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Research Article

An Efficient Class of Repeated Measurements Designs to Control the Residual Effects Using Periods of Three Different Sizes

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Abstract: Repeated measurements designs (RMDs) are always economical but with the use of these designs, there may arise residual effects. Minimal strongly balanced RMDs are well known to estimate the treatment effects and residual effects independently. In the situation, where these designs cannot be constructed, minimal nearly strongly balanced RMDs are used which is an efficient class of RMDs to control the residual effects. In this article, efficient minimal circular nearly strongly balanced RMDs are constructed in periods of three different sizes.

Key Words: Repeated Measurements Design, Carry Over Effects, Residual Effects, Strongly Balanced RMDs, Minimal Designs.

1. INTRODUCTION

Repeated measurements designs (RMDs) are popular in experiments of psychology, pharmacology, medicine, and animal sciences, but residual effects may arise in RMDs. Effect which a treatment has during its period of application (treatment effect) may persist into the preceding period. Such effect is called residual effect or carry over effect. Balanced or strongly balanced RMDs control these effects efficiently. RMD is minimal balanced (MBRMD) with respect to the residual effects if each treatment is immediately preceded once by all other treatments (excluding itself). RMD is minimal strongly balanced (MSBRMD) if each treatment is immediately preceded once by all other treatments (including itself). Using method of cyclic shifts, minimal circular strongly balanced RMDs (MCSBRMDs) can only be constructed through its Rule I which provides constructions of these designs for some specific cases of number of treatments (v) and period size (p). For most of the remaining cases, Rule II provides construction of minimal circular nearly strongly balanced RMDs (MCNSBRMDs). If each treatment is immediately preceded once with all other treatments (including itself) except v-1 which is not preceded with itself then it is MCNSBRMD. If experimental subjects are human or animals, they may die or recover during the experiments. In such cases, proposed designs should be used in unequal period sizes.

Williams [1-2] constructed MBRMDs for v even with $p = v$, where p is the period size and v is the number of treatments. Pearce [3] used RMDs in experiments in biology. The application of RMDs with unequal period sizes in experiments of industry and agriculture was described by the Kageyman [4]. Magda [5] gave the idea of a circular balanced RMD (CBRMD). Constantine and Hedayat [6], and Afsarinejad [7-8] constructed MBRMDs for $p < v$. Afsarinejad [9] also introduced a simple method to obtain MBRMDs and MSBRMDs in periods of unequal sizes. Using cyclic shifts, Iqbal and Jones [10], and Sharma *et al.* [11] constructed efficient RMDs in equal and two distinct periods. Iqbal and Tahir [12,14], and Sharma *et al.* [13] presented some CSBRMDs through cyclic shifts. Bailey *et al.* [15]

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constructed universally optimal weakly balanced RMDs for $p = v$. Bashir *et al.* [16], and Rajab *et al.* [17] generated some CBRMDs in equal period sizes. Rasheed *et al.* [18] generated MCSBRMDs for even p_1, $3 \leq p_2 \leq 10$ and $2 \leq p_2 \leq 9$. Daniyal *et al.* [19], Ahmed *et al.* [20], and Rasheed *et al.* [21] constructed some MCSBRMDs. Khan *et al.* [22] generated some minimal circular weakly BRMDs in equal period sizes. Some infinite series of minimal circular strongly partially BRMDs are developed by Jabeen *et al.* [23]. Using Rule I, recently, Rasheed *et al.* [24] constructed MCNSBRMDs in periods of three different sizes with smallest of size two. MCNSBRMDs are efficient designs but these are not available for $p_3 = 3$. Construction of MCNSBRMDs in three distinct period sizes with smallest of size three will be an innovational work. In this article, therefore, MCNSBRMDs are constructed in periods of three different sizes for $v \leq 100$, $4 \leq p_1$ (odd) ≤ 9, $4 \leq p_2 \leq 8$ and $p_3 = 3$.

Our proposed designs are highly efficient (i) to estimate the residual effects and treatment effects independently, (ii) to control the residual effects. Proposed designs have their application in the field of medicine, agriculture and animal sciences.

2. MATERIALS AND METHODS

Procedures are explained to find the efficiency for residual effects and efficiency of separability in the following sections.

2.1 Efficiency for Residual Effects

The canonical efficiency factors are the harmonic mean of non-zero Eigen values of their respective C^* (information matrix) are expressed by James and Wilkinson [25], and Pearce *et al.* [26]. Design possessing high value of E_r will be efficient to estimate the residual effects.

2.2 Efficiency of Separability (ES)

The following relation is given by Divecha and Gondaliya [27] for Es.

$$Es = \left[1 - \left\{ \frac{(l_1 + 4l_2)v - (l_1 + 2l_2)^2}{(v-1)(l_1 + 2l_2)^2} \right\}^{\frac{1}{2}} \right] \times 100\%,$$

where

l_1: No. of treatments preceded once by other treatment.

l_2: No. of treatments preceded twice by other treatment.

3. CONSTRUCTION OF EFFICIENT MCNSBRMDS IN THREE DISTINCT PERIOD SIZES

In this Section, using the method of cyclic shifts (Rule II) introduced by Iqbal [28], MCNSBRMDs are constructed from the following sets of shifts in three distinct period sizes for $v = ri+s+2$, here $p_1 = r$ (odd), $p_2 = s$, and $p_3 = 3$. In these designs, only one pair (v-1, v-1) do not appear while all others appear once which means v-1 does not appear as its own preceded treatment.

$S_u = [q_{u1}, q_{u2}, \ldots, q_{u(r-1)}]$; $\quad u = 1, 2, \ldots, i$.
$S_{u+1} = [q_{(u+1)1}, q_{(u+1)2}, \ldots, q_{(u+1)(s-1)}]$,
$S_{u+2} = [q_{(u+2)1}]t$.

Here,

- Each element of sets lies between 0 and v-2.
- In S*, all of 0, 1, 2, …, v-2 appear once.

Here, S* contains (i) each element of all sets, (ii) complement of the sum of elements in each set except the last set with single element, where complement of 'a' is v-1-a.

Example 3.1: $S_1 = [1, 2, 3, 4]$, $S_2 = [6, 7, 8]$ and $S_3 = [5]t$ provides following MCNSBRMD for $v = 11$ in $p_1 = 5$, $p_2 = 4$ and $p_3 = 3$.

Here, S* = [1, 2, 3, 4, 0, 6, 7, 8, 9, 5] contains each of 0, 1,…,9 exactly once. Hence given sets of shifts provide MCNSBRMD. The complete design is obtained through Rule II from the given sets of shifts in the following manners, see Table 1, 2, and 3.

Table 1: Blocks generated from $S_1 = [1, 2, 3, 4]$

Periods	Subjects									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1	2	3	4	5	6	7	8	9
2	1	2	3	4	5	6	7	8	9	0
3	3	4	5	6	7	8	9	0	1	2
4	6	7	8	9	0	1	2	3	4	5
5	0	1	2	3	4	5	6	7	8	9

Table 2: Blocks generated from $S_2 = [6, 7, 8]$

Periods	Subjects									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0	1	2	3	4	5	6	7	8	9
2	6	7	8	9	0	1	2	3	4	5
3	3	4	5	6	7	8	9	0	1	2
4	1	2	3	4	5	6	7	8	9	0

Table 3: Blocks generated from $S_3 = [5]t$

Periods	Subjects									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	0	1	2	3	4	5	6	7	8	9
2	5	6	7	8	9	0	1	2	3	4
3	10	10	10	10	10	10	10	10	10	10

Tables 1, 2, & 3 jointly present the MCNSBND for $v = 11$ in $p_1 = 5$, $p_2 = 4$ and $p_3 = 3$.

Catalogues of MCNSBRMDs are presented in Appendix for $v \leq 100$, $5 \leq p_1 \leq 9$, $4 \leq p_2 \leq 8$ and $p_3 = 3$.

4. RESULTS AND DISCUSSION

To estimate the treatment and residual effects independently, MCSBRMDs are 100 % effective but these are not available for all combinations of v and p. Our proposed MCNSBRMDs have at least (i) 97 percent efficiency of Separability, making them the best alternatives to MCSBRMDs for independently estimating residual effects and treatment effects, and (ii) 81 percent efficiency of residual effects, making them effective to control residual effects. As a future work, a catalogue of efficient MCNSBRMDs will be constructed in three distinct sizes with $p_3 > 3$.

5. CONCLUSION

In the present study, minimal circular nearly strongly balanced RMDs (MCNSBRMDs) have been constructed to control the residual effects. These designs are the best alternatives of minimal circular strongly balanced RMDs (MCSBRMDs) which are useful in the field of medical, agriculture and animal sciences.

6. ACKNOWLEDGEMENT

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7. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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APPENDİX

MCNSBRMDs for $v \leq 100$, $5 \leq p_1$ (odd) ≤ 9, $4 \leq p_2 \leq 8$ and $p_3 = 3$, where $p_1 > p_2$.

v	p_1	p_2	p_3	Sets of Shifts	*E*s	*E*r
11	5	4	3	[1,2,3,4]+[6,7,8]+[5]t	0.97	0.87
21	5	4	3	[12,2,3,4]+[5,7,9,8]+[15,1,13,14]+[16,6,18]+[10]t	0.98	0.83
31	5	4	3	[1,2,3,4]+[13,7,8,18]+[11,12,6,5]+[16,17,9,19]+[21,22,23,24]+ [10,27,28]+[15]t	0.99	0.82
41	5	4	3	[1,2,3,12]+[19,6,7,8]+[26,4,9,5]+[15,16,17,18]+[21,13,23,24]+ [11,27,28,29]+[30,31,32,33]+[35,37,38]+[20]t	0.99	0.82
51	5	4	3	[1,2,3,4]+[5,6,7,8]+[10,11,48,22]+[16,17,18,19]+ [13,21,23,14]+[26,27,28,20]+[36,32,33,34]+[42,37,38,39]+ [41,35,43,31]+[45,46,47]+[25]t	0.99	0.81
61	5	4	3	[1,2,3,5]+[4,7,8,10]+[11,12,13,15]+[14,17,18,20]+ [19,32,23,24]+[26,58,28,29]+[6,48,33,34]+[45,37,25,38]+ [50,47,43,44]+[40,42,21,41]+[16,57,53,54]+[46,52,27]+[30]t	0.99	0.81
71	5	4	3	[11,18,3,48]+[6,7,8,9]+[20,38,13,14]+[16,17,2,1]+ [21,22,23,24]+[25,27,28,29]+[26,61,33,15]+[36,37,49,19]+ [30,42,43,44]+[45,46,47,68]+[41,52,53,54]+[39,56,57,58]+ [32,62,63,64]+[65,66,67]+[35]t	0.99	0.81
81	5	4	3	[1,2,3,4]+[5,7,8,9]+[56,17,13,14]+[15,16,18,19]+ [34,39,23,35]+[26,27,33,49]+[31,38,47,48]+[24,36,37,41]+ [32,42,43,44]+[46,28,21,20]+[65,52,53,64]+[11,57,58,59]+ [61,62,63,54]+[66,67,68,69]+[71,72,73,74]+[75,77,78]+[40]t	0.99	0.81
91	5	4	3	[1,2,3,4]+[28,67,8,17]+[11,12,13,14]+[20,24,18,19]+ [21,22,23,26]+[16,27,6,76]+[31,32,33,34]+[36,37,38,75]+ [41,42,43,44]+[89,46,86,49]+[69,5,53,58]+[66,65,68,64]+ [61,62,63,59]+[57,56,52,54]+[71,72,73,74]+[39,29,77,78]+ [79,81,82,83]+[87,48,30]+[45]t	0.99	0.81
13	7	4	3	[1,3,2,4,5,9]+[7,8,11]+[6]t	0.97	0.90
27	7	4	3	[1,2,3,4,5,11]+[9,8,10,14,25,20]+[6,16,15,17,19,24]+ [21,22,23]+[13]t	0.99	0.88
41	7	4	3	[1,2,3,4,5,25]+[7,8,9,10,11,12]+[15,16,17,18,19,39]+ [34,22,13,6,26,27]+[28,29,30,31,24,37]+[14,33,38]+[20]t	0.99	0.87
55	7	4	3	[1,2,3,4,5,39]+[8,9,48,11,12,13]+[14,15,17,18,37,19]+ [21,23,24,25,26,53]+[47,43,31,32,33,34]+[35,36,20,38,40,41]+ [16,30,22,45,46,29]+[10,51,52]+[27]t	0.99	0.87
69	7	4	3	[1,2,3,4,5,6]+[7,8,9,10,11,25]+[14,15,16,17,18,19]+ [22,23,13,24,26,29]+[28,20,31,32,21,33]+[30,36,38,27,40,41]+ [42,43,44,45,46,52]+[49,50,51,48,53,54]+[57,58,59,55,61,62]+ [63,64,65]+[34]t	0.99	0.86
83	7	4	3	[1,2,3,4,5,6]+[7,9,10,11,12,13]+[14,15,16,17,18,19]+ [60,23,24,25,26,54]+[29,42,31,32,33,81]+[35,36,37,51,39,27]+ [43,44,45,46,47,48]+[28,38,52,53,30,77]+[56,57,58,59,22,76]+ [63,64,8,66,67,68]+[70,71,72,73,69,75]+[40,78,79]+[41]t	0.99	0.86
97	7	4	3	[1,2,3,4,5,6]+[7,8,9,11,12,53]+[15,16,17,18,19,52]+ [21,22,23,24,25,50]+[29,30,60,32,49,34]+[35,37,38,39,14,41]+ [43,44,45,46,47,95]+[33,26,36,20,69,76]+[56,57,58,59,61,62]+ [63,42,65,66,67,68]+[70,71,72,73,74,80]+[77,78,79,81,82,83]+ [85,86,87,88,89,90]+[91,93,94]+[48]t	0.99	0.86
15	9	4	3	[3,2,1,4,6,5,8,13]+[10,11,12]+[7]t	0.98	0.91

v	p_1	p_2	p_3	Sets of Shifts	*E*s	*E*r
33	9	4	3	[1,2,3,4,5,6,7,8]+[10,11,12,13,14,21,30,17]+ [19,20,15,22,23,24,25,26]+[27,29,31]+[16]t	0.99	0.90
51	9	4	3	[1,2,3,4,5,6,7,22]+[10,11,48,21,14,28,16,17]+ [18,19,20,13,8,23,24,49]+[27,15,29,30,31,32,33,44]+ [36,38,37,40,39,41,42,43]+[45,46,47]+[25]t	0.99	0.90
69	9	4	3	[1,2,3,4,5,6,7,40]+[9,10,11,12,13,14,15,16]+ [18,19,20,21,22,23,24,25]+[28,29,30,31,26,33,67,35]+ [48,37,38,39,41,42,43,44]+[46,47,49,66,62,51,52,58]+ [54,55,56,57,53,64,60,50]+[63,59,65]+[34]t	0.99	0.87
87	9	4	3	[1,2,3,4,5,6,7,8]+[9,10,11,12,13,14,15,16]+ [18,19,21,22,23,24,25,26]+[44,28,29,31,32,33,34,35]+ [81,37,38,39,40,41,42,85]+[45,46,47,48,49,83,52,60]+ [54,56,55,57,58,59,53,61]+[62,64,65,66,67,68,69,70]+ [36,73,82,75,76,77,79,84]+[17,74,50]+[43]t	0.97	0.89
21	7	5	3	[1,2,3,4,5,6]+[7,8,9,11,12,13]+[15,16,17,18]+[10]t	0.98	0.66
35	7	5	3	[1,2,3,4,5,6]+[7,8,9,21,11,12]+[15,30,33,26,19,20]+ [22,23,24,25,18,14]+[29,16,31,32]+[17]t	0.99	0.87
49	7	5	3	[1,2,3,4,5,6]+[8,9,10,11,12,13]+[15,16,18,17,19,20]+ [22,23,47,25,26,28]+ [44,45,31,32,7,35]+ [36,37,38,14,40,41]+[42,43,29,30]+[24]t	0.99	0.87
63	7	5	3	[1,2,3,4,5,6]+[8,9,10,11,12,13]+[15,16,18,17,19,25]+ [22,23,24,20,26,27]+ [29,30,55,32,34,40]+ [36,37,38,39,56,7]+[42,43,21,45,46,51]+ [58,47,52,5 3,54,48]+[57,50,59,49]+[31]t	0.99	0.87
77	7	5	3	[1,2,3,4,5,6]+[14,9,10,11,12,13]+[15,28,30,40,19,74]+ [41,23,24,25,26,27] +[29,37,47,32,33,34]+ [36,17,75,39,18,8]+[43,44,45,46,31,53]+ [49,50,51 ,52,48,54]+[57,58,59,60,61,21]+ [56,65,66,67,70,69]+[71,72,73,20]+[38]t	0.99	0.87
91	7	5	3	[1,2,3,4,5,6]+[8,70,10,11,12,13]+[27,36,17,18,19,20]+ [22,23,28,25,26,35] +[29,63,31,32,33,58]+ [16,37,38,39,84,41]+[52,44,89,46,47,40]+ [50,51,49,53,54,55]+[77,59,34, 60,61,62]+ [30,64,65,66,67,68]+[71,72,73,74,75,76]+ [78,79,80,81,82,83]+[85,86,87,88]+[45]t	0.99	0.84
25	9	5	3	[1,3,2,16,5,6,7,8]+[10,11,23,13,18,15,4,17]+ [19,20,21,22]+[12]t	0.99	0.91
43	9	5	3	[1,2,3,4,5,6,7,14]+[9,11,12,13,8,15,17,16]+ [18,19,20,41,22,23,24,33]+[28,29,39,31,32,26,34,35]+ [36,37,38,30]+[21]t	0.99	0.90
61	9	5	3	[1,2,3,4,5,6,7,8]+[10,11,12,13,15,14,16,20]+ [18,17,19,21,22,25,23,35]+[54,58,29,59,31,32,33,38]+ [37,39,34,40,41,42,43,48]+[46,47,44,49,50,51,52,53]+ [27,55,56,57]+[30]t	0.99	0.90
79	9	5	3	[1,2,3,4,5,6,7,8]+[10,11,12,13,14,15,16,17]+ [19,56,21,22,23,24,25,26]+[27,29,28,30,31,32,34,35]+ [36,37,38,77,74,41,43,44]+[62,9,47,49,73,51,52,54]+ [53,55,20,57,58,59,61,60]+[72,50,65,33,67,68,69,76]+ [63,64,40,75]+[39]t	0.99	0.89
97	9	5	3	[1,2,15,88,5,6,7,8]+[9,10,11,12,13,14,3,28]+ [34,78,21,22,23,24,25,26]+[16,29,71,31,44,33,18,19]+ [36,37,38,39,64,53,42,43]+[46,47,95,49,87,63,52,41]+ [51,55,56,57,58,90,61,74]+[80,40,65,66,67,68,69,76]+ [20,4,86,75,54,77,30,79]+[82,83,84,85,62,50,73,72]+ [91,17,93,94]+[48]t	0.97	0.89

v	p_1	p_2	p_3	Sets of Shifts	Es	Er
15	7	6	3	[2,1,3,4,5,13]+[9,8,10,11,12]+[7]t	0.98	0.91
29	7	6	3	[1,2,3,4,5,6]+[8,9,10,11,12,13]+[27,15,16,17,18,19]+ [20,22,23,24,25]+[14]t	0.99	0.88
43	7	6	3	[1,2,24,4,5,6]+[7,8,10,11,12,13]+[14,15,16,17,18,19]+ [29,9,3,25,26,35]+[30,22,31,32,33,34]+[36,37,38,39,40]+[21]t	0.99	0.87
57	7	6	3	[1,2,3,4,5,6]+[8,9,10,11,12,13]+[14,15,16,17,18,25]+ [22,23,24,19,26,33]+[48,37,47,32,27,34]+[36,30,38,39,40,41]+ [42,43,44,45,46,31]+[50,51,52,53,54]+[28]t	0.99	0.87
71	7	6	3	[1,2,3,4,5,6]+[8,9,10,11,12,13]+[16,14,17,18,19,20]+ [52,62,24,25,26,27]+[22,30,31,32,33,34]+[15,37,38,39,40,42]+ [41,61,44,46,47,56]+[50,51,29,53,54,43]+[48,57,58,59,60,45]+ [21,65,66,67,68]+[35]t	0.99	0.87
85	7	6	3	[1,2,3,4,5,6]+[21,8,9,11,12,46]+[14,16,17,18,20,19]+ [22,80,24,25,26,27]+[29,65,31,32,33,34]+[62,37,38,43,39,41]+ [50,44,45,13,47,54]+[7,49,40,51,52,53]+[57,58,59,60,10,36]+ [55,15,30,66,67,68]+[70,71,72,73,74,75]+[78,79,23,81,82]+ [42]t	0.99	0.87
99	7	6	3	[1,2,3,4,5,6]+[8,9,10,11,12,13]+[14,15,16,17,18,19]+ [23,29,24,25,26,27]+[62,30,31,89,33,57]+[36,37,38,39,40,41]+ [43,44,45,46,47,48]+[56,51,52,53,54,55]+[34,58,59,60,61,22]+ [64,65,66,67,68,69]+[72,70,73,74,75,76]+[0,78,79,81,82,83]+ [85,86,87,88,32,28]+[92,93,94,95,96]+[49]t	0.97	0.84
17	9	6	3	[1,2,3,4,5,6,7,12]+[10,11,7,13,14]+[8]t	0.88	.99
35	9	6	3	[1,2,3,4,5,6,7,8]+[10,11,12,13,14,15,33,19]+ [18,20,16,30,42,22,23,26]+[27,28,29,21,31]+[17]t	0.94	0.90
53	9	6	3	[1,2,3,4,5,6,7,8]+[9,10,11,12,13,14,15,20]+ [34,17,21,22,23,32,25,51]+[27,50,30,31,24,33,18,19]+ [37,38,39,40,41,42,43,48]+[45,46,47,44,49]+[26]t	0.99	0.90
71	9	6	3	[1,2,3,4,5,6,7,8]+[9,10,12,11,13,14,15,16]+ [19,20,21,22,23,24,25,38]+[53,28,29,62,31,32,46,69]+ [37,26,39,17,66,42,43,44]+[45,33,47,48,49,68,51,52]+ [54,55,56,57,59,58,60,61]+[63,64,65,41,67]+[35]t	0.99	0.90
89	9	6	3	[1,2,3,4,5,6,7,8]+[9,11,12,13,14,15,16,17]+ [18,19,21,22,23,24,25,26]+[28,29,30,31,32,33,34,48]+ [36,37,38,39,40,41,27,43]+[80,46,47,35,49,78,42,53]+ [55,56,57,58,59,60,61,68]+[63,64,65,66,67,62,70,71]+ [20,85,82,75,76,77,50,79]+[81,74,83,84,73]+[44]t	0.99	0.90
27	9	7	3	[1,2,3,5,4,6,7,8]+[24,10,11,12,25,14,15,19]+ [18,17,20,21,22,23]+[13]t	0.99	0.92
45	9	7	3	[1,3,43,4,5,6,7,8]+[2,9,12,13,14,15,16,17]+ [19,20,18,21,24,23,25,26]+[28,29,27,39,31,33,32,35]+ [37,38,30,40,41,42]+[22]t	0.99	0.90
63	9	7	3	[1,2,3,4,5,6,7,8]+[9,10,11,12,13,61,15,38]+ [18,19,20,21,22,23,24,25]+[27,28,29,30,32,33,34,35]+ [36,57,60,55,56,41,42,43]+[45,46,48,49,50,51,52,53]+ [39,54,47,37,58,59]+[31]t	0.99	0.81
81	9	7	3	[1,2,3,4,5,6,7,8]+[50,11,12,53,14,15,16,17]+ [19,67,21,22,23,24,25,26]+[28,29,30,72,32,33,34,35]+ [37,38,39,79,41,42,43,45]+[46,47,48,49,10,51,9,60]+ [55,56,57,58,59,18,61,62]+[64,65,66,20,68,69,74,71]+ [73,70,75,76,77,78]+[40]t	0.99	0.90

v	p_1	p_2	p_3	**Sets of Shifts**	***E*s**	***E*r**
99	9	7	3	[1,2,3,4,5,6,7,8]+[10,11,12,13,14,15,18,17]+ [19,20,21,23,22,24,25,26]+[28,39,30,71,32,33,34,35]+ [36,37,38,61,40,41,42,53]+[46,47,48,97,50,51,52,54]+ [43,57,55,58,59,60,29,31]+[63,64,65,66,67,69,70,56]+ [72,73,74,76,75,77,78,79]+[81,80,83,84,85,87,89,88]+ [91,92,93,94,95,96]+[49]t	0.99	0.90
19	9	8	3	[1,3,2,4,5,6,8,7]+[11,10,12,13,14,16,15]+[9]t	0.98	0.93
37	9	8	3	[9,2,3,4,5,6,8,7]+[11,10,12,13,14,15,16,17]+ [19,20,21,22,23,24,27,25]+[26,29,30,31,32,33,34]+[18]t	0.99	0.91
55	9	8	3	[1,2,3,4,5,6,7,8]+[10,11,12,13,14,16,15,17]+ [53,20,21,34,23,24,25,26]+[28,29,41,31,32,33,35,22]+ [46,37,38,39,40,30,42,43]+[36,47,48,49,50,51,52]+[27]t	0.99	0.90
73	9	8	3	[1,2,3,4,5,6,7,11]+[23,8,12,13,14,15,16,17]+ [18,19,20,21,22,10,25,9]+[28,29,30,31,32,42,34,35]+ [46,38,39,68,59,24,43,44]+[47,37,48,49,50,51,52,53]+ [55,56,57,58,41,60,61,62]+[63,64,65,66,67,69,70]+[36]t	0.99	0.87
91	9	8	3	[1,2,3,4,5,6,54,16]+[9,11,12,21,14,15,8,17]+ [63,48,13,22,23,24,25,34]+[28,29,58,31,32,33,7,81]+ [37,38,39,40,41,42,43,44]+[60,47,62,49,50,51,52,53]+ [55,57,56,20,59,46,27,30]+[64,65,66,67,68,69,70,71]+ [72,74,75,76,77,78,79,80]+[82,83,84,85,86,87,88]+[45]t	0.99	0.83