow analysis when the extra load is connected to UET Peshawar Microgrid is shown in Figure 13.

Moreover, balanced load of nearby houses is also connected to the generator. The percentage generation of generators is given in Table 12. The percentage loading, percentage terminal voltage as shown in Table 13 while the total connected load and the total generation is shown in Table 14.

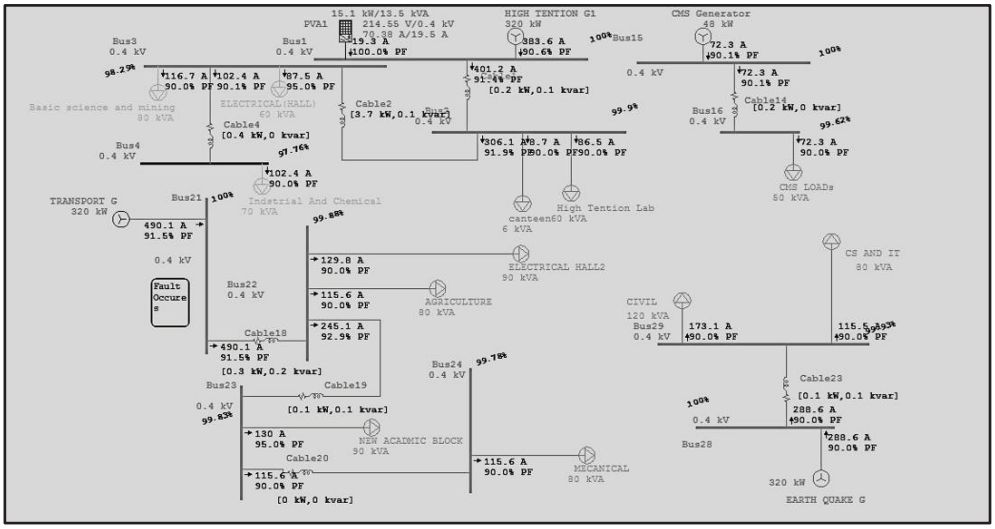

Fig. 11. Without Microgrid state

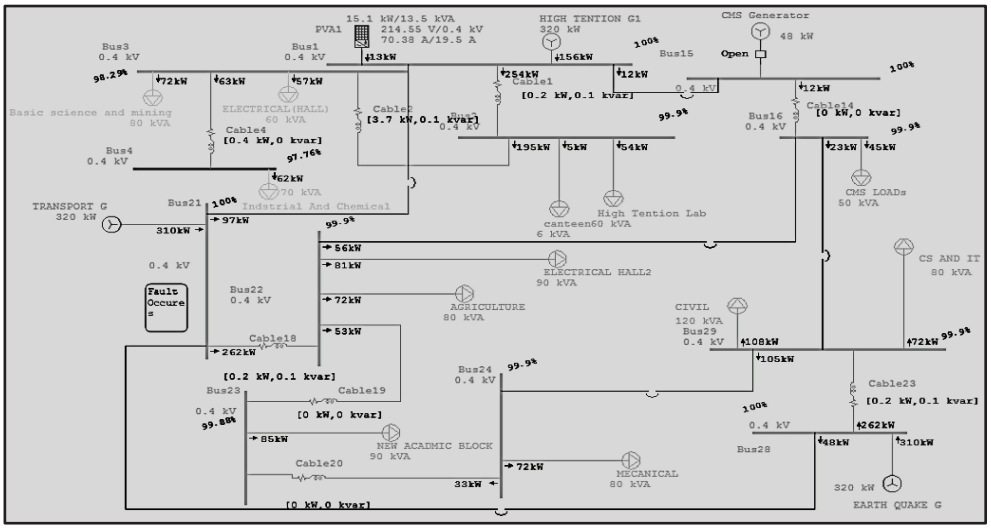

Fig. 12. Load flow analysis without the 60KVA generator

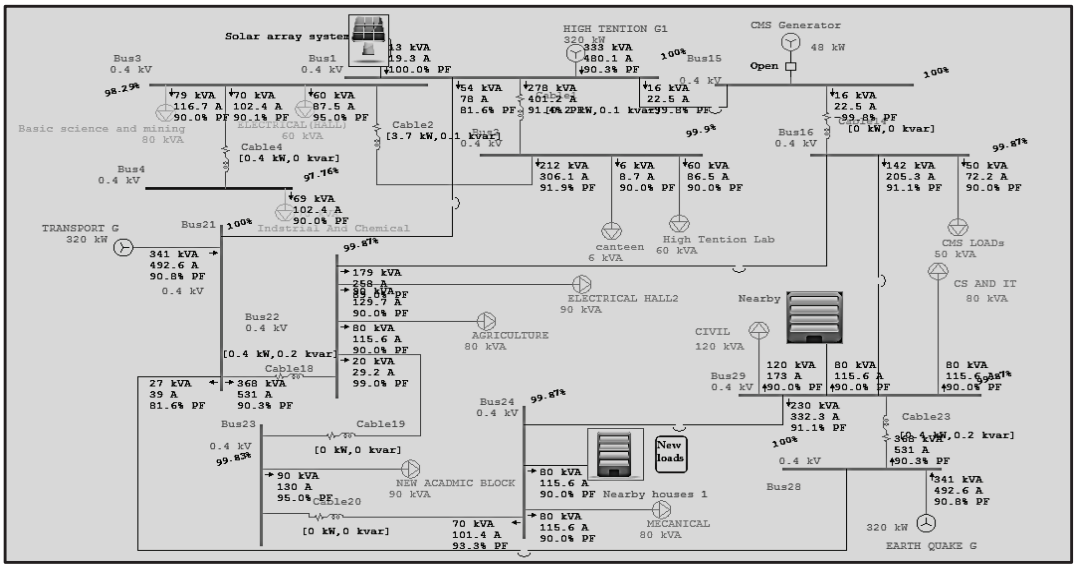

Fig. 13. Load flow analysis when the extra load is connected

Table 6. Generators percentage generation

Generator ID	Rating (kW)	Rated (kV)	kW	kvar	Amp	%PF	% Generation
CMS Generator	48	0.4	45.138	21.469	72.33	90.07	94
Earth Quake G	320	0.4	180	87.168	288.6	90	56.2
High Tension G1	320	0.4	241	112	383.6	90.6	75.2
Transport G	320	0.4	311	137	490.1	91.48	97.1

Table 7. Percentage loading, percentage terminal voltage

Load ID	Rating (kVA)	Rated (kV)	kW	kvar	Amp	%PF	% Loading	% V(terminal)
Agriculture Department	80	0.4	71.966	34.855	115.6	90	100.1	99.88
Basic Sciences Department	80	0.4	71.511	34.634	116.7	90	101.1	98.29
Canteen	6	0.4	5.398	2.614	8.665	90	100.1	99.9
Civil Department	120	0.4	108	52.235	173.1	90	99.9	99.93
CMS Loads	50	0.4	44.933	21.762	72.33	90	100.2	99.62
CS&IT Department	80	0.4	71.98	34.862	115.5	90	100	99.93
Electrical Department Hall 1	90	0.4	80.811	39.139	129.8	90	99.9	99.88
Electrical Department Hall 2	60	0.4	56.613	18.608	87.51	95	101.1	98.29
High Tension Laboratory	60	0.4	53.897	26.103	86.52	90	99.9	99.9
Industrial & Chemical Departments	70	0.4	62.441	30.241	102.4	90	101.4	97.76
Mechanical Department	80	0.4	71.936	34.84	115.6	90	100.1	99.78
New Academic Block	90	0.4	85.44	28.083	130	95	100.1	99.83

Table 8. Total connected load and total generation

Buses	12
Branches	8
Generators	4
Power Grids	0
Loads	12
Load-MW	0.79
Load-Mvar	0.359
Generation-MW	0.79
Generation-Mvar	0.359
Loss-MW	0.005
Loss-Mvar	0.001

Table 9. Generators percentage generation

Generator ID	Rating (kW)	Rated (kV)	kW	kvar	Amp	%PF	% Generation
Earth Quake G	320	0.4	310	120	479.6	93.3	96.9
High Tension G1	320	0.4	156	120	284	79.43	48.8
Transport G	320	0.4	310	120	479.6	93.3	96.9

Table 10. Percentage loading, percentage terminal voltage

Load ID	Rating (kVA)	Rated (kV)	kW	kvar	Amp	%PF	% Loading	% V(terminal)
Agriculture Department	80	0.4	71.971	34.857	115.5	90	100.1	99.9
Basic Sciences Department	80	0.4	71.511	34.634	116.7	90	101.1	98.29
Canteen	6	0.4	5.398	2.614	8.665	90	100.1	99.9
Civil Department	120	0.4	108	52.203	173	90	99.9	99.9
CMS Loads	50	0.4	44.982	21.786	72.21	90	100.1	99.9
CS&IT Department	80	0.4	71.971	34.857	115.5	90	100.1	99.9
Electrical Department Hall 1	90	0.4	80.839	39.152	129.8	90	99.9	99.9
Electrical Department Hall 2	60	0.4	56.613	18.608	87.51	95	101.1	98.29
High Tension Laboratory	60	0.4	53.897	26.103	86.52	90	99.9	99.9
Industrial & Chemical Departments	70	0.4	62.441	30.241	102.4	90	101.4	97.76
Mechanical Department	80	0.4	71.971	34.857	115.5	90	100.1	99.9
New Academic Block	90	0.4	85.459	28.089	130	95	100.1	99.88

Table 11. Total connected load and total generation

Load-MW	0.79
Load-Mvar	0.359
Generation-MW	0.79
Generation-Mvar	0.359
Loss-MW	0.005
Loss-Mvar	0.001

Table 12. Generators percentage generation

Generator ID	Rating (kW)	Rated (kV)	kW	kvar	Amp	%PF	% Generation
Earth Quake G	320	0.4	310	143	492.6	90.83	96.9
High Tension G1	320	0.4	300	143	480.1	90.31	93.9
Transport G	320	0.4	310	143	492.6	90.83	96.9

Table 13. Percentage loading, percentage terminal voltage

Load ID	Rating (kVA)	Rated (kV)	kW	kvar	Amp	%PF	% Loading	% V(terminal)
Agriculture Department	80	0.4	71.963	34.854	115.6	90	100.1	99.87
Basic Sciences Department	80	0.4	71.511	34.634	116.7	90	101.1	98.29
Canteen	6	0.4	5.398	2.614	8.665	90	100.1	99.9
Civil Department	120	0.4	108	52.174	173	90	99.9	99.87
CMS Loads	50	0.4	44.977	21.783	72.22	90	100.1	99.87
CS&IT Department	80	0.4	71.963	34.854	115.6	90	100.1	99.87
Electrical Department Hall 1	90	0.4	80.795	39.131	129.7	90	99.9	99.87
Electrical Department Hall 2	60	0.4	56.613	18.608	87.51	95	101.1	98.29
High Tension Laboratory	60	0.4	53.897	26.103	86.52	90	99.9	99.9
Industrial & Chemical Departments	70	0.4	62.441	30.241	102.4	90	101.4	97.76
Mechanical Department	80	0.4	71.963	34.854	115.6	90	100.1	99.87
Nearby Houses	80	0.4	71.963	34.854	115.6	90	100.1	99.87
Nearby Houses 2	80	0.4	71.963	34.854	115.6	90	100.1	99.87
New Academic Block	90	0.4	85.443	28.084	130	95	100.1	99.83

Table 14. Total connected load and total generation

Load-MW	0.934
Load-Mvar	0.428
Generation-MW	0.934
Generation-Mvar	0.428
Loss-MW	0.005
Loss-Mvar	0.001

4. DISCUSSION & CONCLUSION

From the preceding analysis, a Microgrid is proposed for our local campus of UET, Peshawar while in islanding mode when power is not available due to load shedding or faults occurring in the distribution system. As per our backup resources, three diesel generators with solar system can be interconnected to form such a proposed Microgrid for UET Peshawar. The load flow analysis, short circuit analysis, total harmonic analysis, transient stability analysis of our proposed Microgrid are shown in this paper and are carried out using the ETAP simulation software. The results vividly show that a reduction in number of engaged diesel generating units on campus that normally operate in parallel greatly reduces the per annum cost paid by the University. It is also shown in this paper

that by implementing the proposed Microgrid, the optimum units will dispatch power to the load in an effective way without overloading the operating units.

This will greatly decrease the cost of operating an extra diesel generator at full capacity. Thus, the efficiency of the system will increase and this arrangement will also be eco-friendly due to reduction in diesel related carbon emissions. Furthermore, this proposed arrangement with added features such as precision smart switches can be implemented for further efficiency improvement in future. Finally, it is concluded that a smart Microgrid such as proposed in this paper is an efficient and cost effective way of running on campus power generating resources while in islanding mode.

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Research Article

Automatic Detection of Noisy Signals in sEMG Grids Using Statistical Thresholding

Khalil Ullah[1*], and Khalid Shah[2]

[1]Department of Software Engineering, University of Malakand, Chakdara, Pakistan
[2]Department of Computer Science, Bannu University of Science and Technology, Bannu, Pakistan

Abstract: Electromyogram (EMG) signal is often processed offline, after its acquisition, using digital signal processing algorithms to extract muscle anatomical and physiological information. As most of the signal processing algorithms work on an adequate quality of the signals, thus quality checking of the EMG in real-time during its acquisition is of immense importance. In multi-channel sEMG signals, usually there are some noisy or bad channels. If the noise is of low level, it is of little concern but high level of noise can limit the usefulness of the EMG. To make sure acquisition of a good quality EMG signal in terms of SNR, one way to detect noisy channels is through visual inspection by an expert human operator, however visual inspection of multiple electrodes in real-time is not possible and is also expensive both in terms of time and cost. In this research study, we propose a novel method for automatic detection of noisy channels in multi-channel surface EMG signals based on statistical thresholding of several parameters. The results of the proposed method are in perfect agreement with the ground truth for simulated EMG signals, with an accuracy of 98.6%.

Keywords: EMG, Noisy Channel, Power Line Interference, Statistical Thresholding.

1. INTRODUCTION

In a multi-channel signal detection system, some of the channels are often contaminated by various physiological and non-physiological sources of noise. Typically, these noises come from the acquisition system itself (amplifier noise, saturation, poor electrical contacts), the environment (mains interference, stimulation devices, body vibration etc.) and from the subject (bad skin electrode contact, movement of the skin under electrode, heartbeats, artefactual spikes, Baseline Wander) [1,2]. These noises are inherent in most signal acquisition protocols and are often the limiting factor in the performance of the post EMG signal processing algorithms. In most signal processing techniques, an adequate quality of the sEMG signal is assumed, if this assumption is not met, may lead to invalid extraction of the muscle physiological and anatomical features. Thus, sEMG quality checking is required to have valid signals for further processing and analysis. sEMG quality checking can be performed by an expert human operator, however human expert involvement is sometime impractical in real applications as it is time consuming and not cost effective. Thus, an algorithm is required for automatic detection of these artifacts.

Literature review reveals significant research activities for automatic detection of artifacts in sEMG signals. Wavelet analysis has been used to detect and remove the artifacts from bio-signals [3-6], however due to its computationally intensive nature; wavelet transform can't be used in real time situations [7]. Independent Component Analysis (ICA) is also being used to detect and isolate artifacts from EMG signals [8-10]. Higher Order Statistics (HOS), and Empirical mode decomposition (EMD) are some other approaches used to detect and remove artifacts from bio- signals [11,13]. Due to high computational cost and error in case of signal

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*Corresponding Author: Khalil Ullah <khalil.ullah@uom.edu.pk>

distorted by Colored-Gaussian noise, these methods can't be used. Some of the above methods focus on detection of a specific artifact in EMG signals while others are application dependent. Most of these methods are also based on supervised learning for classification of artifacts. As supervised methods require training of the algorithm for different scenarios, it is time consuming and may not be real time applicable [14,15]. In [16], the authors used a supervised method to detect artifacts in EEG signals, however this method detects time epochs of the EEG signals and is also not applied to multi-channel EMG.

Due to the random nature of sEMG signals, an alternate method, to detect automatically the bad channels, is to use a thresholding of various statistical parameters. Our proposed method is an unsupervised method and it uses various statistical parameters for automatic detection of bad channels. As it is an unsupervised method, it could be applied in real- time without need of training of any classifier. The statistical parameters used in our method are Mean Correlation, Root Mean Square, Hurst Exponent, and Complexity Coefficient. Adaptive thresholding of these parameters has been used to classify a channel as "Good" or "Bad". Our proposed method is a high accuracy of 99.2% by detecting both the noisy and clean signals automatically. The primary contribution of this research work are: 1) the selection of a new set of parameters which can differentiate noise from EMG signal, 2) Use of statistical thresholding to distinguish between a clean signal and the one contaminated by some noise sources.

2. MATERIALS AND METHODS

The proposed algorithm depicted as a flow chart in Figure 1, computes and checks various statistical parameters to verify the quality of the EMG signal in each channel of a multi-channel acquisition system. The surface EMG recordings are affected by several artifacts such as power line interference (PLI), abrupt baseline drift due to movement of patient and skin-electrode impedance, ECG signal, electrode artifacts and amplifier saturation [17,18]. A channel of an electrode array that has any of the above-mentioned artifacts larger in amplitude than EMG with SNR less than 15dB is termed as Bad Channel. A Flag of each parameter is determined for each channel. The value of the Flag of each parameter is set either to 0 (for bad channel) or 1 (for good channel). As we have four parameters, so four flags are obtained for each channel. The values of all these flags are added together and if this sum is greater than two (majority voting) then it means that more than two parameters have identified this channel as good, thus the channel is classified as good channel. Similarly, if a channel is detected as bad by majority of the parameters then the sum of its parameter's Flags is less than two and this channel is classified as bad channel. We use an adaptive thresholding for each parameter after analysis of 320 simulated EMG channels. The selection of threshold for each parameter is discussed in next subsections.

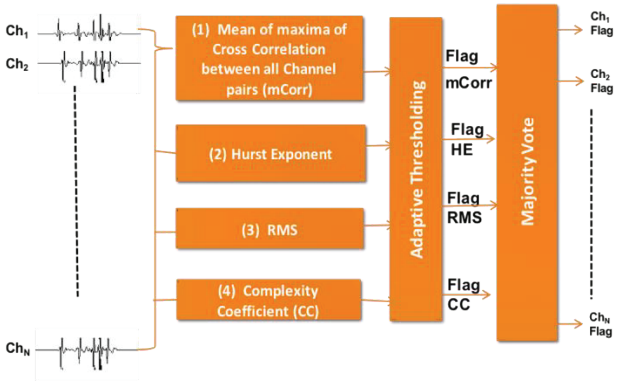

Fig. 1. Flow chart of the proposed method for identification of bad channels based on statistical parameters 1) mean of the maxima of cross correlation of each channel with all other channels, 2) Hurst exponent of each channel, 3) Root mean square value, and 4) The complexity Coefficient of each channel

2.1 Statistical Parameters

The parameters computed and analyzed for each channel are 1) mean of the maxima of the cross-correlation functions (mCorr) of each channel with all the other channels, 2) Hurst Exponent (HE), 3) Root mean square value (RMS) and 4) Complexity Coefficient(CC). The algorithm calculates all the above four parameters for each channel and apply a threshold for each parameter for each channel.

2.1.1 Mean of Maxima of Cross-Correlation

The first parameter used for automatic detection of the bad sEMG channels is the mean correlation (mCorr), which is the average of the maxima of cross correlation of each channel with all other channels. For EMG signal of channel 1 i.e. X¬1, out of the total 16 Channels, the mCorr is computed as follow.

$$mCorr(1) = \frac{1}{16}\sum_{i=1}^{16}(\max\left(R_{X_1X_i}(\tau)\right)) \qquad (1)$$

Where $R_{X_1X_i}(\tau)$ is the cross-correlation of channel X1 with channel Xi. This mCorr is thus computed for each channel 2,3 and so on up to 16.

For muscles, parallel to the skin, the sEMG Channels in a high-density system are highly correlated with neighboring channels therefore a channel with artifacts will have a lower correlation with other channels [19] and thus, the mean correlation of that channel will also be lower. Also, if in worst case most of the channels are noisy then still there will be low correlation between them as the noises are mostly uncorrelated and random. An example of a 16-channel single differential (SD) simulated EMG signals (simulated using the planer Model described in [1]) are shown in Figure 2a. The channels 3 and 14 are respectively contaminated by ECG artifact and electrode movement artifact with SNR of 8dB and 5dB respectively. The cross-correlation of all the channels with all other channels and the mCorr of each channel are shown respectively in Figure 2b and c. From the output in Figure 2b and c it is evident that the cross-correlation and the mean correlation i.e. mCorr of the contaminated channels (channel 3 and 14) is lower than the other channels and can easily be detected automatically by applying a proper threshold. The threshold is determined by finding a boundary condition between good and bad (noisy) channels for a total of 320 simulated channels (100 bad and 220 good channels) as shown in Figure 3. The optimal boundary condition obtained in our case was mCorr = 0.5. The channels having mCorr below this value are detected as bad channels.

2.1.2 Root Mean Square Value

Root Mean Square (RMS) value is commonly used as EMG amplitude indicator. RMS of the EMG signal usually ranges from 0 to 1.5 mV [20]. As most of the noises are additive in nature thus the noisy EMG channels have a higher RMS value. For example, due to sudden spikes, additive white Gaussian noise and movement artifacts the RMS value of the EMG channels significantly increases. The RMS of the EMG signal of a channel i is calculated as follow.

$$RMS = \left(\frac{1}{N}\sum_{1}^{N} {X_i}^2(n)\right)^{\frac{1}{2}} \qquad (2)$$

Here, N is the total number of samples of the channel i. As in our case the signal is simulated for 3 seconds with sampling frequency of 2048 samples/s, so the total number of samples (N) are 6144.

For a simulated SD EMG signal with four channels 2, 4, 8 and 13 contaminated with ECG artifact, electrode movement artifact, PLI and a mixture of PLI and low frequency noise respectively with SNR of 5dB, the RMS value of each channel are computed using eq. 2 as shown in Figure 4. It is found that the noisy channels appear as outlier in the corresponding histogram of the RMS values. Thus the channels which have RMS higher than the mean+2σ of the RMS of all the channels, is classified as bad channel i.e. outlier (see Figure 4b).

2.1.3 Hurst Exponent

Hurst Exponent (HE) is a parameter used to check the randomness of a signals and is also a measure of the long range dependence with in a signal [21], [22]. It is also considered as a measure of self-similarity. Self-similarity means that the random signals like EMG looks similar if it is zoomed in time in and out [23], like fractal index. Various

algorithms are available for the estimation of the HE.

In this study, the Hurst Exponent is estimated using absolute moment method described in [23] and shown in Figure 5. Let X is one channel EMG signal of length N that is divided into M subseries each of length k such that the total number of subseries K = N/k. From each subseries an aggregate series is calculated as,

$$X_m{}^{(k)} = \frac{1}{k} \sum_{j=(m-1)k+1}^{mk} X_j, \quad m = 1,2,\ldots,K \tag{3}$$

The Hurst Exponent is then approximated as,

$$H_m = \frac{1}{K} \sum_{m=1}^{K} \left| X_m^k - \overline{X^k} \right| \tag{4}$$

Here, $\overline{X^k}$ is the mean of the subseries.

To obtain a threshold for distinguishing between a good and a bad channel, we compute H for a total of 20 sets of simulated signals, each containing a total of 16 channel SD EMG signals. Randomly selected 5 out of 16 channels from all 20 sets are contaminated with one of the noises from PLI, ECG artifact, movement artifact, low frequency noise, Guassian and Colored Gaussian, with their SNRs varying from 10dB down to

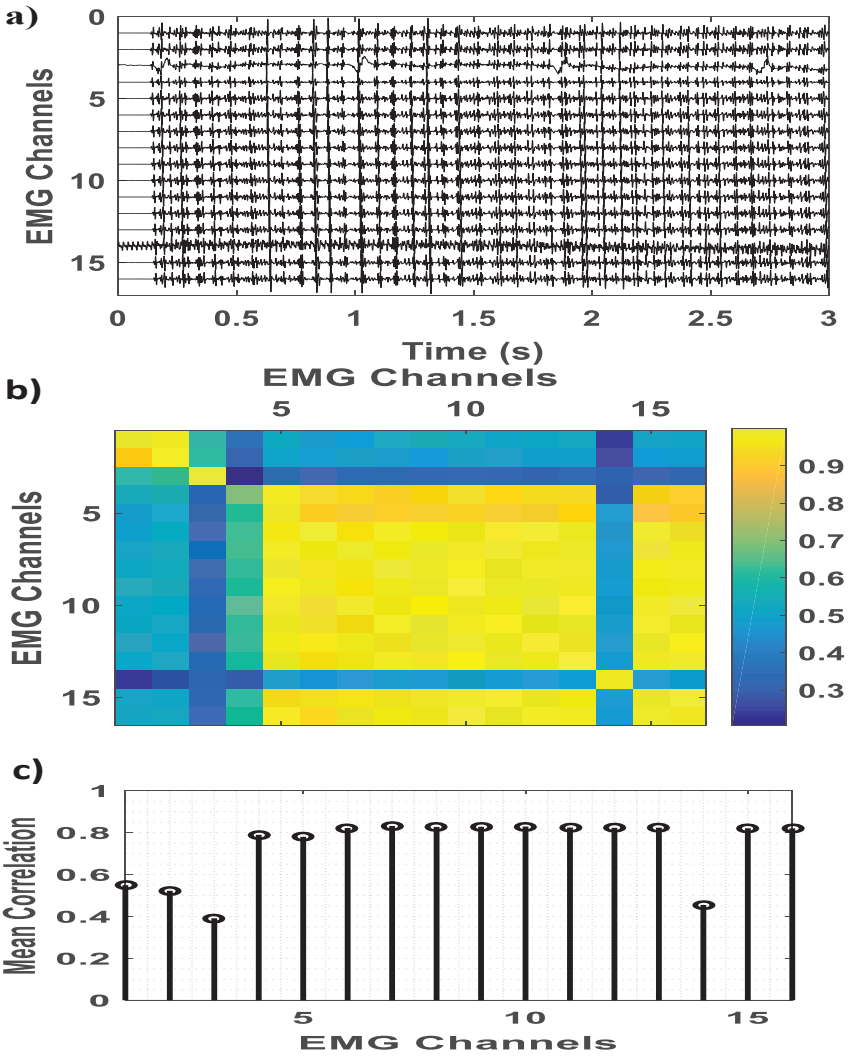

Fig 2. a) Simulated EMG signals with channels 3 and 14 contaminated with ECG artifact and electrode movement with SNR of 8dB and 5dB respectively, b) The maximum of cross correlation of each channel with all other 16 channels i.e. the correlation matrix. It can be seen from the correlation.

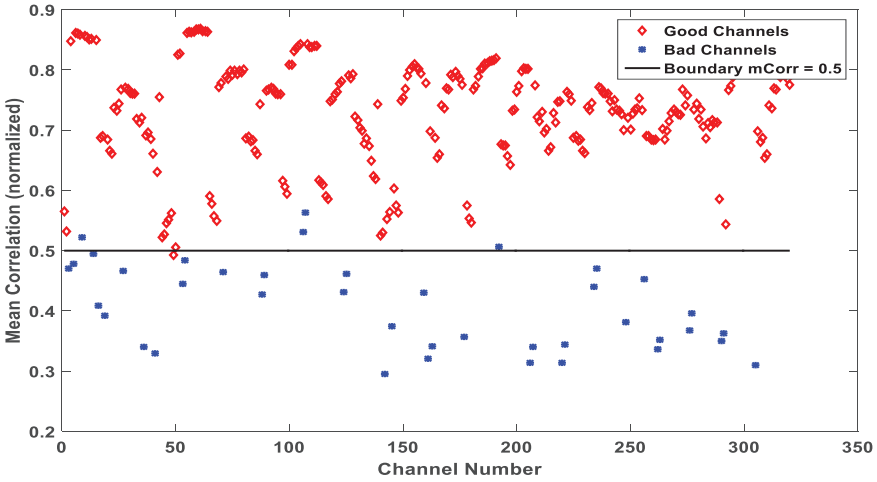

Fig. 3. Mean Correlation coefficient computed for a total of 320 single differential EMG channels. It is clear from the Figure that the threshold for Mean correlation coefficient between bad and good channels is 0.5.

5dB. The threshold for detection of bad and good channels is obtained by computing the mean of the maximum and minimum values of H for all the 16 channels from the total 20 sets of signals. By doing so, initial clusters of both good and bad channels are obtained. Now, to convert this static boundary into a dynamic boundary, another threshold is computed as the average of the minimum value of one cluster with that of maximum of the other cluster. By doing so, the difference of the distance between the threshold and the two clusters is enlarged. This process is repeated until there is no further change in the threshold value. This optimization of the threshold is shown in Figure 6. Once this threshold is optimized, then the channels having H values greater than this threshold are classified as bad channels and the channels having H value lower than this threshold are classified as good channels.

2.1.4 Complexity Coefficient

Complexity coefficient (CCx) is another feature to characterize a signal. It is defined as the ratio of the mobility coefficient ($MC_x = \sigma_{x'}^2/\sigma_x^2$) of the derivative of the signal to the mobility coefficient of the signal itself. For a signal x(t) the complexity coefficient is computed as,

$$CC_x = \frac{MC_{x'}}{MC_x} \qquad (5)$$

CCx is constant for a single sinusoid and is independent of the frequency and peak amplitude. We also empirically observed that CCX is almost constant for simulated EMG signals (see Figure 7a). If the EMG signal is contaminated

It is clear from the histogram that the RMS of the four noisy channels appear as outlier i.e. the RMS are away from mean (0.142) by more than 2σ. with a noise, the value of CCX changes due to change in the corresponding mobility coefficient of the signal and its derivative. Thus, the CCX is sensitive to noise and is chosen.

As one parameter for the detection of bad channels in EMG signals. From Figure 7a, the CC for clean EMG signals is 1.5. As noise is introduced to the simulated signals the CC value deviates from 1.5 (See Figure 7b). The threshold selected after analysis of 320 channels of simulated EMG signals is 2. Any channel with CC value greater than this threshold is marked as bad channel.

3. RESULTS AND DISCUSSION

In this study a new method for automatic detection of bad channels, based on thresholding of four statistical parameters as discussed in section 2, is proposed. To check the performance of the proposed method, 2 sets of simulated single differential EMG signal each consisting of multi-channels are prepared. The first set of EMG signals, consists of 20 simulated single differential EMG signals (each consisting of 16 channels and total of 320 channels), is generated using the planar model developed by Merletti et al [1]. The 2nd set

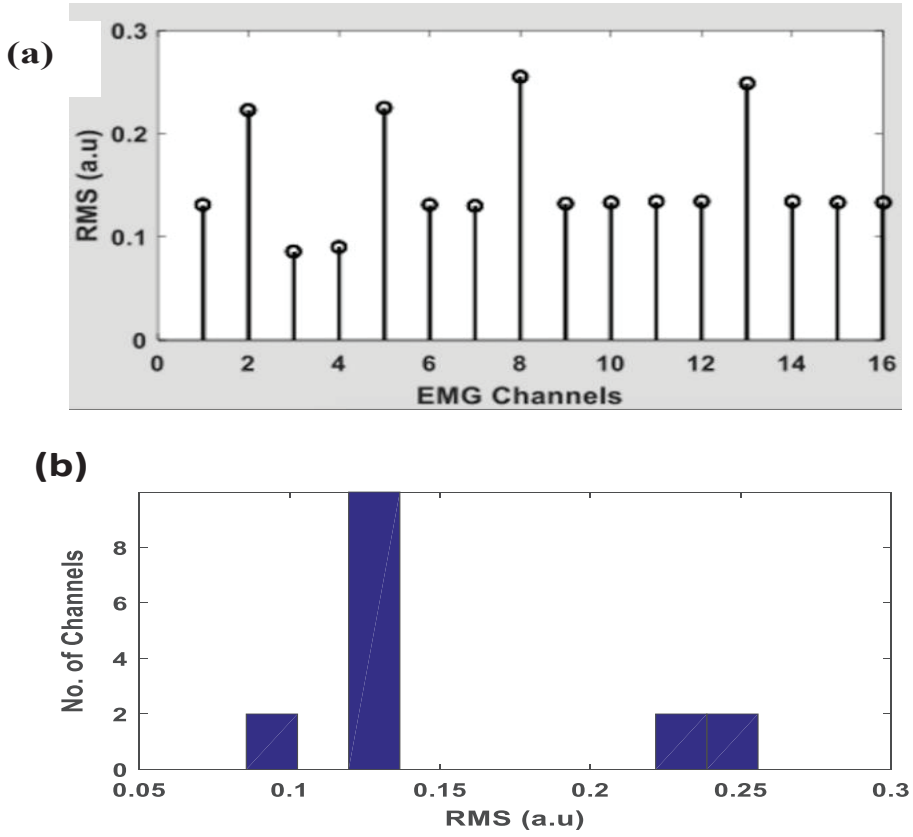

Fig. 4. a) RMS value of each channel of Simulated EMG signal with channel 2 contaminated by ECG artifact, channel 5 with Electrode Moment artifact, channel 8 with PLI and 13 with PLI and low frequency noise with an SNR of 5dB, b)

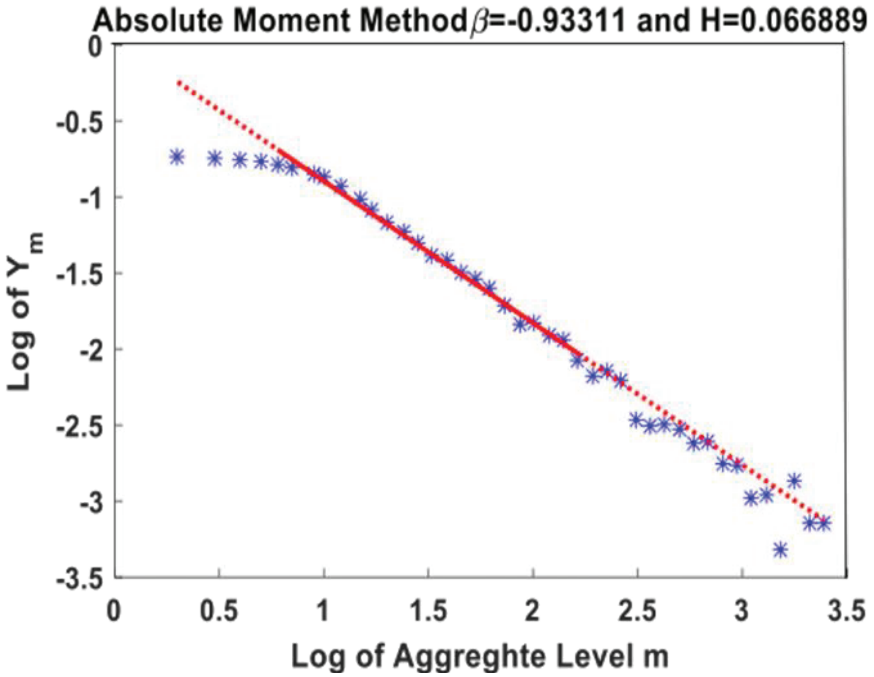

Fig. 5. The logarithmic plot between m and Ym and the fitted line. It should be noted that the slope of this line added with one gives an estimate of the H, the aggregate level m is the size of the non-overlapping blocks which could be from 1 to K.

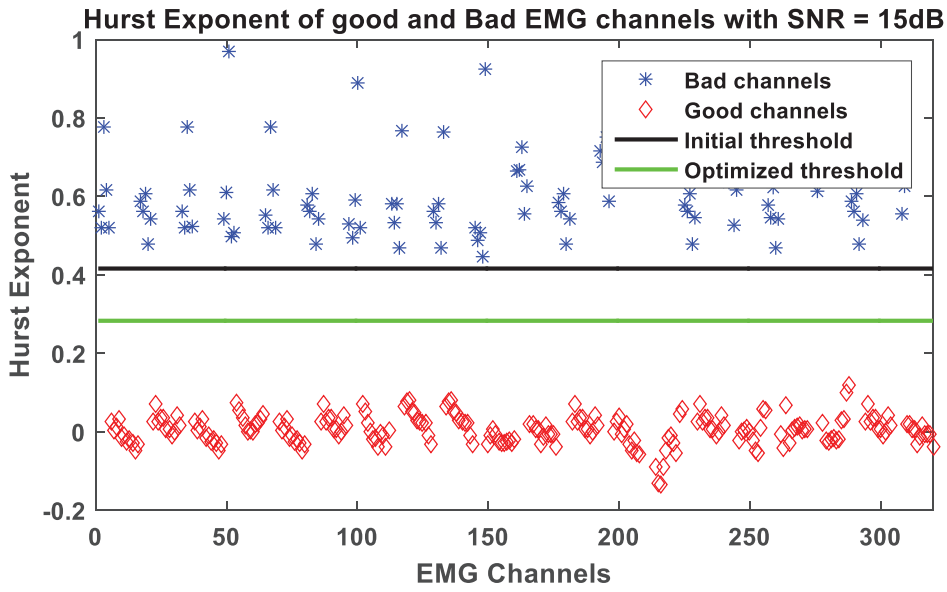

Fig. 6. Hurst Exponent for 320 channels (220 good, 100 bad (contaminated randomly with one of the noises PLI, Electrode movement artifact, ECG artifact and bad contact, WN and CN, with SNR=10dB and 15dB). The initial threshold (-) and the adaptive threshold (-) for the detection of good and bad channels.

consists of simulated EMG signals generated using the multilayer cylindrical description of the volume conductor model, described in [24].

For the first set of 20 simulated EMG signals (320 channels), noise such as power line interference (PLI), electrode movement (EM), white noise (WN), colored noise (CN) and ECG artifacts (ECGA) are added to channel number 2, 5, 8, 13 and 16 of each signal with SNR varying from 15 dB to -5 dB (total of 61 SNR values) respectively. The accuracy, sensitivity and specificity is then computed for each signal of the set of 20 signals across all the SNR values, resulting in 61 values of accuracy, sensitivity and specificity for each signal.

The accuracy, sensitivity and specificity of the proposed algorithm for this set of 20 signals across all SNR values are shown in Figure 8. The results show that the proposed method has a high accuracy in detecting both bad and good channels.

The 2nd set of simulated EMG signals, generated using the cylindrical model, were detected with circular electrodes (diameter 1 mm), arranged in a grid with 5 columns and 40 rows (200 electrodes) with 5 mm inter-electrode distance in both the longitudinal and transverse directions. The center of the grid corresponded to the center of the muscle volume projected on the skin surface. The detection system covered both the muscle (approximately 20 electrodes corresponding to the central portion of each column) and the tendon regions (approximately 10 electrodes over each tendon).

A monopolar recording was simulated for each electrode of the detection system and is then converted to signal differential across the channels. The surface-recorded motor-unit potential was obtained by summing the action potentials of all muscle fibers belonging to individual motor units. EMG signals were simulated at 4096 samples/s. As in this study we investigate only an array of electrodes so only the central column of the detection system is taken. A total of 20 simulated signals with different level of contractions (ranging from 10% to 100% Maximum Voluntary Contractions) were taken.

In the first case all the clean signals were passed through the algorithm for quality checking. The behavior of each of the statistical parameter (quality indicator) are on one side of the threshold value which means that all the channels are identified as good channels. Various artifacts like PLI, movement artifact, real ECG artifact, white noise, colored noise etc. were then added to randomly selected channels with SNR ranging from 10 to -2dB to

make some channels bad. For all these signals the accuracy and sensitivity of the proposed algorithm is also computed at various level of noises as shown in Figure 9. It is evident from the results that even at low level of noise i.e. higher SNR, the algorithm still detects the bad channels with a high accuracy, sensitivity and specificity. It is clear from the results that the parameters values are on opposite side of the threshold for good and bad channels in this case too.

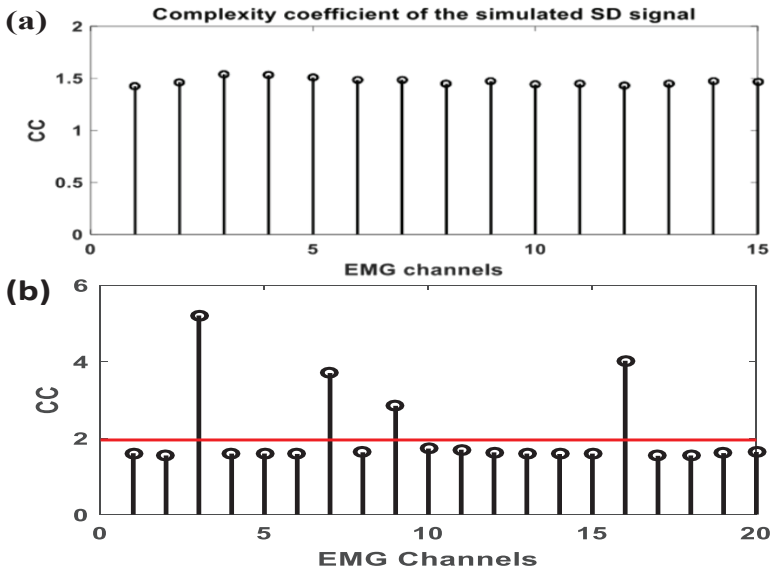

Fig. 7. a) Complexity coefficient of clean (without noise) 16 channel simulated EMG signal, b) Complexity coefficient of simulated EMG with channels 3, 7, 9 and 16 contaminated with different noises.

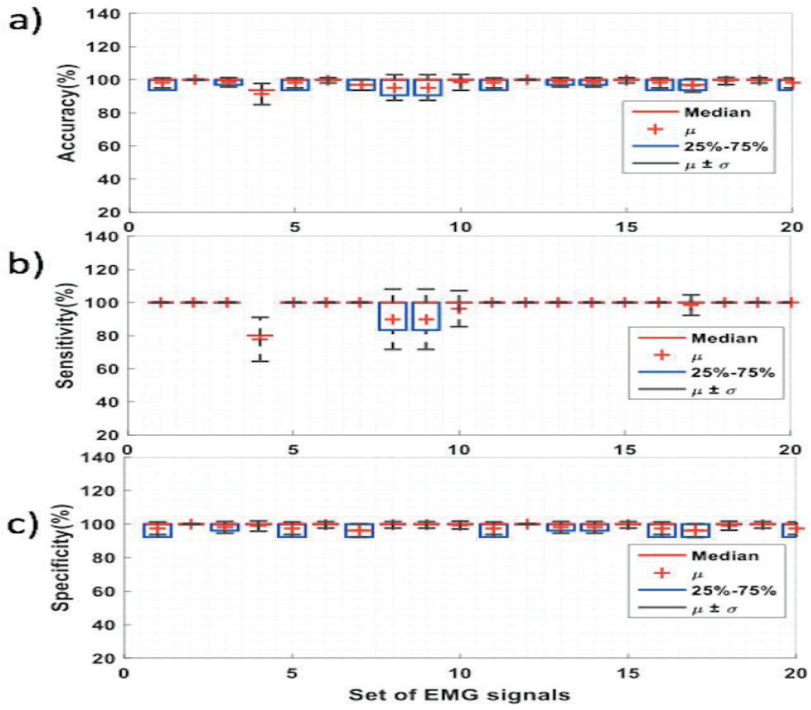

Fig. 8. Accuracy, sensitivity, and specificity of the proposed algorithm for a set of 20 simulated SD EMG signals with SNR varying from 15dB to -5dB. The higher specificity value (98.5492±2.681), shows that the algorithm can preserve most of the good channel as 'good'.

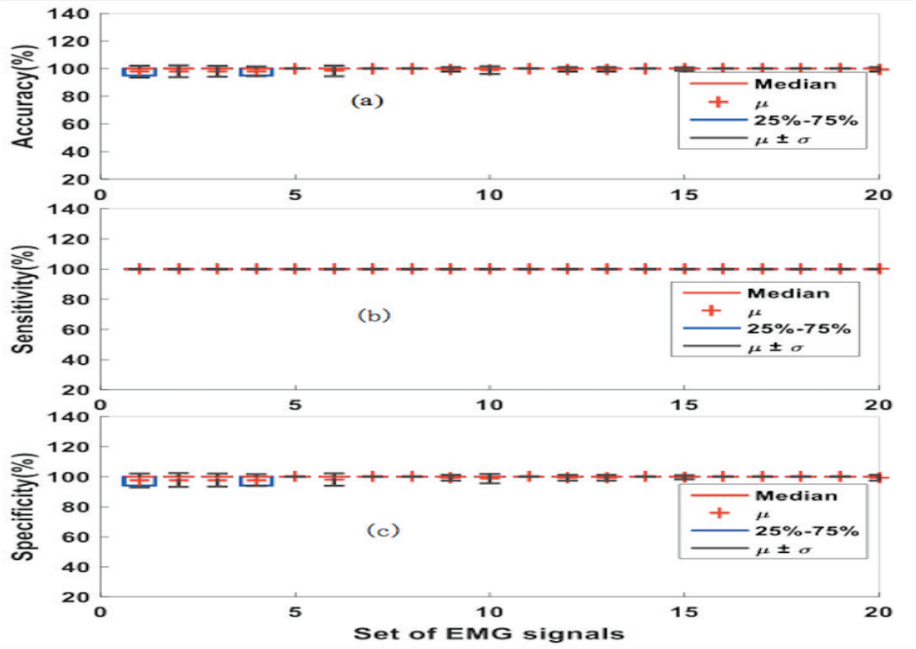

Fig. 9. Accuracy, sensitivity, and specificity of the proposed algorithm for a set of 20 simulated SD EMG signals generated using the cylindrical model [25], each set consisted of 20 channels, with noisy channels of SNR varying from 15dB to -5dB

4. CONCLUSION

This study presents a novel method for the automatic detection of noisy and clean EMG signals which will be helpful to the experimenter while recording EMG signals from a subject. If there are too many noisy signals in an experiment then it is useless for future, so the experimenter will know in realtime to clean the electrodes and make other necessary actions to record clean signals. From the performance of the algorithm on simulated signals, it is concluded that it will be a best choice for automatic detection of bad channels at the time of the acquisition of the signals. As a future study, a classification algorithm can be used after the detection of the noisy channel to also identify the type of the noise and then attenuate it by using a proper filter. As future work this research may be enhanced by recording more experimental signals and use of some machine learning techniques.

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Research Article

A Low-Power, High-Gain Amplifier with Rail-to-Rail Operating Capability: Applications to Biomedical Signal Processing

Hassan Faraji Baghtash*, and Rasoul Pakdel

Faculty of Electrical Engineering, Sahand University of Technology, Sahand New Town, Tabriz, Iran

Abstract: A low-voltage, low-power, rail-to-rail, two-stage trans-conductance amplifier is presented. The structure exploits body-driven transistors, configured in folded-cascode structure. To reduce the power consumption, the transistors are biased in the subthreshold region. The Specter RF simulation results which are conducted in TSMC 180nm CMOS standard process proves the well-performance of the proposed structure. The performance of the proposed structure against process variations is checked through process corners and Monte Carlo simulations. The results prove the robustness of the proposed amplifier against process uncertainties. Some important specifications of the design derived from circuit simulations are 93.36 dB small-signal gain, 14.4 PV^2/Hz input referred noise power, 26.5 kHz unity gain frequency, 20 V/ms slew rate. The proposed structure draws 260 nW power from 0.5 V power supply and is loaded with a 15 pF loading capacitor. The input common mode range of structure is from 0 to 0.5 V.

Keywords: Body-driven, Low-power, Sub-threshold, Low-voltage, Folded Cascade Structure.

1. INTRODUCTION

Today, the request for low-power, low-voltage, battery-operated circuits and systems is increased drastically. Numerous efforts have been made to lower the supply voltage of the structures. This trend is mainly imposed by technology scaling, from one generation to another, following Moore's scaling law. This trend also is encouraged by the increased versatility of battery-powered and portable devices [1, 2]. Emerging applications such as implantable medical devices, which need to operate with extremely low power dissipation (P_{diss}), made low voltage and low power circuit design more interesting than ever.

In the reduced supply voltages, using transistors operating in the subthreshold region is a very good choice, because they inherently overcome the threshold voltage limitation associated with regular application of transistors in the saturation region. Besides, as they have high gm/I ratio, they deliver high efficiency in terms of power consumption, as well.

The operational amplifiers are amongst the most important analog building blocks. As the logic gates in digital circuitries, the OTAs play very critical role in analog circuit and system designs [3]. For amplifiers, there are very common well-known features, of which some are more important in the biomedical applications. A number of these characteristics are: high common-mode rejection ration (CMRR), low input-referred noise (IRN), and low power consumption. The high value of CMRR is mandatory to remove the DC offsets from biological signals. The low noise operating performance is also essential due to the fact that the weak biological signals are always surrounded by a very noisy world. Finally, the power consumption must be kept in the range; as the device is usually in direct contact with the body, hence the heat from the high power consumption may damage the body or the organs in contact with the system [4].

Some efforts have been made to design the amplifier with the abovementioned conditions. For instance, a differential amplifier with devices operating in sub-threshold region is presented in

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*Corresponding Author: Hassan Faraji Baghtash < hfaraji@sut.ac.ir, hfarajii@gmail.com >

[3] to achieve the low power consumption. In [4], the PMOS input transistors are used alongside the switched biasing method to lower the area and IRN. In [5], the body-driven transistors are adopted in folded cascode structure in order to satisfy the design demands.

Literature review reveals that it is not so hard to achieve high gain or high CMRR in a reduced power consumption design. However, it is really difficult to design a high-speed but low-power operational amplifier. The reason behind this is that there is always a tradeoff between the power consumption and speed of operation in operational amplifiers. The [3] tries to maximize the unity-gain frequency (UGF) over power dissipation (UGF/P_{diss}). The paper succeeds to reach the defined figure of merit (FOM) and presents a circuit operating with considerably low power consumption. However, the examinations are done with a very low load capacitance of 0.5 pF, which is at least 30 times less than that's used by similar works. The challenge is dealt with in some recent works, trying to push the borders much further away. For example, in [6-10] body-driven technique is employed to enable low-voltage low-power operation of structures. They all show excellent performance and perform well in very reduced power supplies. Nevertheless, the power consumption is still so high for some energy aware applications such as implantable medicals.

Here, in this work, we present a low-voltage, low-power amplifier structure that tries to boost the unity-gain frequency over power dissipation. Utilizing a PMOS bulk-driven input stage configured in the folded cascode structure, the proposed structure succeeds to reach most of the design parameters. The presented circuit operates properly in the entire input common-mode range (ICMR), providing a full common mode swing range from negative to positive supply rails. The simulation results show that the presented OTA outperforms most similar works in terms of supply voltage, DC gain (G_0), and provided FOM.

The paper is arranged in four sections. After section 1 which introduce the paper, section 2 explains the proposed OTA, the simulation results are given in section 3. In the end, section 4 concludes the paper.

2. THE PROPOSED AMPLIFIER

The transistor level realization of the proposed OTA is depicted in Figure 1. The circuit is constructed from an input stage, a gain stage and an output stage (regions I, II, and III in Figure. 1, respectively). The input stage is designed such that it can offer full common-mode voltage swing from ground to positive supply rail. This is accomplished through bulk-driven input transistors of M1-M2. Normally, in traditional designs, using bulk-driven transistors is not an interesting choice, as it offers less gain-bandwidth product due to the reduced trans-conductance of transistors. However, for the target application of the proposed structure that needs very less gain bandwidth product, it is not a concern at all [11, 12]. In other word, in the biomedical applications, bulk-driven transistors are preferred over gate-driven ones as they offer less trans-conductance variation over input common-mode range. The flicker noise of the structure is handled by employing the large PMOS input transistors instead of NMOS transistors.

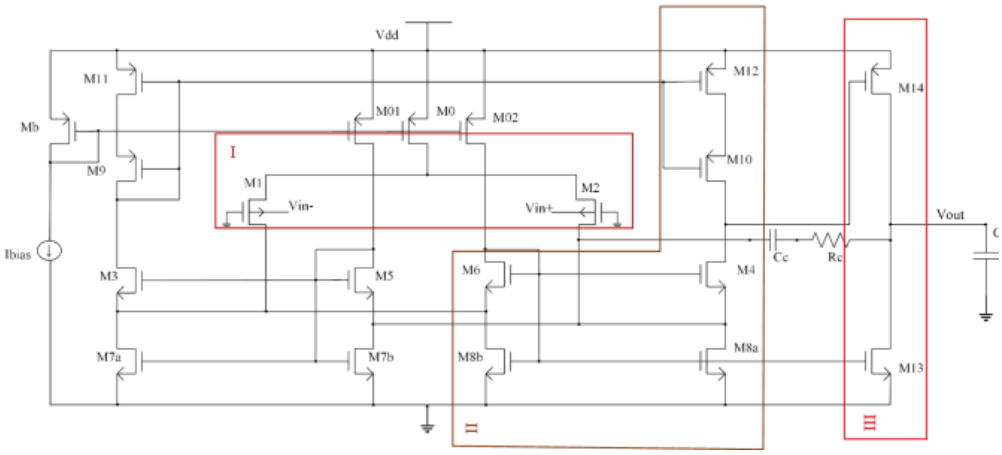

Fig. 1. The transistor-level realization of the proposed OTA

The biasing of the structure is provided by biasing current of 'Ibias' and bias transistors of Mb, M0, M01-M02. Transistors M3-M12 configure six composite transistors which effectively collect the injected current signals from M1-M2, perform amplification, and deliver the amplified signals to the output stage. The composite transistors enable the structure to receive the current signals from input stage and deliver them to the output stage with minimum voltage requirement and increased output impedance and current transfer accuracy. The output stage of the structure is accomplished through a common output stage. All devices are operating in sub-threshold region to deliver the maximum g_m/I_D ratio.

Examining the proposed amplifier, the following equations can be derived, formulating some small signal parameters of the design.

$$A_{dm} = \frac{(N+1)\, g_{mb1} g_{m14}}{(g_{o13} + g_{o14})(L+K)} \tag{1}$$

where A_{dm} is the DC differential voltage gain, N is the mirroring ratio between M7a and M7b or M8a and M8b, and *L* and *K* are given by:

$$L = \frac{g_{o4}(g_{m5} + g_{mb5} + G_O)}{g_{m4} + g_{mb4} + g_{m5} + g_{mb5} + g_{o4} + G_O},$$
$$G_O = g_{o1} + g_{o2} + g_{o5} + g_{o7b} + g_{o8a} \tag{2}$$

$$K = \frac{g_{o10} g_{o12}}{g_{m10} + g_{mb10} + g_{o10} + g_{o12}} \tag{3}$$

The common mode gain can be calculated as:

$$A_{cm} = G_{MX} \times \frac{g_{o0}}{g_{o0} + g_{mb1} + g_{mb2}} \times \frac{g_{m4} g_{m10} g_{m14}}{(g_{o13} + g_{o14})(g_{m7b} g_{o4} g_{o10} + g_{m4} g_{o10} g_{o14})} \tag{4}$$

$$G_{MX} = \frac{g_{mb1}}{g_{m8b} + g_{m3}} \left(g_{m8a} - g_{m3} \frac{g_{m12}}{g_{m11}} \right) + \frac{g_{mb2}}{g_{m4} + g_{m7b}} \left(g_{m4} - g_{m7a} \frac{g_{m12}}{g_{m11}} \right) \tag{5}$$

Finally, the CMRR can be calculated as:

$$CMRR = \frac{A_{dm}}{A_{cm}} \tag{6}$$

The equation (5) reveals that the CMRR can be reach infinity providing that can be satisfied.

$$\frac{g_{m8a}}{g_{m3}} = \frac{g_{m4}}{g_{m7a}} = \frac{g_{m12}}{g_{m11}}$$

3. SIMULATION RESULTS AND DISCUSSIONS

The simulations are carried out using TSMC 180 nm, MS/RF, 1P6M, CMOS technology utilizing Spectre RF simulator. The amplifier is biased with a 0.5 V power supply and loaded with a 15 pF capacitor (C_L). The bias currents and device sizes that are used in circuit simulations are given in Table 1.

Open loop frequency performance of the proposed OTA is pictured at Figure. 2. As depicted in this Figure, the open loop gain is 93.36 dB and the amplifier delivers the unity-gain bandwidth of 26.5 kHz with a 60o phase margin. Fortunately, the obtained DC gain and UGF cover the system requirements in most biomedical implants.

One of the most important parameters of amplifiers in biomedical applications is CMRR. Interestingly, the proposed structure presents more than 128 dB CMRR which is shown in Figure 3. Willingly, the 3-dB bandwidth of the CMRR is about 20 Hz which is rather high and acceptable for most of biomedical applications. This high value of CMRR is due to the balanced configuration of the proposed structure which suppress the common mode signals effectively. This also confirms the discussions from (4) and (6).

Table 1. Transistors Sizing and Element Value

Element	Value (µm/ µm)	Element	Value (µm/ µm)
Mb	150/ 3	M9, M10	380/1.6
M0	450/3	M11, M12	185/3
M01,M02	60/9	M13	180/1.5
M5, M6	19.2/5	Cc	4.5 pF
M7a, M8a	73/3	Rc	700 KΩ
M7b, M8b	30/3	Ibias	35 nA

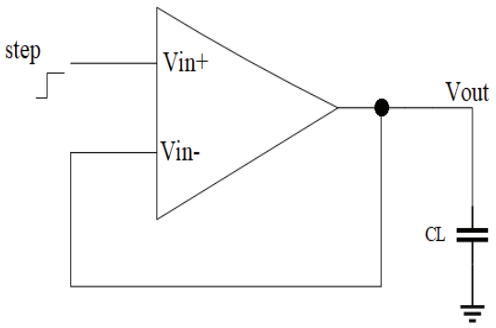

Fig. 6. Unity-gain feedback configuration

The IRN parameter for the OTA structure is pictured in Figure 4. From this figure, it is noticeable that 1/f noise dominates at low frequencies, whereas, in higher frequencies, it is the thermal noise that plays the main role. The input referred noise is evaluated to be 14.14 pV^2/Hz at the core frequency of 1 Hz, which is an acceptable value for most the biomedical applications.

To ensure the full range operation, the input common-mode range is investigated. As shown in Figure 5, the output voltage experiences a full rail-to-rail swing (from 0 to 500 mV) while the input voltage varies from negative rail to the positive rail. This rail-to-rail input common mode range is indeed essential in low-voltage circuits with reduced voltage swing ranges.

To examine the transient performance of the proposed amplifier, it is configured in a unity-gain feedback (see Figure 6) and a step voltage with

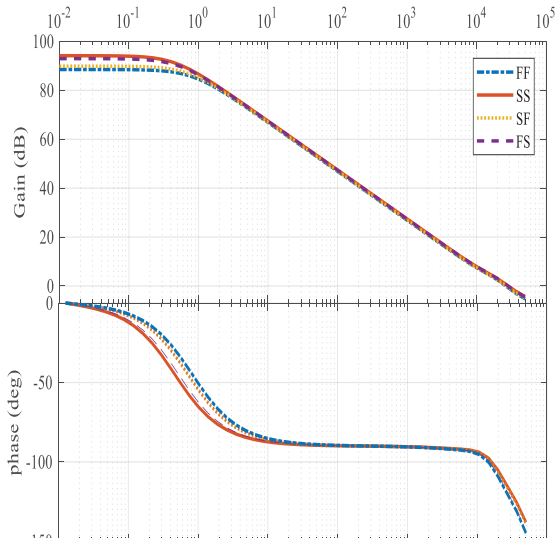

Fig. 8. The transient step response

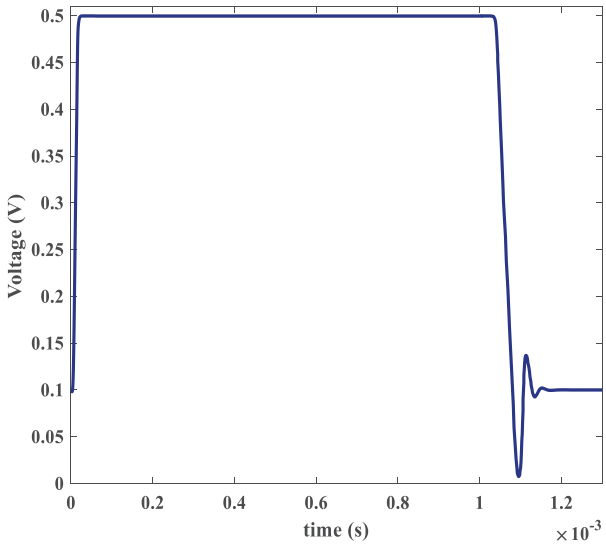

Fig. 7. The transient step response

the amplitude of 0.9 Vpp (100 mV – 1 V) and frequency of 500 Hz is applied to its input node. The transient response is displayed in Figure. 7. This figure exhibits a settling time of 75 µs and an average slew rate value of 20 V/ms.

Figure 8 shows the results of the corner simulations for the proposed amplifier. The frequency performance of OTA structure is examined in this figure. Some abstracts from this figure are given in Table 2. Fortunately, the results (from Figure 8 or Table 2) are promising as they confirm the robustness of the proposed circuit against process deviations.

Finally, the Monte Carlo simulations with 1000 runs are conducted to evaluate the robustness of

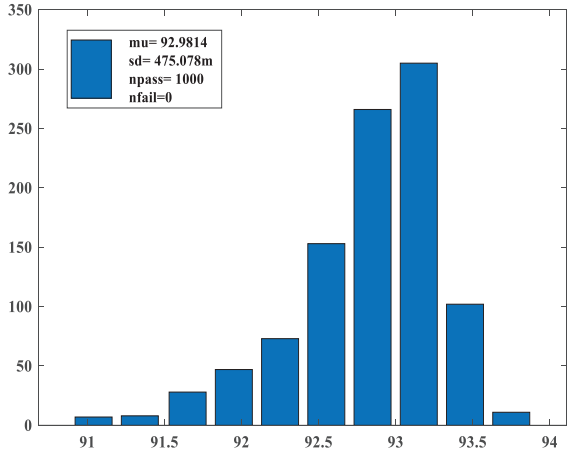

Fig. 9. The spectrum of voltage gain

Table 2. The OTA Parameters Comparison

parameter	This work	[17]	[16]	[15]	[14]	[13]	[12]	[5]	[4]	[3]
VDD (V)	0.5	0.5	0.5	0.5	0.8	0.9	0.6	0.6	±1.5	-
G_0 (dB)	93.36	83.58	65	67.8	59	70	73.5	80	76.2	48
CMRR (dB)	128.36	129	86	-	-	59	67.4	130.2	129.67	-
C_{load} (pF)	15	15	20	15	25	20	15	15	7.5	0.5
UGF (kHz)	26.5	23.8	550	3.26	0.21	5.6	13.02	19.1	10.84	29
PM (°)	60	61.5	50	68.9	83	62	54.1	60	74	60
P_{diss} (nw)	263.8	351	28000	26	40	450	550	400	15174	21
FOM (dB kHz pF/nW)	140.67	140	25.5	127.5	7.74	17.4	26.1	52.3	0.408	33.14
Technology (nm)	180	180	180	180	350	2500	350	180	180	90

Table 2. OTA Parameters in Four Operating Corners

OTA parameter	Corner			
	FF	**FS**	**SF**	**SS**
G_0 (dB)	88.5	92.94	89.88	94.14
PM (°)	60.75	60	64.2	63
UGF (kHz)	26.32	28.35	26.77	28.75

the circuit against process variations. To do this, the DC gain of the amplifier is examined. Figure 9 presents the results. For DC gain, the standard deviation and mean value extracted from Monte Carlo simulations are obtained to be 475.078 mdB and 92.9814 dB, respectively. The results show well-agreement with the results from the corner simulations, validating the circuit performance over process changes, further.

To fairly investigate the performance of proposed circuit with that's of other similar works, we used (7) as the figure of merit (FOM), which is adopted from [5]:

$$FOM = G_0 \times \frac{UGF}{P_{diss}} \times C_L \tag{7}$$

The functionality of the proposed structure is compared with that's of recent relevant circuits at Table 3. The results show better functionality of the presented structure at some important parameters such as Gain, UGF and FOM.

4. CONCLUSION

A high-gain low-power body-driven folded-cascode OTA is presented in this work. The simulation results, proved the advantage and improved functionality of this work compared to other similar works. The results are validated with Spectre circuit simulator employing TSMC 180nm CMOS standard process. Due to the good characteristics of the proposed structure in terms of CMRR, noise, gain, and power consumption, it tenders itself well, for utility in low-voltage applications, especially in the processing of biological signals such as EEG and ECG signals.

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Research Article

Movement Aware Smart Street-lights for Efficient Energy Utilisation

Muhammad Sohail Khan, Abdul Qadeem, Faiz Ali, Bushra Naeem*, Bilal Shabbir, Raza Ali, and Muhammad Ali Shoaib

Faculty of Information & Communication Technology, Balochistan University of IT, Engineering & Management Sciences

Abstract: Now a days, energy is an essential resource as the number of energy resources are sinking day by day. Movement Aware Smart Street light is a simple yet powerful concept, which uses transistor as a switch and replaces the manual system. It instantly switches the lights ON when the sunlight goes below the visible region. As energy is the scarcest source, this requires finding innovative ways to use it efficiently. Big cities consume a large amount of electricity and it is required to save energy by operating the street-lights at the time of need. In this paper, an effective method of street-light operation is presented which detects the sun set and sun rise alongside detection of vehicle movement on roads to utilise the energy only when it is required. Furthermore, a system is proposed which reduces energy consumption by replacing manually operated street-lights as they are not switched OFF even the sunlight appears and also switched ON earlier before sunset. The proposed mehtod has saved the municipal utility budget by 35% - 45% through energy saving. This paper also discussed the elimination of manual operation like ON time and OFF time setting and clearly demonstrates the working of transistor in saturation region and cut-off region.

Keywords: Energy Consumption, Intelligent lighting Control System, Automatic Street-lights, Traffic Density, Sensors.

1. INTRODUCTION

The perception of street-lights from rural to urban areas is mounting rapidly. Street-lights offer safe night time journey for passengers. To grant a safe street-lights to road users is a main responsibility of the city administration. One of the critical apprehensions of developing countries is street-lights because of considered significance of social and economic steadiness. Unnecessary lightening wastes important economic resources every year whereas, on the other hand poor lights create insecure situation. One of the noticeable power losses are road lights (energy consumption is one of the most dangerous streets inciting factors and is also one of the largest energy expenditure) and with the help of advcane technology, 35-45% of the cost of municipal utility budget can be saved. Civilization indexes of any society are development of the transportation facilities. Highways, roads, and streets are the main component of the transportation model which needs to be properly illuminated for proper visibility. Automation systems preference is on manual mode as it reduces unused energy consumption. The energy-saving automated systems play an important role in making daily life opportune for consumers from deck ventilators to washing machines and many other different applications. Above all, road lights play a vital role and have a significant role in lighting which provides security during the night. During the night all the street-lights are ON but at day time if, the lights are ON it is wastage of energy and resources. On normal days street-lights operation requires adequate amount of energy. Energy being a precious resource must be saved and it can be as simple as turning the lights OFF when its not needed. Traditional energy sources like coal, petroleum, hydro-electric power and natural gases are limited energy sources that need to be consumed smartly. In this regard, the intelligent lighting control system reduces the cost of street lighting up to 70% and

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*Corresponding Author: Bushra Naeem <bushra.naeem@yahoo.com>

increase the capacity of the equipment. Traditional lighting was restricted to only two options and is also not effective for this type of operation because the loss of energy due to continuous working to the maximum voltage and the wastage of energy resources. Automation in any industry can increase the efficiency and same goes for street-lights. By manual operation of street-lights, if we turn ON a light it will remain in the ON state for a straight 13 hours on average despite the scenario on the road whether there is any traffic or pedestrian movement or not. By transforming the manual operation to automate we can save an adequate amount of energy. In smart street-light system, the natural light intensity is measured and accordingly the street-lights are turned ON/OFF. Movement on the road is detected by the system which will accordingly keep the lights operational [1-3]. Movement aware smart street-light and properly design mechanism can reduce the cost of street-lights. Implementation of movement aware smart street-light helps to reduce utilization of power, and efficiently consumption of renewable resources for the applications related to street-lights and signals for traffic. The primary goal is to reduce energy usage when cars are not moving on the highway. Lights turned on when cars are on the road and turned off otherwise. So, the system provides a solution for energy savings. In this paper, movement aware smart street-light is proposed and observed that 70% of the energy can be saved as compared to other automatic street-lights and manually controlled systems.

1.1 Literature Review

Due to high energy consumption by street-lights several research based systems are proposed to reduce the amount of energy consumed by street-lights and use them efficiently.

S. Suganya *et al.* [1] have suggested street-light operation by sensing of car motion. It is a system that utilizes the LED technology. The proposed method automatically regulates the street-light operation. K.Santha Sheela *et al.* [2] proposed an algorithm in which street-lights operates according to luminosity and is self-adjustable to different seasons. Prathib Kumar *et al.* [3] studied street lighting scheme, the research was based on the movement of vehicles. It automatically controls the street-lights according to pedestrians and vehicle movement on roads. P. Caroline Cynthia1 et al [4] proposed an automated street-lights control system that detects the movement of objects and vehicles in the streets and accordingly operates the street-light system. The proposed scheme ultimately saves the energy to light-up the system for more hours and increase the lifetime too. K M Harshitha *et al.* [5] proposes a system in which the street-lights glow with maximum intensity during traffic movements and dims the light when there is no movement. The proposed system also identifies the fault location if occurred. Mohd Azaz *et al.* [6] proposed a vehicle detection model which uses laser sensors to detect the vehicle and automatically operate the street-lights by reducing the power consumption in low traffic or zero traffic scenario. A study in [7] designed a system that is based on movement detection and sun rays to operate the street-lights. It avoids the manual operation to control the street-lights. The proposed system is based on Aurdino which used light detection resistor and an infrared sensor. The energy is saved by turning the lights to dim in night hours and it turns to glowing state when it detects the vehicle movement.

Researchers in [8] proposed a microcontroller based system that controls the LED based street-lights. The system glows the street-lights with 5 different levels according to traffic intensity and resultantly saving the 77-85% of energy. Study in [9] developed an automatic system to switch ON/OFF the street-lights intelligently. It uses light sensors that detects the sun rise and sun set times according to geographical area and adjust the brightness of street-lights accordingly. The system saves the energy by operating street-lights intelligently and making the life time of lights longer. Baburajan, S *et al.* [10] proposes a model for street-lights that replaces the conventional lamps with LEDs and a method is introduced to control these LEDs automatically via motion detectors at the same time the operation of LEDs can also be controlled with a mobile app. The faulty lamps can be traced via the mobile app. The battery is charged using solar energy during daytime and provide electricity at night. Y. M. Jagadeesh *et al.* [11] uses infrared sensors to sense the traffic intensity on a road and uses a microcontroller to automatically adjust the street-lights intensity according to the traffic density. The proposed model in [12] uses two sensors LDR and vehicle video sensors. The LDR

adjusts the intensity of the light according to solar activity and vehicle video detectors sense the area for any movement of vehicle in the low light area and based on the value of these sensors the street light operation is performed intelligently by saving energy and providing street light wherever needed. An IoT based system was proposed in [13] which smartly detects the sun light and operates the street-lights . The system can monitor any suspicious activity. The proposed system is linked online with the web to control the operation remotely. In [14] authors propose a solution to timely controls the street-lights and utilize them efficiently to save the energy consumption. The proposed models consider certain factors like sun rise and set timing, 24 hours lightening conditions and weather conditions like dusty, raining or cloudy. The system design is based on nanowatt technology and it uses LED and light sensors. The model in [15] uses photoelectric sensors and light sensors to control the street-lights automatically and save the energy by operating the street-lights relative to sun light and movement in streets.

A smart street lightening project [16] proposed the automatic street-lights. The study proves that the proposed model can save up-to 45% of the energy. In [17], a method to save energy used by manual street-light control system was proposed which uses low cost and efficient components. The system was transformed to automated which controls the light intensity during high and low traffic and detects the solar rays to turn ON and OFF the street-lights. [18] proposes a system to control the street-lights using sensors network and Zigbee and GPRS technology. The street-lights are adjusted according to surrounding lightening condition and efficiently operated to save the energy. Zain Mumtaz et al in [19] and Somnath Rakshit et al in [20] also proposed a street-light controlling system which is based on Aurdino to sense the sun movement and detects the traffic intensity to control the street-lights. The study in [19] also proposes a system which count the number of objects passed the road.

S. Ganesh Moorthi [21] discussed about "Automatic street-light control by detecting vehicles movement". The author has used two types of sensors "IR and LDR sensors [21]" to automate the street-lights. Surrounding of passage and pedestrians is sensed by the sensors. The author has discussed about the lights on highway at night are ON for vehicles at the same time much of the energy is wasted when there is no vehicle or pedestrian. The paper discussed about that the lights needed to be ON at utter night fall. This approach wastes energy. Automatic Street-lights work as key solution to save energy. The paper aimed to detect the movement of vehicles on road to turn ON and to turn OFF the lights when desired to accumulate the energy [21]. Khalid Masood [22] discussed about "Automatic street-light intensity control and road safety module using embedded system "which aimed to develop a safer way for roads by using intelligent traffic street-light system. Based on the movement of vehicles automatic street-light

Table 1. Literature Review

Paper title	Cost	Energy consumption	Techniques used	Year
Street-lights that glows on detecting vehicles movement using sensors [1]	50% Reduction	High	Microcontroller based	2014
Automatic street-lights control based on vehicles detection using Arduino for power saving applications [4]	70% Reduction of fund budget	High	Arduino based	2017
Automatic streetlight control and fault detection [5]	50% Reduction	High	Microcontroller	2017
Arduino based solar street lighting [17]	Minimum	Average	Arduino, solar based	2018
Residential areas streetlights intelligent monitoring management [18]	Low cost	High	ZigBee and GPRS	2017
Automatic streetlights that glows on by detecting objects during night [19]	Low cost	50%	Arduino based	2018
Piezoelectric Transducer and Arduino Based Wirelessly Controlled Energy-Saving Scheme for Street-lights [20]	70% Reduction	Low	Arduino	2019

control is enabled to reduce the power consumption [22] during the hour of inadequate road usage by controlling the light intensity. All the literature available has used some techniques for making the street-light system automated and improving the energy efficiency by utilizing it smartly. Furthermore, the table1 gives a brief comparison between different schemes to control the street-lights smartly. The Table 1 presents the techniques used with comparison to reduce the cost.

2. MATERIALS AND METHODS

The street lighting automatic control system operates on a 12-volt DC current source. The automatic street-light control unit has a photoconductor that changes its resistance to the light range, which turns the lamp ON or OFF using the transistor as a switch. Based on light, a photovoltaic device was used as a transformer to convert photovoltaic energy into electrical energy. Figure 1 and 2 shows the circuit diagrams of the hardware circuit. In the proposed model IR sensor, LDR (Light Detection Resistor) and Aurdino UNO are used. The LDR senses the light intensity in the environment and sends its value to Aurdino which will decide to switch ON/OFF the street-lights according to the value detected. The IR sensor is used to detect the movement in the streets by emitting infrared rays. In the case of motion detection, IR sensors will trigger the street-light and it will turn ON otherwise, they will remain in OFF state. Figure 3-5 shows the detection of vehicle movement and ultimately turning the respective area street-light while the other lights remains off.

The central idea of the circuit is that the change in voltage falls through the resistor which depends on the lighting in the dark or the dark changes the transistor between the broken area or the saturated zone and turns off or lights up the LED indicator as we know the LDR during the time the resistance today is low, so the voltage at the inverted input is higher than the voltage at the inverted input and then the output at pin6 is low and thus the transistor goes to the cutting case which means the LED or the lamp will not glow. These outputs are sent from the LDR to an Arduino controller that specifies this reading time whether day or day if it is detected at night time from the control unit that checks the senor output if infrared sensors are detected and then the LEDs running are still out. We use six IR sensors sequentially when the vehicle approaches from the beginning, the first light is on, and when the first light is turned on, it will be automatically stopped and the operation is done by the second IR sensor and all other devices one by one. By this system, we can save a lot of energy without any manpower.

Inputs in the street lighting system are LDR and photoelectric sensors, after dark, the light sensor will activate the system, be ready to detect anything by infrared sensors, on the road to turn on the street lamps. Lamps will be used as street lighting in this paper. In this section, each circle designed will be discussed. First, LDR and RV1 form one arm of the bridge, while R1-R2 is the other arm. These weapons can be considered as potential separators, with the R1-R2 arm applying a fixed half-supply voltage to the inverted inputs of the amp, and with the LDR-RV1 being applied to a variable voltage at the centre of the terminal. In use, the RV1 is adjusted so that the LDR-

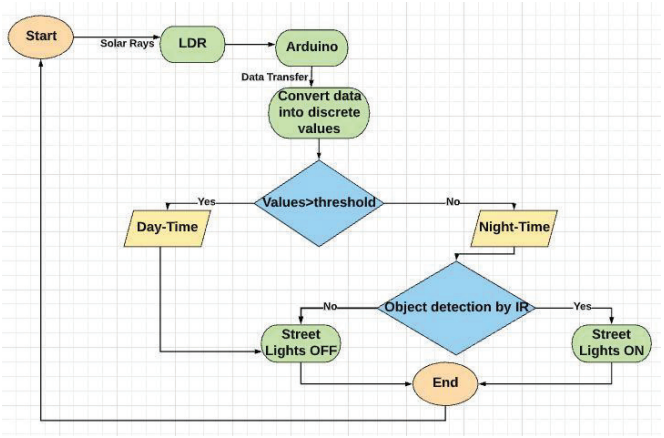

Fig. 1. Model Flow Chart

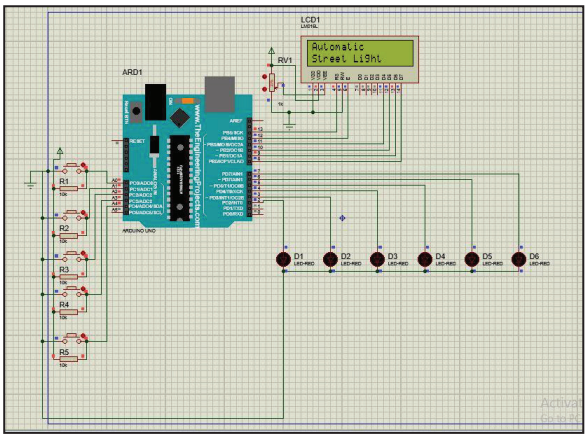

Fig. 2. Proteus simulation design

RV1 is slightly higher than the R1-R2, with the light intensity increasing to the desired trigger arm. In this case, the amp output turns to negative saturation, thus pushing the relay through Q1 and the R3-R4 bias resistors decreasing the intensity of mathematical methods and techniques of improvement in engineering [12]. Below this level, the output of the op amp goes to positive saturation. The circuit is very sensitive, so that it can detect changes in the light level so small that it cannot be seen by the human eye. The circuit can be adjusted to function as a dark active transformer accurately either by inverting the inverted and inverted reference amperes or by transferring RV1 and LDR. Furthermore, the circle reset is used to place Arduino in a known condition. Normally, when you reset Arduino, execution starts from address 0 of program memory. Also, the oscillator circuit was used to supply Arduino with an hour, so Arduino could implement a program using six IR sensors in this paper. Its function is to sense the target that pass across the street, at the same time giving a signal to Arduino to turn on the lamp. The idea of saving energy, where the system is designed to light the lamp at night only and only if anything is passing through the street. Except that the light will be off. The first infrared sensor is used to automatically activate the first Arduino lighting column when any object passes in front of it. Meanwhile, the second option electric sensor turns the second light column on and off after a short delay when the object passes in front of it. The third sensor will activate the third light column when the object passes in front of it, and the second light column will turn off after a little delay.

Fig. 3. Vehicle Detection in Start

Fig. 4. Vehicle Detection in the Middle

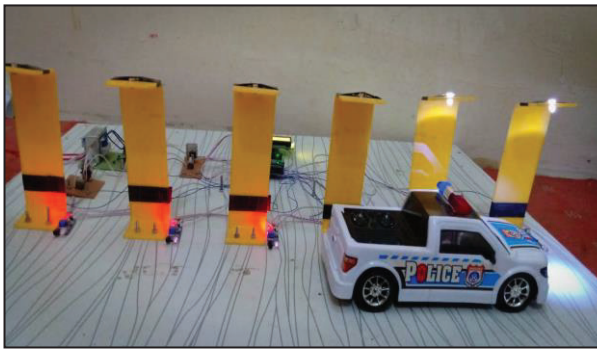

Fig. 5. Vehicle Detection in the End

3. RESULTS AND SIMULATION

The proposed scheme was simulated using MATLAB and targeted results were achieved. The hardware model designed detected the vehicle up-to the mark and lights are illuminated at every stage (start, middle and end). Random traffic is modelled for the proposed scheme to check the efficiency with respect to the traffic intensity in different hours. Figure 6, shows the traffic scheme for the simulation model, busy and low traffic can be seen. Figure 7 shows the energy consumption with respect to modelled traffic intensity scheme. The Figure 8 shows the time and traffic relation of a road. It shows that, in some hours the traffic is more on the road shown in the graphs peak values and in some time traffic is minimum or less traffic on road that is also shown in the graph down values. Graph

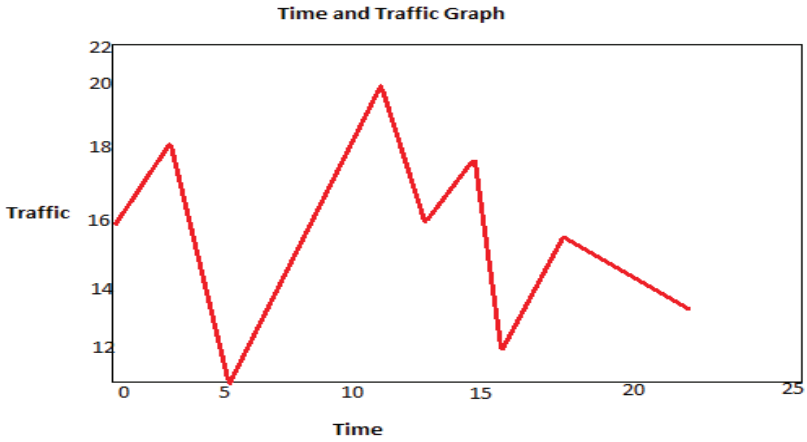

Fig. 6. Traffic Intensity throughout the Day

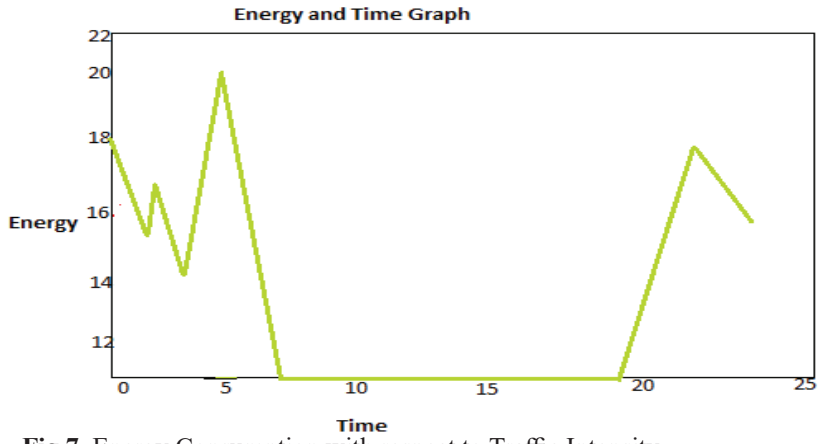

Fig.7. Energy Consumption with respect to Traffic Intensity

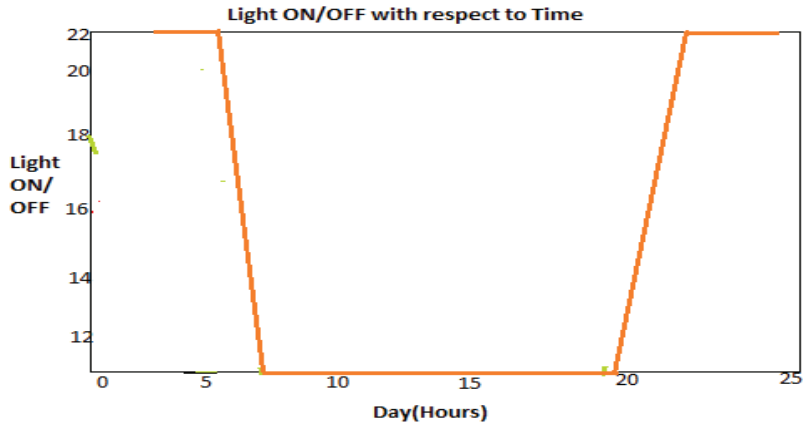

Fig.8. ON/OFF lights

Figure 7 shows the energy consumption at the time due to traffic.If the traffic is more the energy consumption will be greater at night time and if the traffic is minimum the energy consumption will alos be minimum. At the day time there will be no energy consumption because this system is LDR based. The threshold value is selected to control the lights. Smart algorithm designed in MATLAB is considered traffic density in different situations of the day and refers to the system to provide automatic control. Figure 8 shows the duration of lights are ON at day time and OFF at night time. The system is based on LDR, therefore, at day time all the lights are OFF as shown in Figure 9. The traffic intensity on the road at night time is also shown in Figure 10. At night time all the lights will

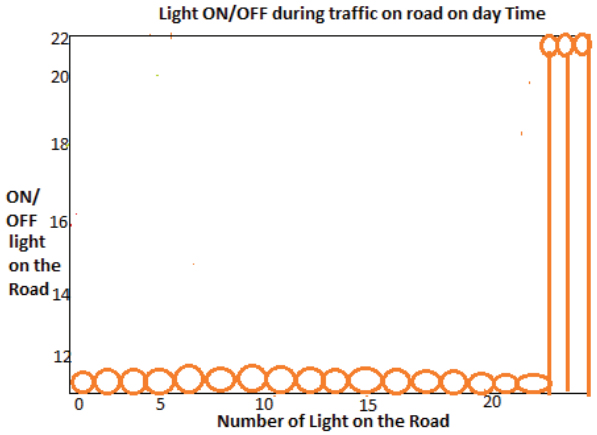

Fig. 9. Day Time Graph

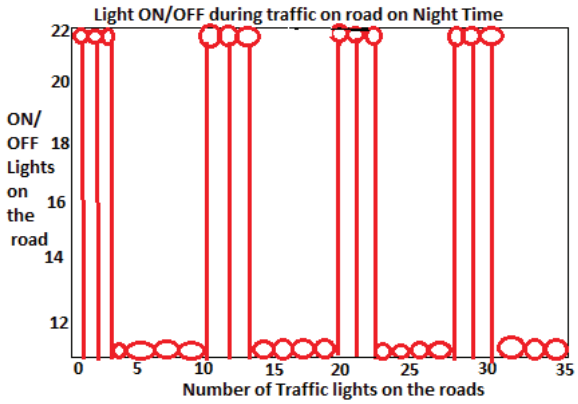

Fig. 10. Night Time Graph

turns ON. The high values shows the ON lights when the sensors detscts vehicles or other objects and the zera values shows that the lights are OFF because of no detection on sensors.

LDR is used in the proposed model due to which it is traffic aware and always turns street-lights ON whenever it senses any movement or traffic in the street. Another thing that should be observed that the energy is consumed only in dark hours when sun light is not present which shows that our model is smart enough to operate only in the scenario when natural light is not available.

4. DISCUSSION & CONCLUSION

The LDR light intensity and traffic density control module in developing countries will be more cost effective and security compared to complex lighting control systems. The intelligent lighting control system applied in this work is easy to use and can increase power. According to natural light intensity and IR sensors, used to detect the movement in the streets, the lights remain turned OFF for no movement and if it detects movement it will turn ON the lights. In this prototype, street-lights were successfully controlled by a microcontroller through commands from the console. In addition, the disadvantages of the street lighting system have been overcome by using the timer, where the system depends on the photoelectric sensor. The proposed model is smart enough to achieve the energy efficiency by controlling the street-lights smartly. LDR is used in the model to detect the solar rays through which the model will turn OFF/ ON the street-lights. The system under experiment can be used for short as well as long roads.

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Research Article

Vulnerability Assessment of Urban Floods in Lahore, Pakistan using GIS based integrated Analytical Hierarchy Approach

Sahar Zia[1], Safdar A. Shirazi[2], and Muhammad Nasar-u-Minallah[3*]

[1]Department of Geography, Lahore College for Women University, Lahore, Pakistan
[2]Department of Geography, University of the Punjab, Lahore, Pakistan
[3]Department of Geography, Govt. Postgraduate College Gojra, Pakistan

Abstract: Urban flooding is getting attention due to its adverse impact on urban lives in mega cities of the developing world particularly Pakistan. This study aims at finding a suitable methodology for mapping urban flooded areas to estimate urban flooding vulnerability risks in the cities of developing countries particularly Lahore, Pakistan. To detect the urban flooded vulnerability and risk areas due to natural disaster, GIS-based integrated Analytical Hierarchy Process (AHP) is applied for the case of Lahore, which is the second most populous city and capital of the Punjab, Pakistan. For the present research, the flood risk mapping is prepared by considering these significant physical factors like elevation, slope, and distribution of rainfall, land use, density of the drainage network, and soil type. Results show that the land use factor is the most significant to detect vulnerable areas near roads and commercial areas. For instance, this method of detection is 88%, 80% and 70% accurate for roads, commercial and residential areas. The methodology implemented in the present research can provide a practical tool and techniques to relevant policy and decision-makers authorities to prioritize and actions to mitigate flood risk and vulnerabilities and identify certain vulnerable urban areas, while formulating a methodology for future urban flood risk and vulnerability mitigation through an objectively simple and organizationally secure approach.

Keywords: Vulnerability, Urban Flood, Drainage, Analytical Hierarchy Approach, Lahore.

1. INTRODUCTION

The globally human population is at risk due to natural disasters and climatic changes. Doubtlessly weather-related phenomenon is significantly increasing both in terms of intensity and frequency [1-2]. Urban flooding is an emerging risk to the human population residing in developing countries of the world [3-5]. Yet, there is no proper technique and a defined method to gauge this growing urban risk due to the uncertain nature of the occurrence of urban flooding in geographic context [6-8]. There are many strategies for flood management and the vulnerability valuation is one of them [9]. However, the risk is commonly measured through vulnerability in case of any possible hazard occurrence. Hazard can be defined as a phenomenon that can damage property, threaten lives in an area with a given period. In short, any event with having a probability of damage can be called a hazard and it can be measured through risk assessment [10-12]. Some of the known risk elements include transport routes, services of public, commercial activities, residential and educational buildings as well as population itself [13]. Furthermore, risk also has a crucial component that is a vulnerability that determines either the exposure to a hazard will have an outcome of disaster or not. The exposure of urban flood was thought to be a risk in the beginning as compared to other types of flood. Recently, it turned into a disaster that started to damage the properties and disrupt the social lives as well as commercial activities yearly in megacities of developing countries particularly. The emerging issue seeks attention due to its direct impact on the population. The fact is that population is triggering this phenomenon. It has been observed that the urban population becomes more vulnerable too due

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*Corresponding Author: Muhammad Nasar-u-Minallah <nasarbhalli@gmail.com >

to dense infrastructure during urban flooding [12, 14]. Therefore, municipal authorities should keep themselves aware of associated vulnerabilities of an area for territorial management of urban flooded hotspots [15-16].

Several studies used models of hydraulic and hydrological to assess the runoff of flood along with runoff in areas that are flood-prone and low lying. For the assessment of flood risk, the information should be collected about the flood probability of occurrence, event magnitude, and management of flood, the depth and location of inundation [17]. Bhadra, Choudhury, & Kar [18], in their study used hydraulic models for the assessing the risk of floods along with techniques of GIS or Digital Elevation Model (DEM) for the purpose of mapping the flooded area as well as depth of inundation. Furthermore, various techniques are used to represent the maps of flood risk and vulnerability in which DEM and flood flows simulation in different periods of times used by various scholars of the world [19]. In addition, it has also been explained that for the extraction of inundation of flood extent in a cost-effective way and time the most effective technique for hilly basin of Dikrong is GIS because they are remotely located areas, where accompanying the conventional survey is very challenging [20]. Furthermore, for demarcating the hazard especially flood-prone risk zones in the region of Papanasam Taluk, GIS is used as the best technique in which areas are distributed into five major regions on the base of the varying extent of a flood [21]. Moreover, a map of flood risk has the capability to recognize the zones which are mostly in danger to the hazard of flood along with it. It also highlighted that any occurrence of flood events will affect various people.

The best methods which based on GIS are considered as highly suitable for mapping the risk of flood [22]. These are generally based on surveys of ground level and observations of the aerial base. On the other hand, when techniques become common, such type of phenomenon is considered expensive and time-consuming. Furthermore, a drawback of aerial photography is that timely photography mostly becomes difficult due to the prohibitive conditions of weather. Therefore, this study is an attempt to ensure a multi-parametric methodology for demarcating the vulnerability of flood in rapidly developing urban area through GIS-based integrated multi-criteria technique of AHP as used for policy and decision making procedure. Diverse and multiple criteria were involved in the evaluation of problems through AHP effectiveness & trade-offs measurement. Occasionally, data is limited and by using such data lead towards recognition across various fields of application [23]. Similarly, a framework is examined through the technique of analysis as multi-criteria which can deal with variables of various kinds for the documentation of the essentials for the decision of complex problems. Furthermore, in the structure of categorized nature, such a technique organize all elements and examine the associations among problematic components [24]. However, such a technique is still not renowned in the management of flood risk perspective [25].

Based on topographic and morphometric data, this study has the aim to create a map of flood risk areas that is easily detected by using the approach of AHP-GIS from available geodatabase. For the risk mapping of an urban flood, analysis through multi-parametric is commonly done that creates map of the composite index for provoking risk. Later, by assigning weights, the urban flood causative criteria are chosen. For current study, research questions are considered: (a) what risk criteria will be more suitable for integration of urban flood risk assessment in an urban environment; (b) how different selected weighted criteria can alter the percentage value of flood risk and its corresponding spatial distribution in an urban area.

2. MATERIAL AND METHODS

2.1 Study Area

The central part of Lahore is one of the cultural attractions not only from Punjab but from all over the country. Geographically, a selected area of Lahore city (as a study) extend from74 degree 12 minutes and 30 seconds E to 74 degrees 29 minutes E longitudinaly and latitudinal extends from 31 degrees 26 minutes N to 31 degree 39 minutes 30 seconds E [26]. This area is hypothesized for the emerging problem of urban flooding as per media reports. UN humanitarian agencies issued a warning that “In the province of Pakistan, Punjab, the heavy rainfall triggered flash flooding in Sialkot and

Kasur districts and also urban flooding in central Lahore and other big cities of Pakistan" as reported in Monday UN News, August 11, 2011. In the same year, an official claimed urban flooding is claimed as a chief cause for the outbreak of dengue fever in central Punjab in the month of September 2011, in Monday Foreign Policy-the global magazine of news and ideas, September 19, 2011.

Later, government efforts have been witnessed i.e. a MOU was signed between the Government of Punjab and Google to get cooperation in the IT sector to monitor several activities in Punjab. Under the agreement term and condition, Google provided technical assistance to the Arfa Software Technology Park. This cooperation between the Punjab government and Google was a good initiative even in providing relief to the people affected by the floods in Punjab in combination to add assistance to other sectors as reported in Pakistan Today, February 7, 2012. The paralyzed situation in Punjab cities in the month of August due to devastating rains is reported. All activities were disrupted due to persistent and heavy rain. This situation caused streets and roads into virtual rivulets in the entire city. In addition, deep rainwater covered the low-lying areas paralyzing the city. Knee-deep water was seen at Anarkali, Lakshmi Chowk, GhariShahu, Waris Road, Barkat Market, The Mall and Johar Town as reported in Dawn, September 10, 2012; Dawn, July 21, 2013; Dawn, August 15, 2013; dawn August 26, 2016.

In news, officials of WASA ensured the supervision of drainage operations. It has also been emphasized that Pakistan has a dire need to take structural and non-structural measures for hazardous events. It has been demonstrated that non-structural such as awareness campaigns found useful. A flood cannot be completely controlled or prevented however, damages can be reduced by concerned stakeholders working together as reported in Nation, August 7, 2015. Media reported for the urban flooding situations till 2015 was found in general perspective but in 2016 to onward, very keen interest has been seen on urban flooding in detail. Each event was recorded, for instance, 3rdJuly 2018 and 16th July 2019 were heightened by WASA for long hours rain and its adverse impacts to disrupt the city life.

2.2 Data and its Sources

Keeping in view, the purpose of the study, different sources of data have been incorporated, including topographic sheet, satellite imageries, land use and land cover base maps, drainage system capacity and rainfall. Later, ground-truthing has been done with the help of geo-tagging during field surveys to check the accuracy.

2.3 AHP as a Multi-Criteria Decision Analysis Tool

In a flooding place, the extent of susceptibility is determined through the process of mapping the vulnerability of flood. This process includes the selection of factors which are bio-physical as well as socio-economical; these factor as a combining form along with the preferences of decision-makers allow the user to create an index of composition sustainability. As a result, this process gave the problem of multi-criteria or multi-parameters making of the decision. In the mapping of urban flood vulnerability, integrate AHP with analysis of GIS, the proposed approach presents in Fig. 1 with a schematic outline. In Figure 1, steps compromises on primary data, with manipulation of data in the environment of GIS, analysis for multi-criteria decisions, and analysis for the sensitivity of the model. Such steps are further categorized into two stages of primary and secondary for the process. In addition, Figure 2 shows different datasets for the prediction that show in a hierarchal structure.

2.4 Flood Vulnerability Mapping Variables and Analysis

Elements of topographic and morphometric along with analysis of variables are present in this section, which become notable factors for developing decision making by AHP-GIS. Notably, for AHP that is multi-parametric, the variable selection depends upon the types of flood. Furthermore, in this unit the selection of different variables that are utilized in vulnerability analysis along with their classification into classes of intensity and risk. An analysis of decision making, the spatial reference is a profound and important step for criterion choice. Criteria of this study are undermined according to their significance in causing the flood. Such factors include slope and elevation, types of soil, the

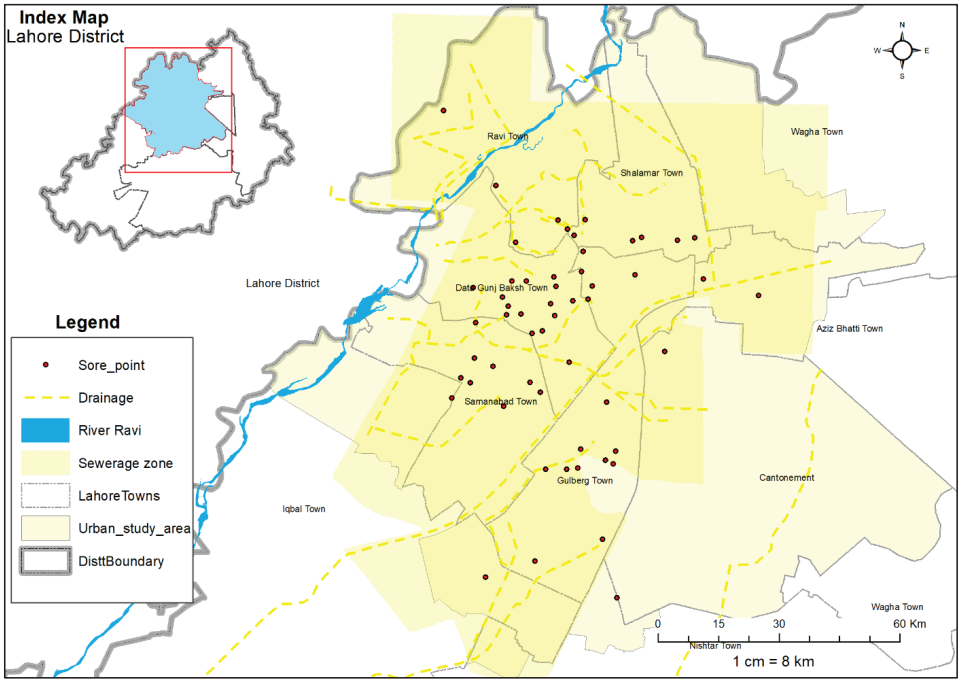

Fig. 1. Depicting geographical location of the study area (Central Lahore)

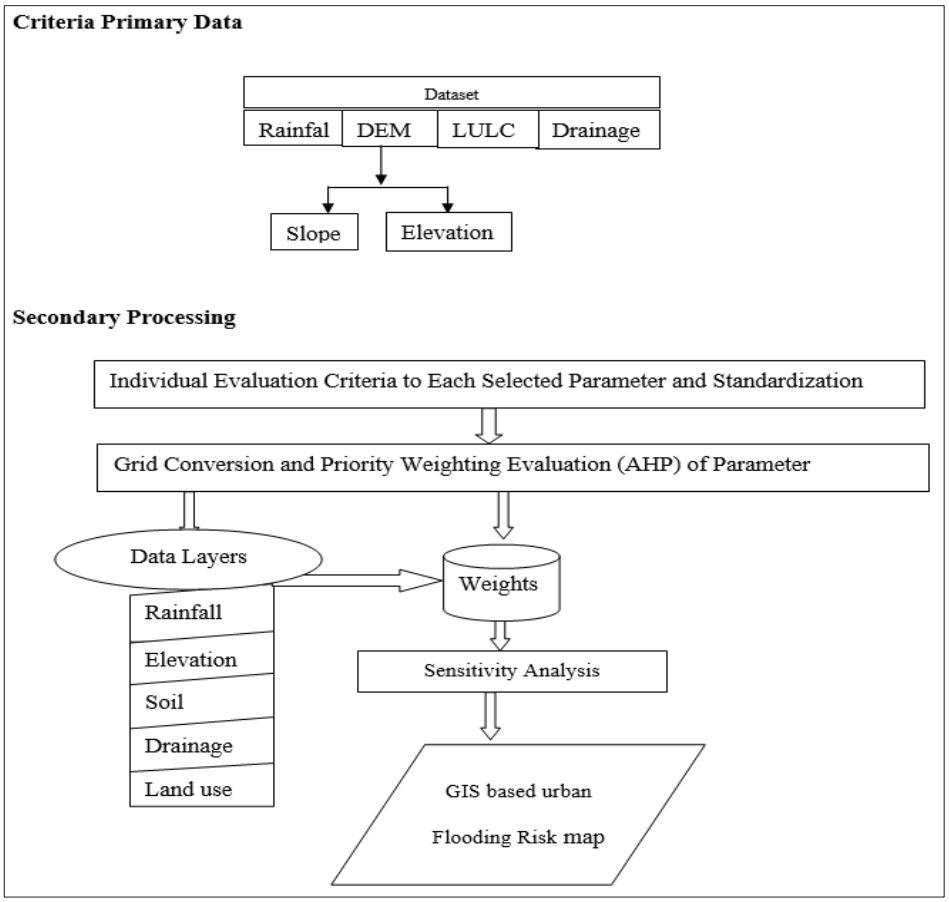

Fig. 2. Depicting the detailed Methodology flow chart

annual distribution of rainfall, density of drainage, and information of land use patterns.

3. RESULTS AND DISCUSSION

3.1 Elevation Profile

The elevation profile is a chief factor to measure urban flood risk. Generally, slope gradient determines direction of flow, duration, and infiltration rate of surface runoff. Both types of elevation profiles, either flat or steep, create environmental disturbances. However, role of human intervening cannot be ignored here for example, construction of metalled roads in plain areas and deforestation on steep slopes i.e. mountains. A flat surface allows rainwater to flow slowly and causes urban flooding in case of metalled roads. On the other hand, steeper slopes are also susceptible to surface runoff, if deforested. These slopes without dense trees, will lead to destabilize the soil and increase the chances of landslides. In present case study, Excessive storm water always gathers in depressions. For example, hotspots of urban flooding are natural local depressions. These local depressions can be generated by DEM. Those cells of DEM that are having a lower height than the nearby ones would be further significant for gathering excessive rainwater. These depressions are known as accumulated areas and have been shown in Figure 3.

A map through the model of digital elevation was prepared along with the use of tools related to hydrological software like Arc-GIS. For the projection of the inundated area, the tool of accumulation with raster data as input is used for the specific study area. Results show that accumulation flow obtained through applying the value of these hold that is 1.5 for selected cells which show the highest flow. Furthermore, results also clarify that mostly sore points lie near or within the area of high accumulation below the image in five classes is reclassified through selecting zone of high accumulation and assign them a value which ranges from 1 to 5 for raster cells that having less tendency of flow flat terrain or steepness identified with a tool of a slope which calculated the slope condition in the affected area. If the value of the slope is low, then cells clearly indicate that area is covered with flat terrain. On the other hand, the high value of raster cells indicates steep slopes.

3.2 Rainfall distribution

An intense amount of rainfall in a particular area for very long time became a reason of the flooding of all types. With heavy rain, flood mostly occurs due to the low capacity of flow channels either they are natural or manmade for drainage the excessive amount of water. Urban flooding mainly has an association with the amount of rainfall, in such areas water cannot infiltrate in soil and start to flow as runoff that gave the result of inundation which spread over the whole region. Thus, in a region it can be described that amount of rainfall clearly indicates the rate of runoff. In this study, it is observed that flooding in urban areas has a major cause that is local rainfall. As a result, it was concluded that the risk and hazard of urban flooding occur through the events of heavy rainfall. That is why events of rainfall that occurred at the local level are integrated by analysis. For the period of eight years such as from 2002 to 2017, take into consideration the amount of mean annual rainfall. For the study area, raster continuous data of rainfall was created with the help of Invasive Distance Weighting (IDW) for the process of interpolation. By using the equal interval, resulting layers of raster data are reclassified into five classes, in which the least rainfall class has a value of 1 while the highest amount of rainfall class is denoted by 5. Figure 5 highlights the outcomes of the raster layers of rain, data layers of interpolated IDW as well as data of rainfall as reclassified.

3.3 LULC Management

In the present research of urban flooding, land use consider as significant aspects. The current situation of land is represented with this factor, along with its pattern also discovered which developed with activities of humans in that area. Land use land cover has a relationship with the rate of infiltration in the soil during the time of the intense event of rainfall due to which consider as significant. Soil cover either the land is barren or cover with agriculture, always has a noteworthy influence on its aptitude in which soil act as an absorbent of water before as well as after the events of rainfall. Soil that presents in barren land openly always increases the rate of infiltration due to the runoff rate become

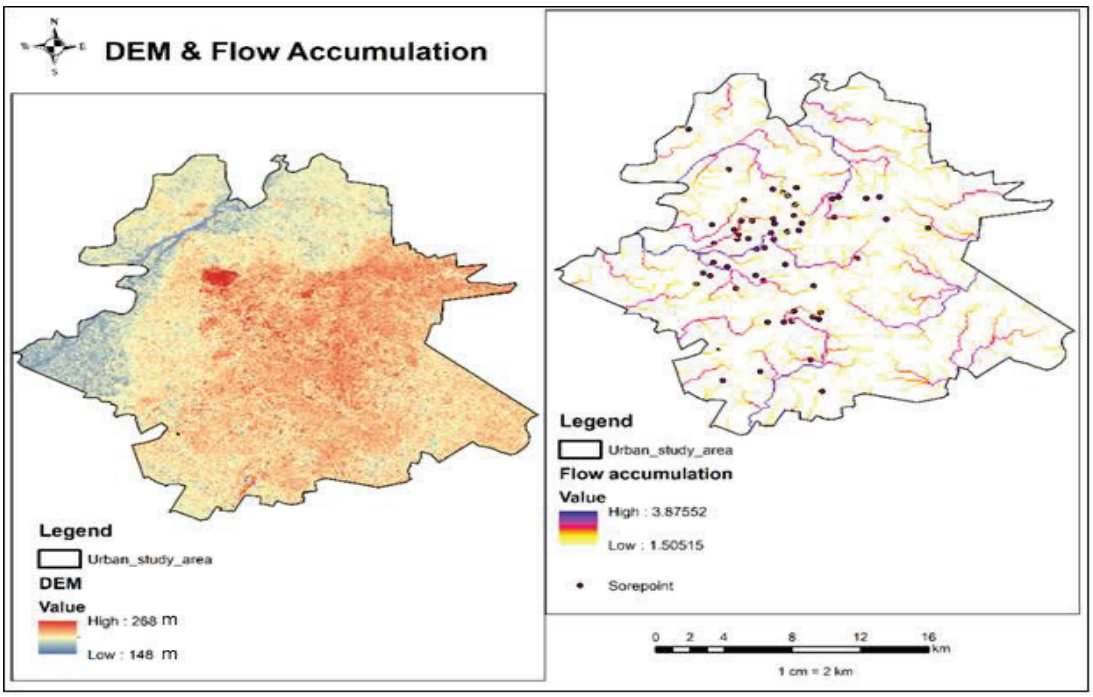

Fig. 3. DEM and flow accumulation

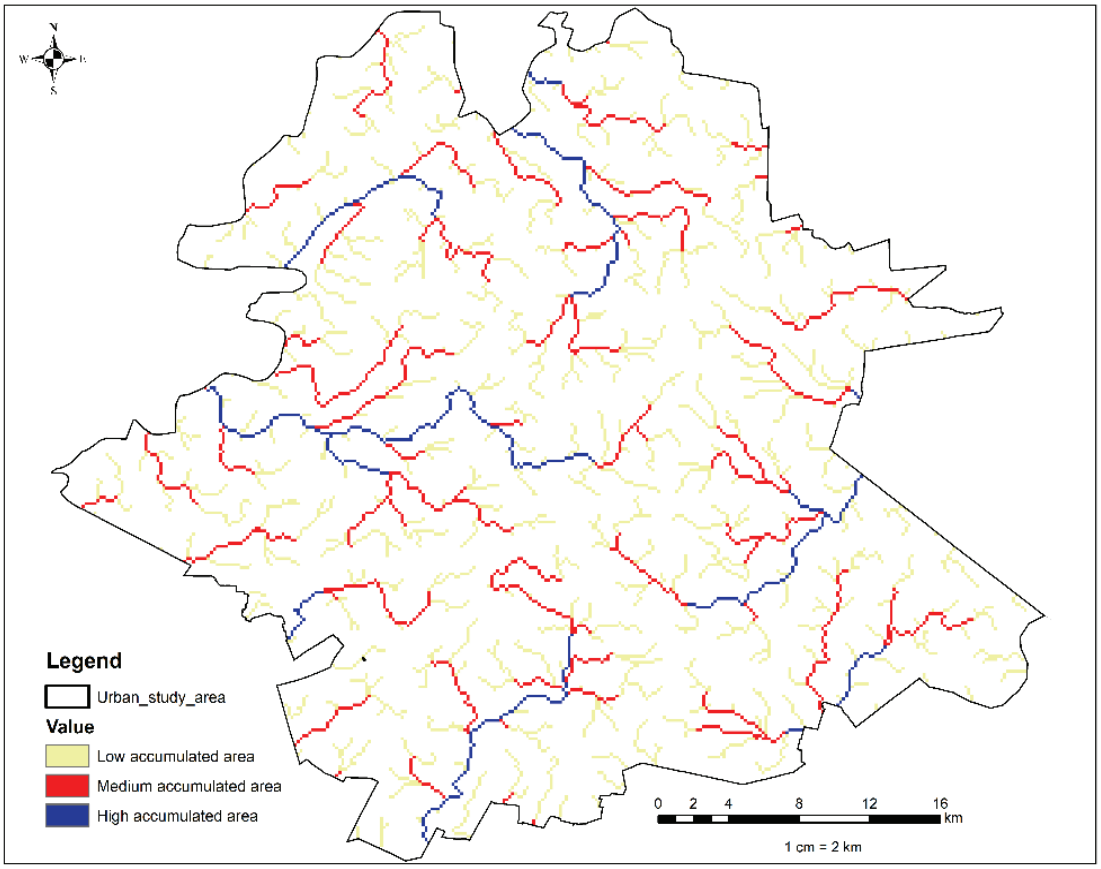

Fig. 4. Projected accumulated area extracted by DEM

low. Meanwhile, building roads which made up of concrete material and consider as impermeable land cover cause diffraction in storm water.

As an outcome, streets in the urban areas remain inundated usually during a season of monsoon. Land uses such as roads, buildings, areas of slums decrease the capacity of rainwater penetration in soil or increase the rate of inundation during intense rainfall. Similarly, types of land use always behave as an agent either the land cover with resistant or cover with absorbent. If the land has resistant quality than holing time decrease due to which chances of flooding also enhance. So that it is concluded that land use land cover is a significant element in determining the chances of events like urban flooding.

With the help of a map based on land use, the classes of land use were prepared for the study area. Map of land use is reassigned through categorizing into two types of land use pervious and impervious. Obtained raster classes of land use were classified again in five classes that move from a high value of absorbent toward land use of least absorbent and this division occurs based on their capacity by considering the land use type. For example, concrete roads are categorized as the least absorbent category. Figure 6 shows the results of the analysis done for land use and land cover.

3.4 Drainage and Sewage Network

In-network of drainage, the capacity of drainage and holding is a chief factor which controls the flooding in the urban area, the density of the drainage network predicts the chances of flooding in urban areas. From this, it is measured that if drainage has the highest capacity for draining the water of storm then probability becomes low about the occurrence of flooding in the urban regions. Maps related to the capacity of drainage originated from drainage map which has information related to the channel diameter of drainage, the tool of density used in the tool of spatial analyst through density is calculated related network drainage based on the pipe's size. Channels of drainage have a varying size which moves from 171 sq. km to 1.8 sq.km. After that, the layer of drainage density is reclassified further into subgroups which are three. Those areas which

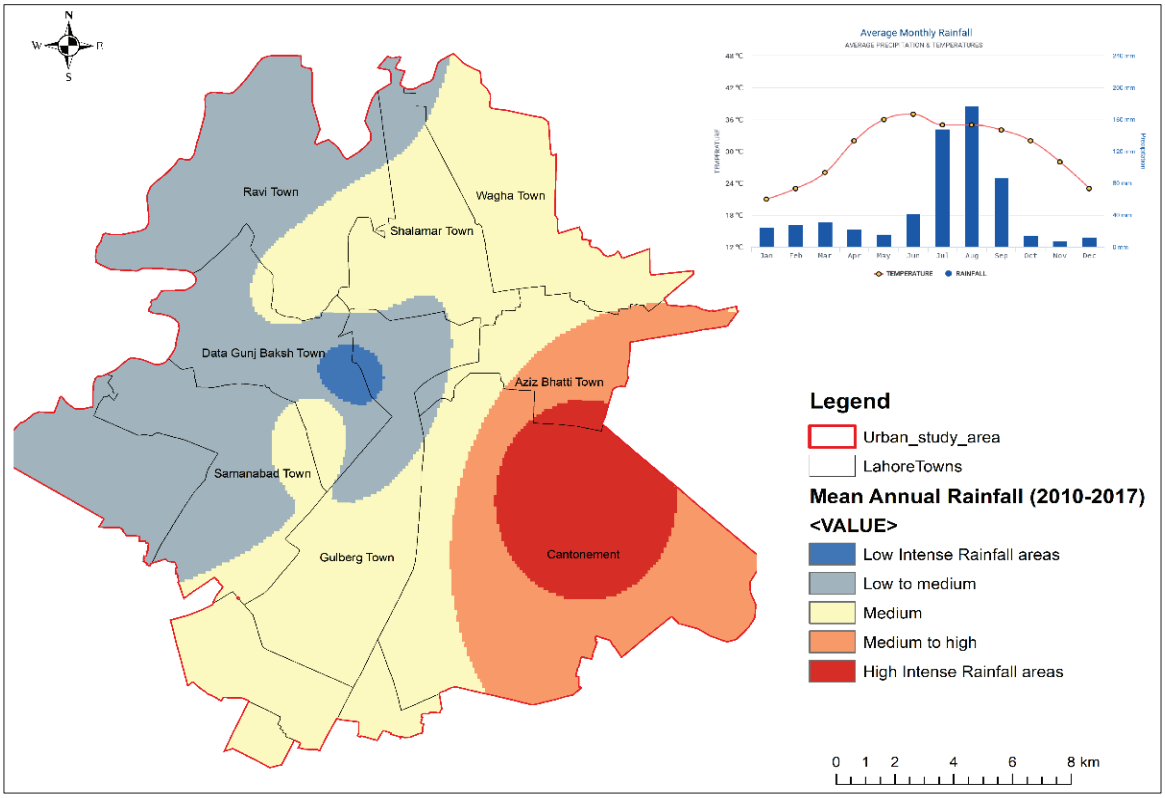

Fig. 5. Spatial rainfall pattern of Central Lahore (study area)

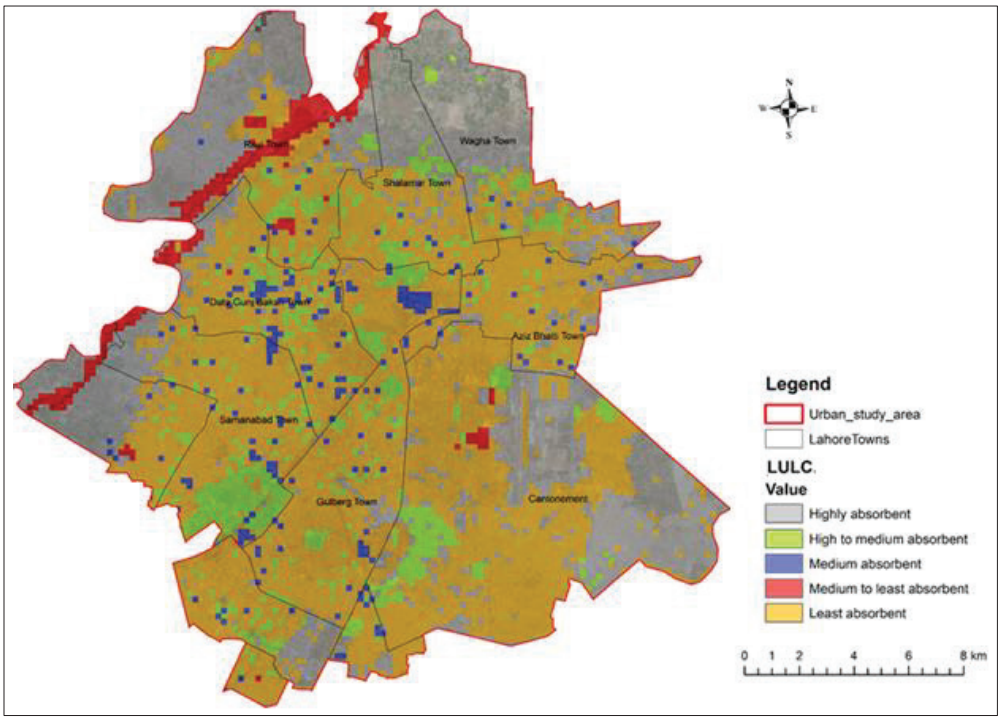

Fig. 6. Spatial rainfall pattern of the study area

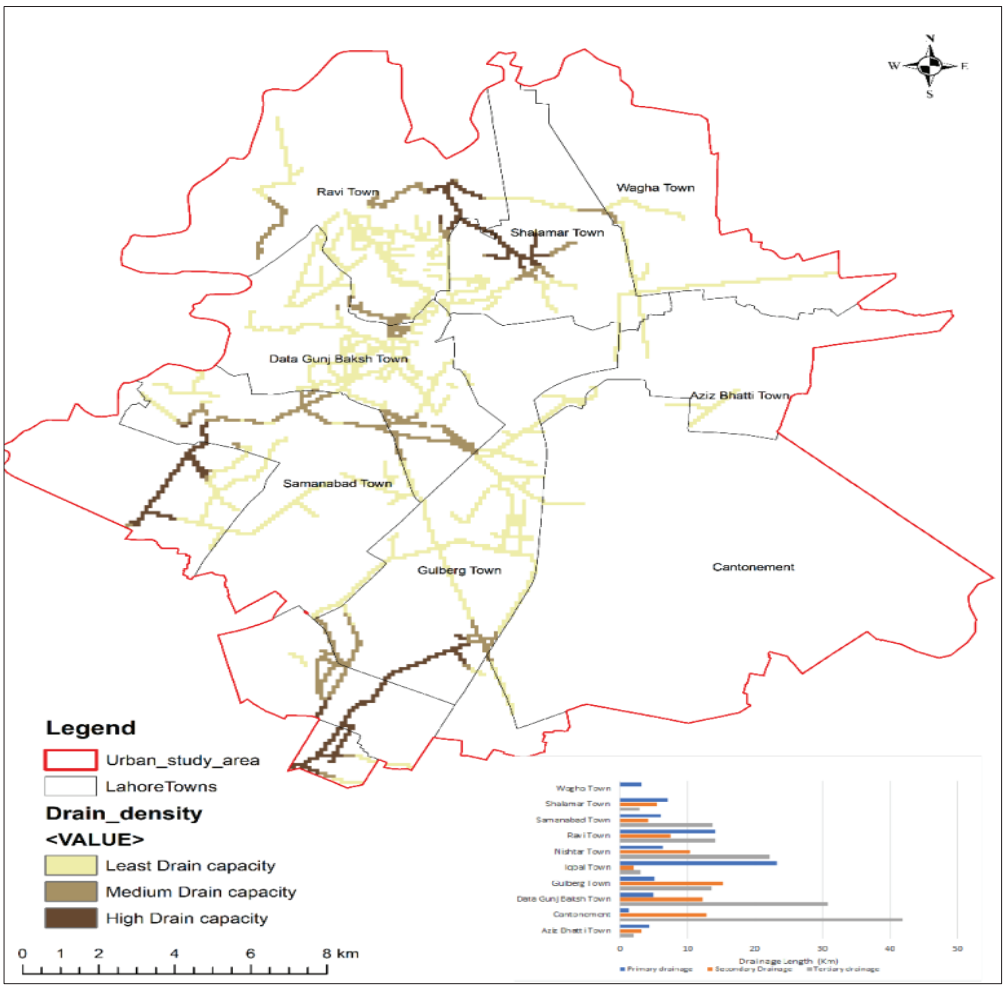

Fig. 7. Drainage network density

are having a low density of drainage are explained as 5. While those areas which show a high value of drainage density representing 1. The main step is ranking and prioritization in AHP.

The priority-setting impacts the usefulness of the decision directly. In the present study, to define the purposes and articulate the decision to recognize vulnerable areas as cited in literature by various scholars [14, 25, 27-28]. The eigenvector is utilized to weight the consistent layers of raster to set criteria in resulting in flood hazard. The outcomes of the comparison pairwise, ranking criterion, average priority vector, and percent contributions are presented in Table 1.

The consistency check CI is calculated from Table 1 by using the formulation matrix (max λ A XX =); where pairwise matrix is (A), and eigenvector of weights is (X). From the answer of max λ, the CI is acquired through the equation number 1:

CI = (5.41-n)/(n-1) = 0.10……Equation (1)

Where n is the deciding factor and n=5

In conclusion, the consistency ratio is calculated from the random index (RI), shown in Table 2 with the following Equation (2):

CR =CI/1.12 = 0.09……..Equation (2)

The obtained CR is 0.09 which is less than the threshold value of 0.1, and point toward high consistency in the judgments pairwise which concludes that the calculated weights are acceptable.

3.5 Identification of Urban flooded areas through AHP

The evaluation of the selected criteria the hierarchical model, LULC is considered as the most important criterion in site selection after ground-truthing with the weight of 5 as shown in Figure. 8. All detected points were mostly along with commercial and residential areas and validated with ground points as shown in Figure 9 (a) Roads, (b) Residential, and (c) Commercial.

Roads, residential and commercial areas are surveyed after the identification of vulnerable areas by the AHP technique. Results show 88 percent accuracy for roads, 70 percent for residential and 80 percent for commercial areas as shown in Table 3.

Therefore, WASA is performing critical activities before and during Monsoon such as setup of emergency camps before Monsoon season, continuous monitoring of the situation, ensured operation of disposal stations, functioning of channels for flow, and dewatering operations from low-lying pockets with the help of mobile squads. Massive program and media awareness campaign to educate people is also initiated last year for citizens to cop-up with flooding situations.

Table 1. Matrix of Pairwise Comparison

	Rainfall	Soil	Slope	Drainage	LU	sum	nth Root	Priority Vector	(%)
Rainfall	1.00	2.00	3.00	4.00	6.00	16.00	3.20	0.378698225	38
Soil	0.50	1.00	2.00	3.00	4.00	10.50	2.10	0.24852071	25
Slope	0.33	0.50	1.00	3.00	4.00	08.83	1.77	0.209072978	21
Drainage	0.25	0.33	0.33	1.00	3.00	04.92	0.98	0.116370809	12
LU/LC	0.17	0.25	0.25	0.33	1.00	02.00	0.40	0.047337278	5
Sum	2.25	4.08	6.58	11.33	18.0	42.25	8.45	1	
sum*PV	0.85	1.01	1.38	1.32	0.85	5.41			

Table 2. Random Index (RI)

1	2	3	4	5	6	7	8	9	10
0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

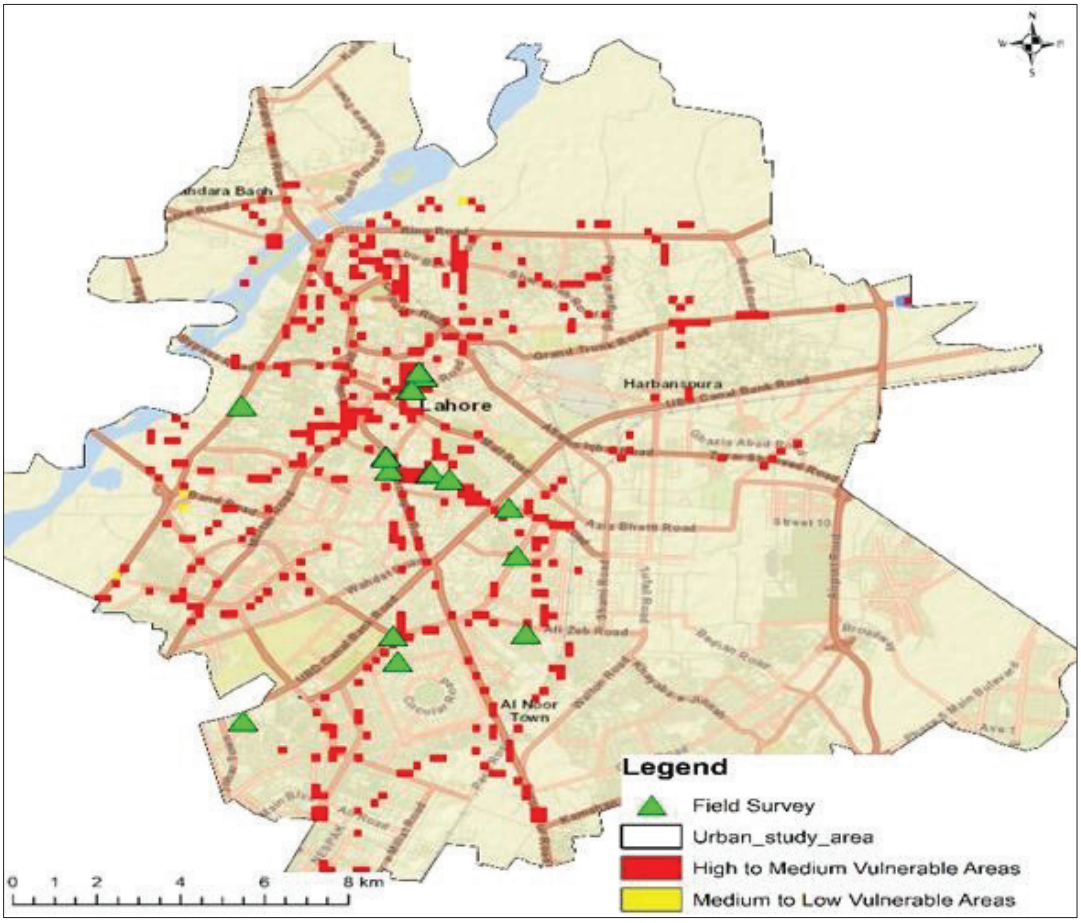

Fig. 8. Vulnerable areas in Lahore

(a) Roads (b) Residential (c) Commercial

Fig. 9. Ground trothing points

Table 3.

Name	x	y	Water depth inches	Land use	Area Sq. km.
Ferozpur Road	74.316	31.544	11	Commercial	0.01
Jublie Town	74.326	31.544	3	Commercial	0.13
Jail Road Samnabad	74.330	31.542	0	Commercial	0.00
Efu House, Jail Rd, Main Gulberg	74.345	31.534	4	Residential	0.01
Block J, Gulberg III	74.349	31.501	4	Residential	0.02
Abdul Haque Rd, Block G, Johar Town	74.281	31.477	5	Residential	0.02
Pind Nizam Pura	74.256	31.618	6	Residential	0.39
Dil Mohammad Road, Jogi mohalla	74.324	31.570	5	Residential	0.00
Main Bazar Gawalmandi	74.323	31.571	5	Residential	0.00
Garden Town	74.318	31.493	4	Residential	0.00
LCWU	74.325	31.543	3	Road	0.97
Main blvd, Block K Gulberg II	74.347	31.521	1	Road	1.18
Aziz Din Road, Sandakhurd	74.280	31.562	12	Road	0.25
Mcleod Rd, GarhiShahu	74.320	31.566	15	Road	0.41
Cooper Rd, Royal Park	74.321	31.566	4	Road	0.41
Lytton Rd, Mazang	74.315	31.548	10	Road	0.97
Qurtaba chowk	74.315	31.548	8	Road	0.05
Main boulevard Garden town, Aibak Block	74.317	31.500	2	Road	0.00

4. CONCLUSION

Urban flooding is a very timely subject for urban areas of developing countries. Solution of this growing threat is not possible without proper identification of risk areas for urban flooding. Therefore, this study employed the AHP technique, a method for evaluating statistically significant hotspots of urban flooded areas. Pair of several variables are selected and criteria is developed by assigning weights to take most appropriate decision in any other hazardous situation. Mostly focussed in the northeast of the present study area. The south eastern part is comprised of a cantonment (a developed settlement), reinforces the concept of informal settlements for causing urban flooding. Thus, Accuracy assessment for the AHP model was found appropriate with an overall accuracy of 80 percent in the case of Lahore to detect urban flooding

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Research Article

Certain Classes of Meromorphic p-Valent Functions Associated with Mittag-Leffler Function

M. K. Aouf [1], A. O. Mostafa[1], and S. M. Madian[2,*]

[1]Department of Mathematics, Faculty of Science, Mansoura University, Egypt
[2]Basic Sciences Department, Higher Institute for Engineering and Technology, New Damictta Egypt

Abstract: In this paper, using definition of subordination and Mittag-Leffler function we introduce classes of p-valent meromorphic functions and obtain some subordination results for these classes.

Keywords: Differential Subordination, Mittag-Leffler Function, Meromorphic p-valent Functions. 2010 Mathematics Subject Classification: 30C45.

1. INTRODUCTION

Let Σ_p denote the class of functions of the form:

$$f(z) = z^{-p} + \sum_{k=1-p}^{\infty} a_k z^k \quad (p \in \mathrm{N} = \{1, 2, ...\}), \tag{1.1}$$

Which are analytic and p-valent in the punctured unit disc $\mathrm{U}^* = \{z : z \in \mathrm{C} \text{ and } 0 < |z| < 1\} = \mathrm{U} \setminus \{0\}$. For functions $f(z) \in \Sigma_p$ given by (1.1) and $g(z) \in \Sigma_p$ given by

$$g(z) = z^{-p} + \sum_{k=1-p}^{\infty} b_k z^k \quad (p \in \mathrm{N}), \tag{1.2}$$

The Hadamard product (or convolution) of $f(z)$ and $g(z)$ is defined by

$$(f * g)(z) = z^{-p} + \sum_{k=1-p}^{\infty} a_k b_k z^k = (g * f)(z). \tag{1.3}$$

For two analytic functions f, $g \in \Sigma_p$, we say that f is subordinate to g, written $f(z) \prec g(z)$ if there exists a Schwartz function $w(z)$, which is analytic in U with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in \mathrm{U}$, such that $f(z) = g(w(z)),\ z \in \mathrm{U}$. Furthermore, if the function $g(z)$ is univalent in U, then we have the following equivalence (see [1] and [2]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(\mathrm{U}) \subset g(\mathrm{U}).$$

The Mittag-Leffler function [3] $\mathrm{E}_\alpha (\alpha \in \mathrm{C},\ \mathrm{Re}(\alpha) > 0)$, is defined by

$$\mathrm{E}_\alpha(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(n\alpha + 1)} z^n. \tag{1.4}$$

A more general function $\mathrm{E}_\alpha(z)$ is $\mathrm{E}_{\alpha,\beta}(z)$ was introduced by Wiman (see [4, 5]) and given by

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*Corresponding Author: S.M. Madian <samar_math@yahoo.com>

$$\mathsf{E}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)}\ \ (z\in\mathrm{C}). \tag{1.5}$$

Srivastava and Tomovski [6] generalized Mittag-Leffler function in the form

$$\mathsf{E}_{\alpha,\beta}^{\gamma,k}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{nk}z^n}{\Gamma(\alpha n+\beta)n!}\ (\alpha,\beta,\gamma\in\mathrm{C},\ \mathrm{Re}(\alpha)>\max\{0,\mathrm{Re}(k)-1\},\ \mathrm{Re}(k)>0). \tag{1.6}$$

Now, by using (1.6) Mostafa and Aouf [7] defined the function $\mathsf{K}_{\alpha,\beta}^{\gamma,k}(z)$ by:

$$\begin{aligned}\mathsf{K}_{\alpha,\beta,p}^{\gamma,k}(z)&=\Gamma(\beta)z^{-p}\mathsf{E}_{\alpha,\beta}^{\gamma,k}(z)\\&=z^{-p}+\sum_{n=1-p}^{\infty}\frac{\Gamma[\gamma+(n+p)k]\Gamma(\beta)z^n}{\Gamma[\alpha(n+p)+\beta]\Gamma(\gamma)(n+p)!}\ (z\in\mathrm{C},\ p\in\mathrm{N})\end{aligned} \tag{1.7}$$

and defined the operator

$$\begin{aligned}\mathsf{K}_{\alpha,\beta,p}^{\gamma,k}f(z)&=\mathsf{K}_{\alpha,\beta,p}^{\gamma,k}(z)*f(z),\\&=z^{-p}+\sum_{n=1-p}^{\infty}\frac{\Gamma[\gamma+(n+p)k]\Gamma(\beta)}{\Gamma[\alpha(n+p)+\beta]\Gamma(\gamma)(n+p)!}a_nz^n.\end{aligned} \tag{1.8}$$

For $\alpha,\beta,\gamma\in\mathrm{C},\ \mathrm{Re}(\alpha)>\max\{0,\mathrm{Re}(k)-1\},\mathrm{Re}(k)>0,\eta\ge0$ and $m\in\mathsf{N}_0=\mathsf{N}\cup\{0\}$, we define the linear operator $\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z):\ \sum_p\to\sum_p$ as follows :

$$\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,0}f(z)=\mathsf{K}_{\alpha,\beta,p}^{\gamma,k}f(z),$$

$$\begin{aligned}\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,1}f(z)&=(1-\eta)\mathsf{K}_{\alpha,\beta,p}^{\gamma,k}f(z)+\eta z^{-p}\,[z^{p+1}\mathsf{K}_{\alpha,\beta,p}^{\gamma,k}f(z)]'\\&=z^{-p}+\sum_{n=1-p}^{\infty}\frac{\Gamma[\gamma+k(n+p)]\,\Gamma(\beta)}{\Gamma(\gamma)\Gamma[\alpha(n+p)+\beta](n+p)!}[1+\eta(n+p)]a_nz^n,\end{aligned}$$

$$\begin{aligned}\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,2}f(z)&=\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,1}[\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,1}f(z)]=(1-\eta)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,1}f(z)+\eta z^{-p}\,[z^{p+1}\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,1}f(z)]'\\&=z^{-p}+\sum_{n=1-p}^{\infty}\frac{\Gamma[\gamma+k(n+p)]\,\Gamma(\beta)}{\Gamma(\gamma)\Gamma[\alpha(n+p)+\beta](n+p)!}[1+\eta(n+p)]^2a_nz^n\end{aligned}$$

and in general

$$\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z)=z^{-p}+\sum_{n=1-p}^{\infty}\frac{\Gamma[\gamma+k(n+p)]\,\Gamma(\beta)}{\Gamma(\gamma)\Gamma[\alpha(n+p)+\beta](n+p)!}[1+\eta(n+p)]^ma_nz^n. \tag{1.9}$$

For $f\in\sum_p$, *it is easy to see that* $\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z)$ *achieve the following relations*

$$(i)\quad z[\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z)]'=\frac{\gamma}{k}\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma+1,k,m}f(z)-\left(\frac{\gamma+pk}{k}\right)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z), \tag{1.10}$$

$$(ii)\quad z\alpha[\mathsf{M}_{\alpha,\beta+1,\eta,p}^{\gamma,k,m}f(z)]'=\beta\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z)-(p\alpha+\beta)\mathsf{M}_{\alpha,\beta+1,\eta,p}^{\gamma,k,m}f(z), \tag{1.11}$$

$$(iii)\quad z[\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z)]'=\frac{1}{\eta}\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m+1}f(z)-\left(p+\frac{1}{\eta}\right)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z), \tag{1.12}$$

$$(\alpha, \beta, \gamma \in \mathrm{C},\ \mathrm{Re}(\alpha) > \max\{0, \mathrm{Re}(k) - 1\}, \mathrm{Re}(k) > 0, \eta > 0, m \in \mathsf{N}_0).$$

Remark 1

- ✓ $\mathsf{M}_{0,\beta,\eta,p}^{1,1,0} f(z) = f(z) \quad (f \in \sum_p)$;
- ✓ $\mathsf{M}_{0,\beta,\eta,p}^{2,1,0} f(z) = (p+1) f(z) + z f'(z) \quad (f \in \sum_p)$;
- ✓ $\mathsf{M}_{0,\beta,\eta,1}^{2,1,0} f(z) = 2 f(z) + z f'(z) \quad (f \in \sum_1)$;
- ✓ $\mathsf{M}_{1,1,\eta,p}^{1,1,0} \left(\frac{1}{z^p (1-z)}\right) = z^{-p} e^z$;
- ✓ $\mathsf{M}_{1,1,\eta,1}^{1,1,0} \left(\frac{1}{z(1-z)}\right) = z^{-1} e^z$;
- ✓ $\mathsf{M}_{2,1,\eta,1}^{1,1,0} \left(\frac{1}{z(1-z)}\right) = z^{-1} \cosh\left(\sqrt{z}\right)$;
- ✓ $\mathsf{M}_{2,1,\eta,p}^{1,1,0} \left(\frac{1}{z^p (1-z)}\right) = z^{-p} \cosh\left(\sqrt{z}\right)$;
- ✓ $\mathsf{M}_{2,2,\eta,1}^{1,1,0} \left(\frac{1}{z(1-z)}\right) = \frac{\sinh\left(\sqrt{z}\right)}{\sqrt{z^3}}$;
- ✓ $\mathsf{M}_{2,2,\eta,p}^{1,1,0} \left(\frac{1}{z^p (1-z)}\right) = \frac{\sinh\left(\sqrt{z}\right)}{z^p \sqrt{z}}$.

We also observe that:

- ✓ $\mathsf{M}_{0,\beta,\eta,p}^{1,1,m} f(z) = D_{\eta,p}^m f(z)$ ([8], [9] and [10] with $l = 1$)
- ✓ $\mathsf{M}_{0,\beta,\eta,1}^{1,1,m} f(z) = D_{\eta}^m f(z)$ (see [11],);
- ✓ $\mathsf{M}_{0,\beta,1,1}^{1,1,m} f(z) = D_{1}^m f(z)$ (see [12]);
- ✓ $\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,0} f(z) = \mathsf{K}_{\alpha,\beta,p}^{\gamma,k} f(z)$ (see [7]).

Unless otherwise mentioned, we assume throughout the paper that:

$$-1 \le B < A \le 1,\ \alpha, \beta, \gamma \in \mathrm{C},\ \mathrm{Re}(\alpha) > \max\{0, \mathrm{Re}(k) - 1\}, \mathrm{Re}(k) > 0,\ \eta > 0,\ m \in \mathsf{N}_0 \quad \text{and} \quad \lambda > 0.$$

2. MATERIALS AND METHODS

Definition 1

For fixed parameters A and B, with $-1 \le B < A \le 1$, we say that $f \in \Sigma_p$ is in the class $\Theta_p^{\eta,k,\alpha}(\lambda, \gamma, \beta, m; A, B)$ if it satisfies

$$-\frac{z^{p+1}}{p}\left\{(1-\lambda)(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f(z))' + \lambda(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma+1,k,m} f(z))'\right\} \prec \frac{1+Az}{1+Bz}. \tag{2.1}$$

Definition 2. *For fixed parameters A and B, with $-1 \le B < A \le 1$, we say that $f \in \Sigma_p$ is in the class $\Omega_p^{\eta,k,\alpha}(\lambda, \gamma, \beta, m; A, B)$ if it satisfies*

$$-\frac{z^{p+1}}{p}\left\{(1-\lambda)(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f(z))' + \lambda(\mathsf{M}_{\alpha,\beta-1,\eta,p}^{\gamma,k,m} f(z))'\right\} \prec \frac{1+Az}{1+Bz}. \tag{2.2}$$

Definition 3

For fixed parameters A *and* B, *with* $-1 \le B < A \le 1$, *we say that* $f \in \Sigma_p$ *is in the class* $\Sigma_p^{\eta,k,\alpha}(\lambda,\gamma,\beta,m;A,B)$ *if it satisfies*

$$-\frac{z^{p+1}}{p}\left\{(1-\lambda)(\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p}f(z))' + \lambda(\mathsf{M}^{\gamma,k,m+1}_{\alpha,\beta,\eta,p}f(z))'\right\} \prec \frac{1+Az}{1+Bz}. \tag{2.3}$$

To prove our main results, we need the following lemmas.

Lemma 1 [13] *For analytic, convex (univalent)* h in U, $h(0)=1$ *and* φ *on the form*

$$\varphi(z) = 1 + c_1 z + c_2 z^2 + \dots, \tag{2.4}$$

analytic in U, if

$$\varphi(z) + \frac{z\varphi'(z)}{\ell} \prec h(z) \quad (\mathrm{Re}\{\ell\} \ge 0;\ \ell \ne 0), \tag{2.5}$$

Then

$$\varphi(z) \prec \psi(z) = \ell z^{-\ell}\int_0^z t^{\ell-1}h(t)dt \prec h(z) \tag{2.6}$$

where $\psi(z)$ is the best dominant of (2.6).

The Gaussian hypergeometric function defined by

$$\begin{aligned} {}_2F_1(a,b;c;z) &= 1 + \frac{ab}{c}.\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}.\frac{z^2}{2!} + \dots. \\ &= \sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(c)_k}\frac{z^k}{k!}, \end{aligned} \tag{2.7}$$

where a,b and c $(c \notin 0,-1,-2,\dots)$, $(d)_k = d(d+1)\dots(d+k-1)$ and $(d)_0 = 1$. We note that the series defined by (2.7) converges absolutely for $z \in \mathsf{U}$ and ${}_2F_1$ represents an analytic function in U (see [14]).

Lemma 2 [14]. For real or complex numbers a,b and c $(c \notin 0,-1,-2,\dots)$ the following identities hold:

$$\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt = \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)}\,{}_2F_1(a,b;c;z)\ (\mathrm{Re}\{b\}>0,\ \mathrm{Re}\{c\}>0,\ z \in \mathrm{C}\setminus(1,+\infty)),$$

and

$${}_2F_1(a,b;c;z) = (1-z)^{-a}\ {}_2F_1\left(a,c-b;c;\frac{z}{z-1}\right)\ (z \in \mathrm{C}\setminus(1,+\infty)), \tag{2.8}$$

where ${}_2F_1(a,b;c;z)$ is the Gaussian hypergeometric function.

Let $F(\varsigma)$ be the class of functions $\Phi(z)$ given by (2.9)

$$\Phi(z) = 1 + b_1 z + b_2 z^2 + \dots,$$

which are analytic in U, and $\mathrm{Re}\{\Phi(z)\} > \varsigma$ $(0 \le \varsigma < 1)$.

Lemma 3 [15].*If* $\Phi \in F(\varsigma)$, then (2.10)

$$\mathrm{Re}\{\Phi(z)\} \ge 2\varsigma - 1 + \frac{2(1-\varsigma)}{1+|z|}\ (0 \le \varsigma < 1).$$

Lemma 4 [16]. If $\Phi_j \in F(\varsigma_j)$ $(0 \le \varsigma_j < 1;\ j = 1,2)$, then

$$\Phi_1 * \Phi_2 \in F(\varsigma_3) \quad (\varsigma_3 = 1 - 2(1-\varsigma_1)(1-\varsigma_2)).$$

The result is the best possible.

3. RESULTS AND DISCUSSION

Theorem 1. *Let* $\operatorname{Re}\left\{\frac{\gamma}{\lambda k}\right\} \ge 0$, if $f \in \Theta_p^{\eta,k,\alpha}(\lambda,\gamma,\beta,m;A,B)$, *then*

$$-\frac{z^{p+1}}{p}\left(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f(z)\right)' \prec q_1(z) \prec \frac{1+Az}{1+Bz}, \tag{3.1}$$

such that

$$q_1(z) = \begin{cases} \frac{A}{B} + \left(1-\frac{A}{B}\right)(1+Bz)^{-1}\ {}_2F_1\left(1,1;\frac{\gamma}{\lambda k}+1;\frac{Bz}{1+Bz}\right) & \text{for} \quad B \ne 0, \\ 1 + \frac{\gamma A}{\lambda k+\gamma} z & \text{for} \quad B = 0, \end{cases} \tag{3.2}$$

is the best dominant of (3.1). and

$$\operatorname{Re}\left(-\frac{z^{p+1}\left(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f(z)\right)'}{p}\right) > \rho, \tag{3.3}$$

where

$$\rho = \begin{cases} \frac{A}{B} + \left(1-\frac{A}{B}\right)(1-B)^{-1}\ {}_2F_1\left(1,1;\frac{\gamma}{\lambda k}+1;\frac{B}{B-1}\right) & \text{for} \quad B \ne 0, \\ 1 - \frac{\gamma A}{\lambda k+\gamma} & \text{for} \quad B = 0. \end{cases} \tag{3.4}$$

The estimate (3.3) is the best possible.

Proof. Assume

$$\varphi(z) = -\frac{z^{p+1}\left(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f(z)\right)'}{p} \quad (z \in \mathsf{U}), \tag{3.5}$$

where φ is given by (2.4). Differentiating (3.5) with respect to z and using (1.10), we get

$$-\frac{z^{p+1}}{p}\left\{(1-\lambda)(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f(z))' + \lambda(\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma+1,k,m} f(z))'\right\} = \varphi(z) + \frac{\lambda k}{\gamma} z\varphi'(z) \prec \frac{1+Az}{1+Bz}.$$

From Lemma 2, we have

$$\varphi(z) \prec q_1(z) = \frac{\gamma}{\lambda k} z^{-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}\left(\frac{1+At}{1+Bt}\right) dt. \tag{3.6}$$

This proves (3.2) of Theorem 1. In order to prove (3.3), we need to show that

$$\inf_{|z|<1}\{\operatorname{Re}(q_1(z))\} = q_1(-1).$$

We have

$$\operatorname{Re}\left\{\frac{1+Az}{1+Bz}\right\} \ge \frac{1-Ar}{1-Br} \quad (|z| \le r < 1).$$

Putting

$$q_1(z)=\int_0^1 G(z,\zeta)\,d\nu(\zeta).$$

Then

$$\mathrm{Re}\{q_1(z)\}\geq\int_0^1\frac{1-A\zeta r}{1-B\zeta r}\,d\nu(\zeta)=q_1(-r)\quad(|z|\leq r<1).$$

Assuming $r\to 1^-$ in the above inequality, we obtain (3.6). The result in (3.3) is the best possible and q_1 is the best dominant of (3.1). This completes the proof of Theorem 1.

Using (1.11) and (1.12) respectively instead of (1.10) we can prove the theorems 2-3, respectively.

Theorem 2. *Let* $\mathrm{Re}(\frac{\beta-1}{\lambda\alpha})\geq 0$, if $f\in\Omega_p^{\eta,k,\alpha}(\lambda,\gamma,\beta,m;A,B)$, *then*

$$\varphi(z)\prec q_2(z)\prec\frac{1+Az}{1+Bz},\tag{3.7}$$

where φ given by (3.5),

$$q_2(z)=\begin{cases}\frac{A}{B}+\left(1-\frac{A}{B}\right)(1+Bz)^{-1}\ {}_2F_1(1,1;\frac{\beta-1}{\lambda\alpha}+1;\frac{Bz}{1+Bz}) & \text{for}\quad B\neq 0,\\ 1+\frac{\beta-1}{\lambda\alpha+\beta-1}\,Az & \text{for}\quad B=0,\end{cases}\tag{3.8}$$

is the best dominant of (3.7). And

$$\mathrm{Re}\{\varphi(z)\}>\sigma,\tag{3.9}$$

where

$$\sigma=\begin{cases}\frac{A}{B}+\left(1-\frac{A}{B}\right)(1-B)^{-1}\ {}_2F_1(1,1;\frac{\beta-1}{\lambda\alpha}+1;\frac{B}{B-1}) & \text{for}\quad B\neq 0,\\ 1-\frac{\beta-1}{\lambda\alpha+\beta-1}\,A & \text{for}\quad B=0.\end{cases}\tag{3.10}$$

The estimate (3.9) is the best possible.

Theorem 3. *Let* $\mathrm{Re}(\frac{1}{\lambda\eta})>0$, if $f\in\Sigma_p^{\eta,k,\alpha}(\lambda,\gamma,\beta,m;A,B)$, *then*

$$\varphi(z)\prec q_3(z)\prec\frac{1+Az}{1+Bz},\tag{3.11}$$

where φ given by (3.5),

$$q_3(z)=\begin{cases}\frac{A}{B}+\left(1-\frac{A}{B}\right)(1+Bz)^{-1}\ {}_2F_1(1,1;\frac{1}{\lambda\eta}+1;\frac{Bz}{1+Bz}) & \text{for}\quad B\neq 0,\\ 1+\frac{1}{\lambda\eta+1}\,Az & \text{for}\quad B=0,\end{cases}\tag{3.12}$$

is the best dominant of (3.11). And

$$\mathrm{Re}\{\varphi(z)\}>\varepsilon,\tag{3.13}$$

where

$$\varepsilon=\begin{cases}\frac{A}{B}+\left(1-\frac{A}{B}\right)(1-B)^{-1}\ {}_2F_1(1,1;\frac{1}{\lambda\eta}+1;\frac{B}{B-1}) & \text{for}\quad B\neq 0,\\ 1-\frac{1}{\lambda\eta+1}\,A & \text{for}\quad B=0.\end{cases}\tag{3.14}$$

The estimate (3.13) is the best possible.

Taking $\varphi(z)=z^p\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m}f(z)$ we can prove Theorems 4-6 below by using the same manner in the proof of Theorem 1.

Theorem 4. *Let* $-1 \le B < A \le 1$. If $f \in \sum_p$ *satisfies*

$$z^p \left\{(1-\lambda)\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z) + \lambda \mathsf{M}^{\gamma+1,k,m}_{\alpha,\beta,\eta,p} f(z)\right\} \prec \frac{1+Az}{1+Bz}, \tag{3.15}$$

then

$$z^p \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z) \prec q_1(z) \prec \frac{1+Az}{1+Bz} \tag{3.16}$$

is the best dominant of (3.15). And

$$\mathrm{Re}\left(z^p \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z)\right) > \rho, \tag{3.17}$$

where q_1 *and* ρ are given by (3.2) and (3.4), respectively.

Theorem 5. *If* $f \in \sum_p$ satisfi*es*

$$z^p \left\{(1-\lambda)\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z) + \lambda \mathsf{M}^{\gamma,k,m}_{\alpha,\beta-1,\eta,p} f(z)\right\} \prec \frac{1+Az}{1+Bz}, \tag{3.18}$$

then

$$z^p \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z) \prec q_2(z) \prec \frac{1+Az}{1+Bz} \tag{3.19}$$

is the best dominant of (3.18). And

$$\mathrm{Re}\left(z^p \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z)\right) > \sigma, \tag{3.20}$$

where q_2 *and* σ are given by (3.8) and (3.10), respectively.

(3.21)

Theorem 6. *If* $f \in \sum_p$ *satisfies*

$$z^p \left\{(1-\lambda)\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z) + \lambda \mathsf{M}^{\gamma,k,m+1}_{\alpha,\beta,\eta,p} f(z)\right\} \prec \frac{1+Az}{1+Bz}, \tag{3.22}$$

then

$$z^p \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z) \prec q_3(z) \prec \frac{1+Az}{1+Bz} \tag{3.23}$$

is the best dominant of (3.21). And

$$\mathrm{Re}\left(z^p \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f(z)\right) > \varepsilon,$$

where q_3 *and* ε are given by (3.12) and (3.14), respectively.

(3.24)

Theorem 7. *If* $Re\left\{\frac{\gamma}{\lambda k}\right\} \ge 0$ and $f_j(z) \in \sum_p$ *satisfies*

$$z^p \left\{(1-\lambda)\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f_j(z) + \lambda \mathsf{M}^{\gamma+1,k,m}_{\alpha,\beta,\eta,p} f_j(z)\right\} \prec \frac{1+A_j z}{1+B_i z} \quad (j=1,2), \tag{3.25}$$

such that

$$\Psi(z) = \mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p}(f_1 * f_2)(z) \tag{3.26}$$

and

$$\varsigma = 1 - \frac{4(A_1 - B_1)(A_2 - B_2)}{(1-B_1)(1-B_2)}\left[1 - \frac{1}{2}\ {}_2F_1(1,1;\frac{\gamma}{\lambda k}+1;\frac{1}{2})\right]. \tag{3.27}$$

The result is the best possible when $B_1 = B_2 = -1$.

Proof. Let $f_j(z) \in \sum_p$ satisfy (3.24),

$$\Phi_j(z) = z^p\left\{(1-\lambda)\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f_j(z) + \lambda \mathsf{M}^{\gamma+1,k,m}_{\alpha,\beta,\eta,p} f_j(z)\right\} \quad (j = 1,2), \tag{3.28}$$

such that $\Phi_j(z) \in F(\varsigma_j)$ $\left(\varsigma_j = \frac{1-A_j}{1-B_j}\right)$ $(j = 1,2)$. By using (1.10) in (3.28), we obtain

$$\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p} f_j(z) = \frac{\gamma}{\lambda k} z^{-p-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}\Phi_j(t)dt \quad (j = 1,2), \tag{3.29}$$

from (3.26) and (3.29), we have

$$\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p}\Psi(z) = \left(\frac{\gamma}{\lambda k} z^{-p-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}\Phi_1(t)dt\right) * \left(\frac{\gamma}{\lambda k} z^{-p-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}\Phi_2(t)dt\right) \quad (j = 1,2)$$

$$= \frac{\gamma}{\lambda k} z^{-p-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}\Phi_0(t)dt, \tag{3.30}$$

such that

$$\Phi_0(z) = z^p\left\{(1-\lambda)\mathsf{M}^{\gamma,k,m}_{\alpha,\beta,\eta,p}\Psi(z) + \lambda \mathsf{M}^{\gamma+1,k,m}_{\alpha,\beta,\eta,p}\Psi(z)\right\}$$

$$= \frac{\gamma}{\lambda k} z^{-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}(\Phi_1 * \Phi_2)(t)dt. \tag{3.31}$$

Since $\Phi_1(z) \in F(\varsigma_1)$ and $\Phi_2(z) \in F(\varsigma_2)$, it follows from Lemma 4 that:

$$(\Phi_1 * \Phi_2)(z) \in F(\varsigma_3) \ (\varsigma_3 = 1 - 2(1-\varsigma_1)(1-\varsigma_2)). \tag{3.32}$$

According to Lemma 3, we get

$$\mathrm{Re}\{(\Phi_1 * \Phi_2)(z)\} \geq 2\varsigma_3 - 1 + \frac{2(1-\varsigma_3)}{1+|z|}. \tag{3.33}$$

Now by using (3.33) in (3.31) and Lemma 2, we have;

$$\mathrm{Re}\{\Phi_0(z)\} = \frac{\gamma}{\lambda k}\int_0^1 u^{\frac{\gamma}{\lambda k}-1}(\Phi_1 * \Phi_2)(uz)du$$

$$\geq \frac{\gamma}{\lambda k}\int_0^1 u^{\frac{\gamma}{\lambda k}-1}\left(2\varsigma_3 - 1 + \frac{2(1-\varsigma_3)}{1+u|z|}\right)du$$

$$> \frac{\gamma}{\lambda k}\int_0^1 u^{\frac{\gamma}{\lambda k}-1}\left(2\varsigma_3 - 1 + \frac{2(1-\varsigma_3)}{1+u}\right)du$$

$$= 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left[1 - \frac{\gamma}{\lambda k}\int_0^1 u^{\frac{\gamma}{\lambda k}-1}(1+u)^{-1}\,du\right]$$

$$= 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left[1 - \frac{1}{2}\ {}_2F_1(1,1;\frac{\gamma}{\lambda k}+1;\frac{1}{2})\right] = \varsigma$$

which completes the proof of (3.25).

To prove the result is best possible, we assume $B_1 = B_2 = -1,\ \ f_j(z) \in \sum_p,$ which satisfy (3.24),

$$\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f_j(z) = \frac{\gamma}{\lambda k} z^{-p-\frac{\gamma}{\lambda k}} \int_0^z t^{\frac{\gamma}{\lambda k}-1}\frac{1+A_j t}{1-t}\,dt\ ,\left(\Phi_j(t) = \frac{1+A_j t}{1-t}\right)(j=1,2)$$

and $(\Phi_1 * \Phi_2)(z) = 1 + \frac{(1+A_1)(1+A_2)z}{1-z}$. Thus by using (3.31) and Lemma 2, we have

$$\Phi_0(z) = \frac{\gamma}{\lambda k}\int_0^1 u^{\frac{\gamma}{\lambda k}-1}\left(1 - (1+A_1)(1+A_2) + \frac{(1+A_1)(1+A_2)z}{1-uz}\right)du$$

$$= 1 - (1+A_1)(1+A_2) + (1+A_1)(1+A_2)(1-z)^{-1}\ {}_2F_1(1,1;\frac{\gamma}{\lambda k}+1;\frac{z}{z-1})$$

$$= 1 - (1+A_1)(1+A_2) + (1+A_1)(1+A_2)\ {}_2F_1(1,1;\frac{\gamma}{\lambda k}+1;\frac{1}{2}) \quad (\text{as } z \to -1).$$

This completes the proof of Theorem 7.

Theorem 8. *If* $\mathrm{Re}\left\{\frac{\beta-1}{\lambda\alpha}\right\} \geq 0$ *and* $f_j(z) \in \sum_p$ *satisfies*

$$z^p\left\{(1-\lambda)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f_j(z) + \lambda \mathsf{M}_{\alpha,\beta-1,\eta,p}^{\gamma,k,m} f_j(z)\right\} \prec \frac{1+A_j z}{1+B_j z} \quad (j=1,2),$$

Then,

$$z^p\left\{(1-\lambda)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} \Psi(z) + \lambda \mathsf{M}_{\alpha,\beta-1,\eta,p}^{\gamma,k,m} \Psi(z)\right\} \prec \frac{1+\left(1-\frac{2\varpi}{p}\right)z}{1-z},$$

such that $\Psi(z)$ given by (3.26) *and*

$$\varpi = 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left[1 - \frac{1}{2}\ {}_2F_1(1,1;\frac{\beta-1}{\lambda\alpha}+1;\frac{1}{2})\right].$$

The result is the best possible when $B_1 = B_2 = -1.$

Proof. The proof is similar to that of Theorem 7, so we omit it.

Theorem 9. *If* $\mathrm{Re}\left\{\frac{1}{\lambda\eta}\right\} > 0$ *and* $f_j(z) \in \sum_p$ *satisfies*

$$z^p\left\{(1-\lambda)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} f_j(z) + \lambda \mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m+1} f_j(z)\right\} \prec \frac{1+A_j z}{1+B_j z} \quad (j=1,2),$$

then

$$z^p\left\{(1-\lambda)\mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m} \Psi(z) + \lambda \mathsf{M}_{\alpha,\beta,\eta,p}^{\gamma,k,m+1} \Psi(z)\right\} \prec \frac{1+\left(1-\frac{2\omega}{p}\right)z}{1-z},$$

such that $\Psi(z)$ given by (3.26) *and*

$$\omega = 1 - \frac{4(A_1 - B_1)(A_2 - B_2)}{(1 - B_1)(1 - B_2)}\left[1 - \frac{1}{2}\ {}_2F_1(1,1;\frac{1}{\lambda\eta} + 1;\frac{1}{2})\right].$$

The result is the best possible when $B_1 = B_2 = -1$.

Proof. The proof is similar to that of Theorem 7, so we omit it.

4. CONCLUSIONS

In conclusion, we given new operator and defined three classes by using this operator. We calculated differential subordination result and introduced special cases. We can apply the new operator in different topics in the future.

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Pakistan Academy of Sciences

Obituary

Prof. Dr. Syed Irtifaq Ali (1930-2021)

Prof. Dr. Syed Irtifaq Ali was a renowned botanist, Fellow PAS, former Vice Chancellor of Karachi University (KU), and Professor Emeritus. Prof. Ali passed away on February 18, 2021, after a brief illness. He was born in Lucknow, India, in 1930, completed his matriculation from Allahabad in 1946. In 1950, he received his Bachelor of Science degree and Master's degree in botany in 1952 from Allahabad University and later migrated to Pakistan. He completed his Ph.D. in botany in 1958 and DSc in 1979 from the University of London.

Dr. Irtifaq Ali also worked as a Professor of botany at the University of Tripoli, Libya from 1972 to 1976. He was also the pioneer and co-editor of Flora of Libya. The distinguished professor joined KU in 1958 as a lecturer at its botany department and later served as its chairman. He continued to work at KU's Plant Conser¬vation Centre till his death. The professor also served as the Director General of Dr. A.Q. Khan Institute of Biotechnology and Genetic Engineering and the dean of KU's science faculty (1981 to 1990).

He was a renowned top-ranking (plant) taxonomist in the region and considered an authority on the plants of Southeast Asia in general and Pakistan in particular. One of Prof Ali's greatest contributions as a researcher was The Flora of Pakistan — the most comprehensive and authentic scientific work ever on Pakistan's flora. Initiated over four decades ago and completed in 2020, the book has 224 volumes containing details of over 6,000 indigenous and commonly cultivated plants. He has contributed to the field of plant sciences at the national and international level and provided basic information on the plant wealth of Pakistan, which is widely used by phytochemists, pharmacists, ecologists, agriculturists, and foresters.

For his contribution and distinguished career, he was conferred with Sitara-e-Imtiaz in 1988 and honored with the Emeritus status in 2011. He was also awarded the title of the Higher Education Commission Distinguished National Professor in 2004. He will be missed by the PAS Fellows.

May the departed soul rest in eternal peace. Aameen!

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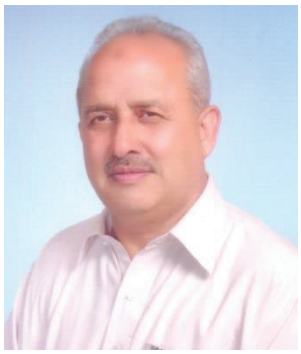
Pakistan Academy of Sciences

Obituary

Prof. Dr. Habib Ahmad
(1959-2021)



The remarkable Prof. Dr. Habib Ahmad left this mortal world on 7th April 2021 for his journey to the hereafter. He was an Eminent Scientist, Distinct Teacher, great Cyto-geneticist, eminent Plant Taxonomist, Fellow of the Pakistan Academy of Sciences, and above all a best Human Being. He was serving as Professor Emeritus of Genetics in the Department of Genetics, Hazara University Mansehra Pakistan. He was born on 13th July 1959 in Matta Swat in the home of his father Muhammad Saeed (known as Serae Mulvi Seb)– the first teacher of the Swat State and a pronounced philanthropist and educationist in the era of Swat State. He passed his SSC & FSc from Matta Swat and BSc from the historical Post Graduate Jahanzeb College Saidu Sharif Swat in 1982. He got his MSc and MPhil degrees from the Department of Botany, University of Peshawar in the years 1985 and 1991, respectively. He did his Ph.D. from the University of the Punjab Lahore in 2003. He remained on various positions including Scientific Officer, Cytogenetics Program NARC, Islamabad(1986 - 1990), Lecturer Botany Higher Education Department, Khyber Pakhtunkhwa (1990 - 2002), Technical Coordinator WWF-Pakistan (2002 - 2005), Professor of Botany, Hazara University (2005 - 2010), Tenured Professor of Genetics, Hazara University (2010 - 2019), and Professor Emeritus of Genetics, Hazara University (2019 - 2021). He also remained Vice Chancellor, Islamia College Peshawar (2016 to 2020), Vice Chancellor, Hazara University (2015 - 2016), Dean of Sciences Hazara University (2006 - 2010 & 2013 – 2016), and Chairman Botany & Genetics (2006-2010).

Based on his MPhil and Ph.D. studies and the initial job at NARC, he was able to introduce a Rapeseed Variety 'Hasnain-2013' that was approved by the Seed Council Government of KPK on March 5, 2013. This variety is being sown widely in Pakistan and is one of the popular rapeseed variety in Pakistan. He completed 28 research projects, supervised 47 Ph.D., 148 MPhil & 84 MSc/BS students, published 325 journal research articles, 51 Book chapters/monographs & more than 100 abstracts, and explored 35 New Genes/ Nucleotide Sequences in his career. He visited more than 15 different countries in the world for various academic purposes. One of his collaborative research projects with US researchers 'Ethnogenetic elaboration of NWFP through dental morphology and DNA analysis' gave rise to one of his prominent research articles in the prestigious journal of Science (IF = 41.84) titled "The first horse herders and the impact of early Bronze Age steppe expansions into Asia".

For his splendid research career, he was awarded the Presidential award Tamgha-e-Imtiaz (2010), KP R&D efforts award (2015), PAS Gold Medal (2019), HEC Best Teacher Award (2015), Productive Scientist of Pakistan Award (2011, 2012, 2013, 2014, 2015 & 2016), IBC Cambridge Leading Scientist Award 2011, UNESCO Teacher of the Year Award 2007, and Best Researcher of Hazara University Award 2006. He remained Fellow of the Pakistan Academy of Sciences, President Pakistan Botanical Society, President Society Conservation Biology of Pakistan, Chief Patron Crying Sky Ambassadors Kennesaw State University (CSA-Pakistan Chapter), Honorary Fellow Zoological Society of Pakistan, member of many national as well as international scientific societies including International Association for Plant Biotechnology (UK), Pak-US Working Group for Science and Technology (Biology/Biotech), Biotech Advisory Committee, DoST Khyber Pakhtunkhwa, National Core Group in Genetics, Government of Pakistan, National Testing Services, Government of Pakistan, Member: Board of Directors Society of

Conservation Biology (USA), Genetics Society of America (USA), International Association of Ecology (USA), Society of Conservation Biology (Asian Group), Society of Conservation Biology (North America), Genetics Society of Pakistan, Ethnobotanical Association of Pakistan, Pakistan Botanical Society, Weed Science Society of Pakistan, Environmental Protection Society Swat, Board of Advisors Ethnobotany Project, WWF.

People like Prof. Habib Ahmad him born once in centuries. It will not be an exaggeration if we claim him as one of the perfect human beings. He was candlelight for many in the field of education and research, especially, the deprived masses of society. We are sure that all his students and followers will take his noble cause ahead in society to educate people and work for the welfare of people.

We pray to the almighty ALLAH that best blessings may shower upon his gallant soul and may he is in the eternal Peace of the best of paradises. Aameen summa Aameen!

Khan Bahadar Marwat (PhD)
Fellow, Pakistan Academy of Sciences

Shujaul Mulk Khan (PhD)
Quaid-i-Azam University Islamabad
Member Pakistan Academy of Sciences

Instructions for Authors

Manuscript Format

The manuscript may contain Abstract, Keywords, INTRODUCTION, MATERIALS AND METHODS, RESULTS, DISCUSSION (or RESULTS AND DISCUSSION), CONCLUSIONS, ACKNOWLEDGEMENTS, CONFLICT OF INTEREST and REFERENCES, *and any other information that the author(s) may consider necessary*.

Abstract (font size 10; max 250 words)**:** Must be self-explanatory**,** stating the rationale, objective(s), methodology, main results, and conclusions of the study. Abbreviations, if used, must be defined on the first mention in the Abstract as well as in the main text. Abstract of review articles may have a variable format.

Keywords (font size 10)**:** Three to eight keywords**,** depicting the article.

INTRODUCTION: Provide a clear and concise statement of the problem, citing relevant recent literature, and objectives of the investigation.

MATERIALS AND METHODS: Provide an adequate account of the procedures or experimental details, including statistical tests (if any), concisely but sufficient enough to replicate the study.

RESULTS: Be clear and concise with the help of appropriate Tables, Figures, and other illustrations. Data should not be repeated in Tables and Figures, but must be supported with statistics.

DISCUSSION: Provide interpretation of the RESULTS in the light of previous relevant studies, citing published references.

ACKNOWLEDGEMENTS: (font size 10)**:** In a brief statement, acknowledge the financial support and other assistance.

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2. W. Bialek, and S. Setayeshgar. Cooperative sensitivity and noise in biochemical signaling. *Physical Review Letters* 100: 258–263 (2008).
3. R.K. Robert, and C.R.L.Thompson. Forming patterns in development without morphogen gradients: differentiation and sorting. *Cold Spring Harbor Perspectives in Biology* 1(6) (2009).
4. D. Fravel. Commercialization and implementation of biocontrol. *Annual Reviews of Phytopathology* 43: 337359 (2005).

b. Books

5. W.R. Luellen. Fine-Tuning Your Writing. *Wise Owl Publishing Company, Madison, WI, USA* (2001).
6. U. Alon, and D.N. Wegner (Ed.). An Introduction to Systems Biology: Design Principles of Biological Circuits. *Chapman & Hall/CRC, Boca Raton, FL, USA* (2006).

c. Book Chapters

7. M.S. Sarnthein, and J.D. Stanford. Basal sauropodomorpha: historical and recent phylogenetic developments. In: The Northern North Atlantic: A Changing Environment. P.R. Schafer, & W. Schluter (Ed.), *Springer, Berlin, Germany*, pp. 365–410 (2000).
8. J.E. Smolen, and L.A. Boxer. Functions of Europhiles. In: Hematology, 4th ed. W.J. Williams., E. Butler and M.A. Litchman (Ed.), *McGraw Hill, New York, USA*, pp. 103–101 (1991).

d. Reports

9. M.D. Sobsey, and F.K. Pfaender. Evaluation of the H2S method for Detection of Fecal Contamination of Drinking Water, Report WHO/SDE/WSH/02.08, *Water Sanitation and Health Programme, WHO, Geneva, Switzerland* (2002).

e. Online references

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10. L. Branston. SENSPOL: Sensors for Monitoring Water Pollution from Contaminated Land, Landfills and Sediment (2000). http://www.cranfield.ac.uk/biotech/senspol/ (accessed 22 July 2005)

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